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A NEW SUBGRID CHARACTERISTIC LENGTH FOR LES

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INTRODUCTION

Large-eddy simulation (LES) equations result from applying a spatial commutative filter, with filter length δ, to the Navier-Stokes equations

$$\partial_t \overline{\boldsymbol{u}} + (\overline{\boldsymbol{u}} \cdot \nabla) \overline{\boldsymbol{u}} = \nu \nabla^2 \overline{\boldsymbol{u}} - \nabla \overline{p} - \nabla \cdot \tau(\overline{\boldsymbol{u}}), \qquad \nabla \cdot \overline{\boldsymbol{u}} = 0, \quad (1)$$

where $\overline{\boldsymbol{u}}$ is the filtered velocity and $\tau(\overline{\boldsymbol{u}})$ is the subgrid stress (SGS) tensor and aims to approximate the effect of the under-resolved scales, *i.e.* $\tau(\overline{\boldsymbol{u}}) \approx \overline{\boldsymbol{u} \otimes \boldsymbol{u}} - \overline{\boldsymbol{u}} \otimes \overline{\boldsymbol{u}}$. Because of its inherent simplicity and robustness, the eddy-viscosity assumption is by far the most used closure model, $\tau(\overline{\boldsymbol{u}}) \approx -2\nu_e S(\overline{\boldsymbol{u}})$. Then, the eddy-viscosity, ν_e, is usually modeled as follows

$$\nu_e = (C_m \delta)^2 D_m(\overline{\boldsymbol{u}}). \quad (2)$$

In the last decades most of the research has focused on either the calculation of the model constant, C_m (*e.g.* the dynamic modeling approach), or the development of more appropriate model operators $D_m(\overline{\boldsymbol{u}})$ (*e.g.* WALE, Vreman's, Verstappen's, σ-model, S3PQR [1],...). Surprisingly, little attention has been paid on the computation of the subgrid characteristic length, δ, which is also a key element of any eddy-viscosity model. Despite the fact that in some situations it may provide very inaccurate results, three and a half decades later, the approach proposed by Deardorff [2], *i.e.*, the cube root of the cell volume (see Eq. 3), is by far the most widely used to computed the subgrid characteristic length, δ. Its inherent simplicity and applicability to unstructured meshes is probably a very good explanation for that. Alternative methods to compute δ are summarized and classified in Table 1 according to a list of desirable properties for a (proper) definition of δ. Namely,

$$\delta_{\mathrm{vol}} = (\Delta x \Delta y \Delta z)^{1/3}, \qquad \delta_{\mathrm{Sco}} = f(a_1, a_2)\delta_{\mathrm{vol}}, \quad (3)$$

$$\delta_{\max} = \max(\Delta x, \Delta y, \Delta z), \delta_{L^2} = \sqrt{\frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{3}}, \quad (4)$$

$$\delta_{\boldsymbol{\omega}} = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\boldsymbol{\omega}|^2}, \quad (5)$$

$$\tilde{\delta}_{\boldsymbol{\omega}} = \max_{n,m=1,\dots,8} \frac{|\boldsymbol{l}_n - \boldsymbol{l}_m|}{\sqrt{3}}, \ \delta_{\mathrm{SLA}} = \tilde{\delta}_{\boldsymbol{\omega}} F_{\mathrm{KH}}(VTM), \quad (6)$$

where $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z) = \nabla \times \boldsymbol{u}$ is the vorticity and $f(a_1, a_2) = \cosh \sqrt{4/27[(\ln a_1)^2 - \ln a_1 \ln a_2 + (\ln a_2)^2]}$ ($a_1 = \Delta x/\Delta z$, $a_2 = \Delta y/\Delta z$, assuming that $\Delta x \leq \Delta z$ and $\Delta y \leq \Delta z$) is the correcting function proposed by Scotti [3]. The function $0 \leq F_{\mathrm{KH}}(VTM) \leq 1$ has been recently proposed by Shur *et al.* [4] to correct the $\tilde{\delta}_{\boldsymbol{\omega}}$ definition proposed by Mockett *et al.* [5], both in the context of Detached Eddy Simulation.

	δ_{vol}	δ_{Sco}	$\delta_{\max}$	δ_{L^2}	$\delta_{\boldsymbol{\omega}}$	$\tilde{\delta}_{\boldsymbol{\omega}}$	δ_{SLA}	δ_{lsq}
Ref.	[2]	[3]	[6]		[7]	[5]	[4]	
	Eq.3	Eq.3	Eq.4	Eq.4	Eq.5	Eq.6	Eq.6	Eq.10
P0	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
P1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
P2	No	No	No	No	Yes	Yes	Yes	Yes
P3	Yes	No	No	No	No*	Yes	Yes	Yes
P4	+++	++	++++	+++	++	+	+	+++

Table 1: Properties of different definition of the subgrid characteristic length, δ. Namely, **P0**: $\delta \geq 0$, locality and frame invariant; **P1**: boundedness, *i.e.*, given a structured Cartesian mesh where $\Delta x \leq \Delta y \leq \Delta z$, $\Delta x \leq \delta \leq \Delta z$; **P2**: sensitive to flow orientation; **P3**: applicable to unstructured meshes; **P4**: well-conditioned and low computational cost.

