A new subgrid characteristic length for LES

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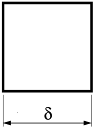

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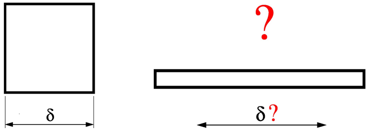

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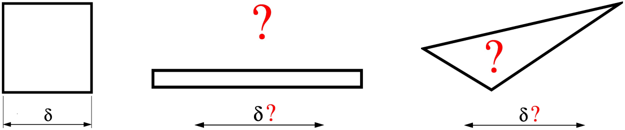
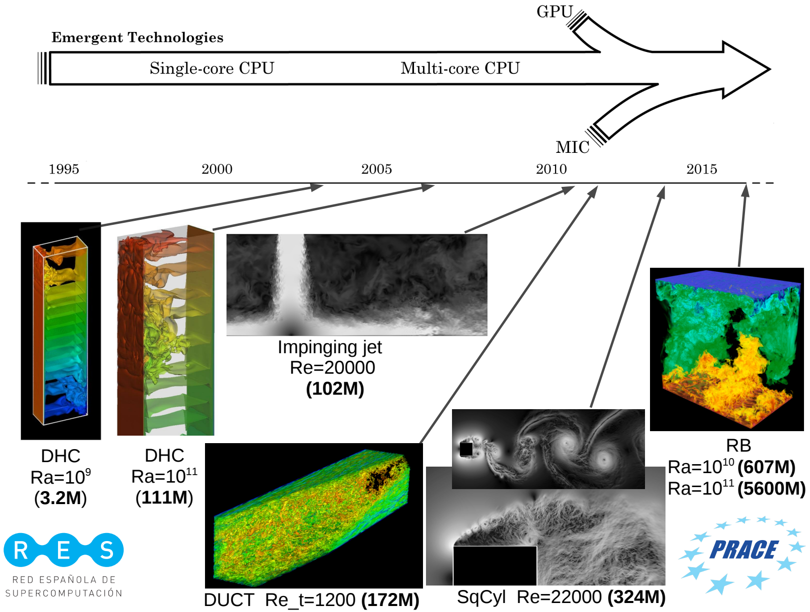


Contents


GPU
Emergent Technologies
Single-core CPU
Multi-core CPU
MIC
1995
2000
2005
2010
2015
Impinging jet
Re=20000
(102M)
RB
Ra=10^10 (607M)
Ra=10^11 (5600M)
DHC
Ra=10^9
(3.2M)
DHC
Ra=10^11
(111M)
RES
RED ESPAÑOLA DE SUPERCOMPUTACIÓN
DUCT Re_t=1200 (172M)
SqCyl Re=22000 (324M)
PRACE

Subgrid characteristic length for LES: state of the art

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nabla^2 \overline{u} - \nabla \overline{p} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_e S(\overline{u})$$

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$$\delta ?$$

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Subgrid characteristic length for LES: state of the art

- In the context of **LES**, most popular (by far) is:

$$\boxed{\delta_{\mathrm{vol}} = (\Delta x \Delta y \Delta z)^{1/3}} \Longleftarrow \textbf{Deardorff (1970)}$$

$$\delta_{\mathrm{Sco}} = f(a_1, a_2)\delta_{\mathrm{vol}}, \qquad \delta_{L^2} = \sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)/3}$$

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- In the context of **DES**:

$$\boxed{\delta_{\mathrm{max}} = \max(\Delta x, \Delta y, \Delta z)} \Longleftarrow \textbf{Sparlart et al. (1997)}$$

Recent flow-dependant definitions

$$\delta_\omega = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\omega|^2} \Longleftarrow \textbf{Chauvet et al. (2007)}$$

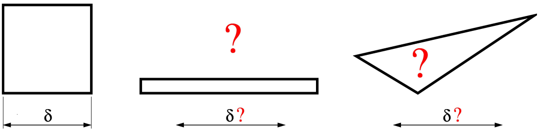
$$\tilde{\delta}_\omega = \frac{1}{\sqrt{3}} \max_{n,m=1,\dots,8} |l_n - l_m| \qquad \Longleftarrow \textbf{Mockett et al. (2015)}$$

$$\delta_{\mathrm{SLA}} = \tilde{\delta}_\omega F_{\mathrm{KH}}(VTM) \qquad \Longleftarrow \textbf{Shur et al. (2015)}$$

Building a new subgrid characteristic length for LES

Research question:

- Can we find a **simple and robust** definition of δ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?



