An energy-preserving unconditionally stable method for DNS and LES simulations on unstructured grids

F.X.Trias[1], D.Santos[1], J.Hopman[1], A.Gorobets[2] and A.Oliva[1]

[1] Heat and Mass Transfer Technological Center, Technical University of Catalonia, C/Colom 11, 08222 Terrassa (Barcelona), francesc.xavier.trias@upc.edu

[2] Keldysh Institute of Applied Mathematics, 4A, Miusskaya Sq., Moscow 125047, Russia

ABSTRACT

The essence of turbulence are the smallest scales of motion. They result from a subtle balance between convective transport and diffusive dissipation. Mathematically, these terms are governed by two differential operators differing in symmetry: the convective operator is skew-symmetric whereas the diffusive is symmetric and positive-definite. On the other hand, accuracy and stability need to be reconciled for simulations of turbulent flows in complex geometries. With this in mind, an energy-preserving discretization for unstructured grids was proposed in Ref. [1]: it exactly preserves the symmetries of the underlying differential operators on collocated grids. Hence, unlike other formulations, the discrete convective operator transports energy from a resolved scale of motion to other resolved scales without dissipating energy, as it should do from a physical point-of-view. Therefore, we think that apart from being a right approach for large-scale DNSs of turbulence, it also forms a solid basis for testing subgrid scale LES models.

The discretization is based on only five operators (*i.e.* matrices): the cell-centered and staggered control volumes (diagonal matrices), Ω_c and Ω_s, the face normal vectors, N_s, the cell-to-face interpolation, $\Pi_{c\rightarrow s}$ and the cell-to-face divergence operator, M. Therefore, it constitutes a robust approach that can be easily implemented in already existing codes such as OpenFOAM® [2]. Moreover, for the sake of cross-platform portability and optimization, CFD algorithms must rely on a very reduced set of (algebraic) kernels (*e.g.* sparse-matrix vector product, SpMV; dot product; linear combination of vectors). This imposes restrictions and challenges that need to be addressed such as the inherent low arithmetic intensity of the SpMV, the reformulation of flux limiters [3] or the efficient computation of eigenbounds to determine the time-step, Δt. Results showing the benefits of symmetry-preserving discretizations will be presented together with novel methods aiming to keep a good balance between code portability, numerical robustness and performance.

References

[1] F. X. Trias, O. Lehmkuhl, A. Oliva, C.D. Pérez-Segarra, and R.W.C.P. Verstappen. Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured meshes. *Journal of Computational Physics*, 258:246–267, 2014.

[2] E. Komen, J. A. Hopman, E. M. A. Frederix, F. X. Trias, and R. W. C. P. Verstappen. A symmetry-preserving second-order time-accurate PISO-based method. *Computers & Fluids*, 225:104979, 2021.

[3] N. Valle, X. Álvarez-Farré, A. Gorobets, J. Castro, A. Oliva, and F. X. Trias. On the implementation of flux limiters in algebraic frameworks. *Computer Physics Communications*, 271:108230, 2022.