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Starting point:

$$\underbrace{G \equiv \nabla \overline{u}}_{\text{physical space}} \qquad \underbrace{G_\delta \equiv G\Delta}_{\text{computational space}}$$

where for a Cartesian grid $\Delta \equiv \begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}$

Building a new subgrid characteristic length for LES

Idea: δ, appears in a natural way when we consider the leading term of the Taylor series expansion of the subgrid stress tensor,

$$\underbrace{\tau(\overline{u}) = \frac{\delta^2}{12} GG^T + \mathcal{O}(\delta^4)}_{\text{physical space}} \qquad \underbrace{\tau(\overline{u}) = \frac{1}{12} G_\delta G_\delta^T + \mathcal{O}(\delta^4)}_{\text{computational space}}$$

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A **least-square minimization** leads to[2]

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}}$$

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- Sensitive to flow orientation, *e.g.* for rotating flows ($G = \Omega$)

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{\omega_x^2(\Delta y^2 + \Delta z^2) + \omega_y^2(\Delta x^2 + \Delta z^2) + \omega_z^2(\Delta x^2 + \Delta y^2)}{2|\omega|^2}}$$

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- Applicable to unstructured grid

Idea: $\dfrac{\partial \phi}{\partial x} \approx \dfrac{\phi_{i+1} - \phi_i}{\Delta x} \implies$ you can compute G; then, you can compute δ_{lsq}!

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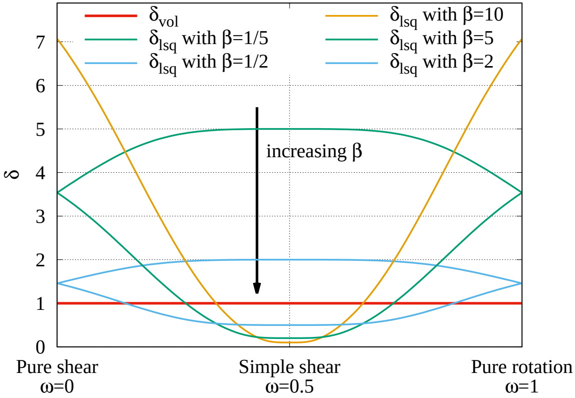
- Easy and cheap

Building a new subgrid characteristic length for LES

Properties of new definition

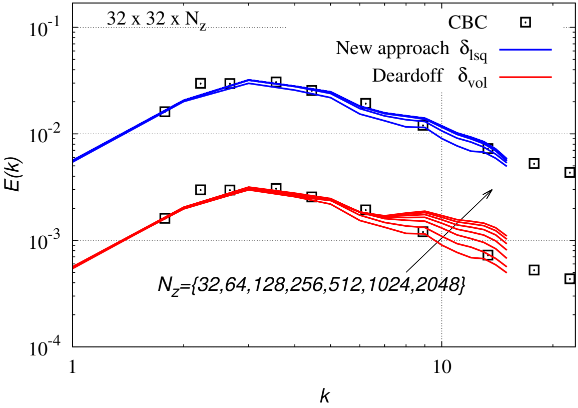
$$\Delta = \begin{pmatrix} \beta & 0 \\ 0 & \beta^{-1} \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 1 \\ 1-2\omega & 0 \end{pmatrix}$$



Results for LES

Decaying isotropic turbulence

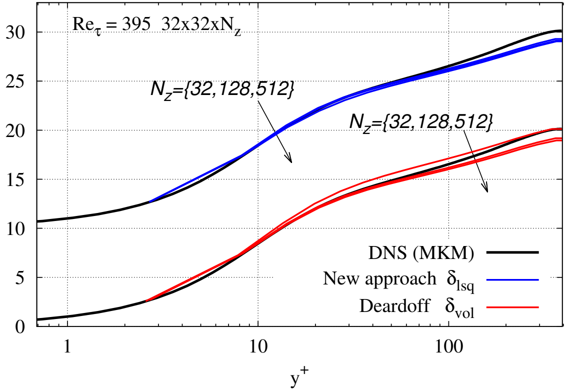
Comparison with classical Comte-Bellot & Corrsin (CBC) experiment



Results for LES

Turbulent channel flow

$Re_\tau = 395$ DNS Moser et al. LES $32 \times 32 \times N_z$



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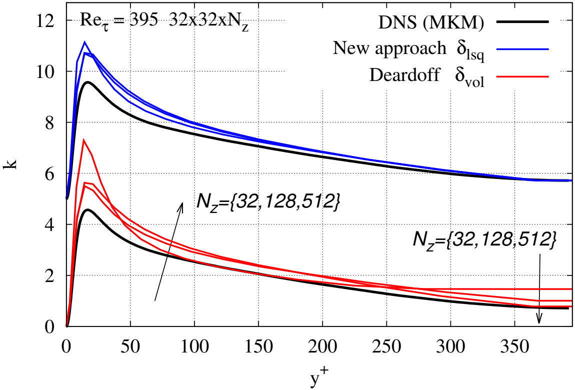
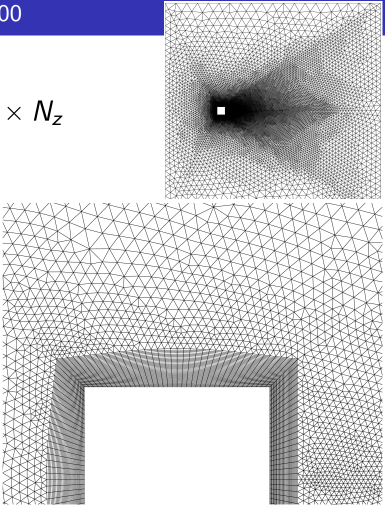
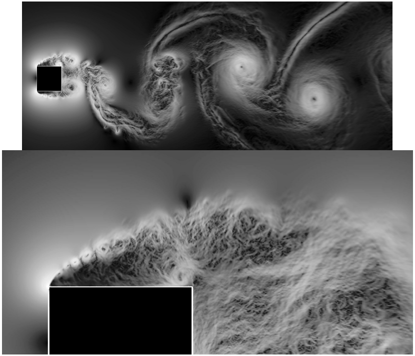


Results for LES

Turbulent flow around square cylinder at $Re = 22000$



DNS[3] 324M grid points LES $19524 \times N_z$



[3] F.X.Trias, A.Gorobets, A.Oliva. *Turbulent flow around a square cylinder at Reynolds number 22000: a DNS study*, **Computers&Fluids**, 123:87-98, 2015.

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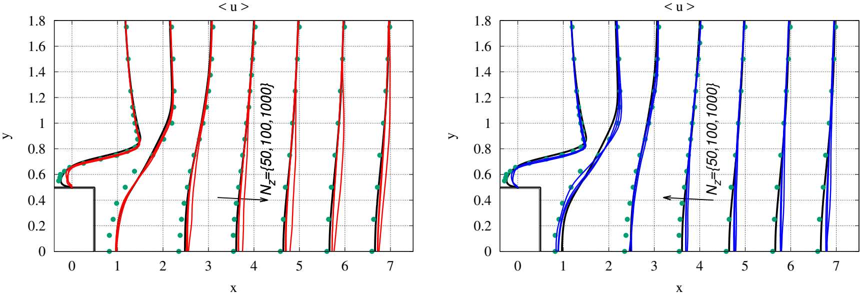
LES[4] $19524 \times N_z$



Deardorff δ_{vol} New approach δ_{lsq}

[4] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

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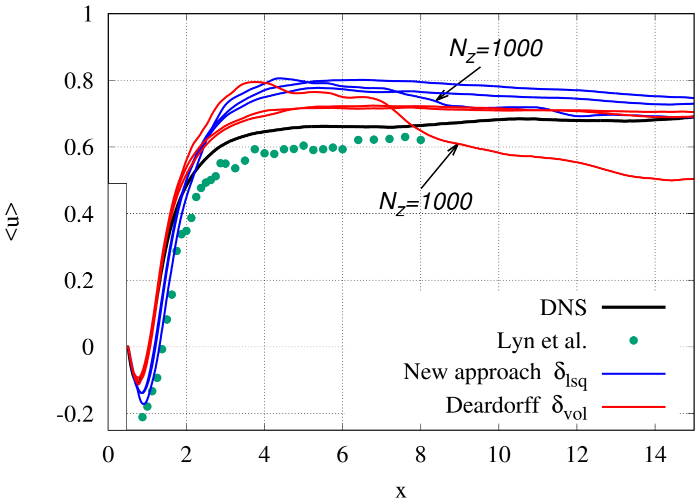

Concluding remarks

- A new definition for δ has been proposed

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}}$$

- It is locally defined, well-bounded, cheap and easy to implement
- LES tests: HIT, turbulent channel flow ✓
- LES on unstructured grids: turbulent flow around square cylinder ✓
- DES tests: turbulent jet (not shown here) ✓

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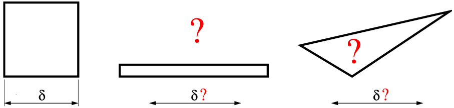

Takeaway message:

- Definition of δ can have a big effect on simulation results

Thank you for your attention