These properties are based on physical, numerical, and/or practical arguments. This list is completed with the definition of δ_{lsq} given in Eq.(10) and introduced in the next section. According to property **P2**, they can be classified into two main families; namely, (i) definitions of δ that solely depend on geometrical properties of the mesh, and (ii) definitions of δ that are also dependent on the local flow topology. The latter is characterized by the gradient of the resolved velocity field, $\mathsf{G} \equiv \nabla \overline{\boldsymbol{u}}$. On the other hand, the local mesh geometry for a Cartesian grid is contained in the following second-order diagonal tensor,

$$\boldsymbol{\Delta} \equiv \mathrm{diag}(\Delta x, \Delta y, \Delta z). \quad (7)$$

Hereafter, we take $\Delta x \leq \Delta y \leq \Delta z$ without loss of generality. Therefore, methods solely based on the geometrical properties of the mesh are fully characterized by the tensor $\boldsymbol{\Delta}$. Apart from the geometric information contained in $\boldsymbol{\Delta}$, the other methods are also dependent on the flow topology, G.

BUILDING A NEW SUBGRID CHARACTERISTIC LENGTH

The subgrid characteristic length, δ, appears in a natural way when we consider the leading term of the Taylor series expansion of the subgrid stress tensor,

$$\tau(\overline{\boldsymbol{u}}) = \frac{\delta^2}{12} \mathsf{G}\mathsf{G}^T + \mathcal{O}(\delta^4). \quad (8)$$

This is the gradient model proposed by Clark *et al.* [9], where in this case, δ denotes the filter length. On the other hand, for anisotropic filter lengths, the Taylor expansion of τ gives

$$\tau(\overline{\boldsymbol{u}}) = \frac{1}{12} \mathsf{G}_\delta \mathsf{G}_\delta^T + \mathcal{O}(\delta^4), \quad (9)$$

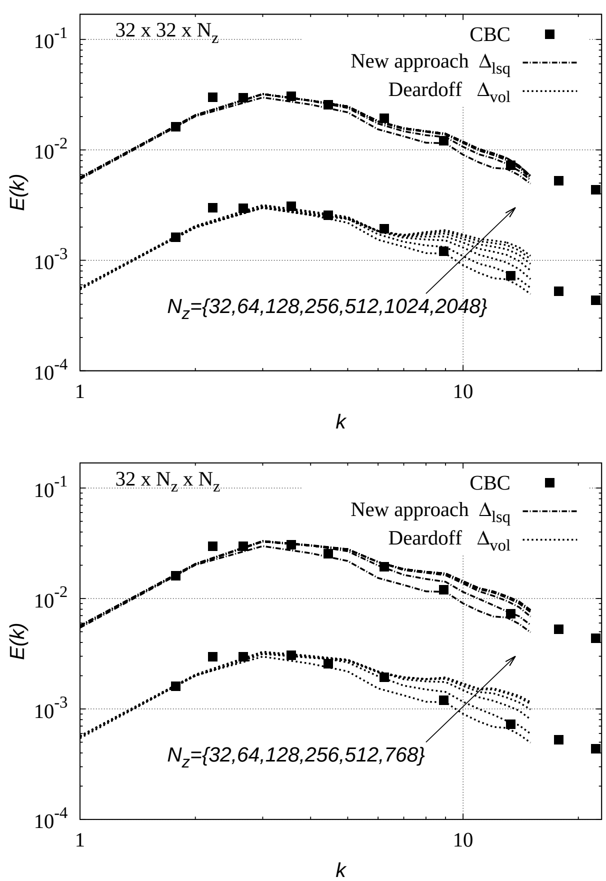

Figure 1: Energy spectra for decaying isotropic turbulence corresponding to the experiment of Comte-Bellot and Corrsin [8]. Results obtained with the new definition δ_{lsq} proposed in Eq.(10) are compared with the classical definition proposed by Deardorff given in Eq.(3). For clarity, latter results are shifted one decade down.

where $\mathsf{G}_\delta \equiv \mathsf{G}\boldsymbol{\Delta}$. Minimizing the difference between the leading term tensors of Eqs.(8) and (9), *i.e.* $(\delta^2/12)\mathsf{G}\mathsf{G}^T \approx (1/12)\mathsf{G}_\delta\mathsf{G}_\delta^T$ using a least-squares minimization leads to

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{\mathsf{G}_\delta\mathsf{G}_\delta^T : \mathsf{G}\mathsf{G}^T}{\mathsf{G}\mathsf{G}^T : \mathsf{G}\mathsf{G}^T}}. \tag{10}$$

This definition of δ fulfills a set of desirable properties. Namely, it is locally defined and well bounded, $\Delta x \leq \delta_{\mathrm{lsq}} \leq \Delta z$; therefore it meets properties **P0** and **P1**. Moreover, it is sensitive to flow orientation (property **P2**) and applicable to unstructured meshes (property **P3**). In this regard, for purely rotating flows, *i.e.*, $\mathsf{S} = 0$ and $\mathsf{G} = \Omega$, δ_{lsq} reduces to

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{\omega_x^2(\Delta y^2 + \Delta z^2) + \omega_y^2(\Delta x^2 + \Delta z^2) + \omega_z^2(\Delta x^2 + \Delta y^2)}{2|\boldsymbol{\omega}|^2}}, \tag{11}$$

which resembles the definition of $\delta_{\boldsymbol{\omega}}$ given in Eq.(5) proposed by Chauvet *et al.* [7]. Actually like the definition $\delta_{\boldsymbol{\omega}}$ given in Eq.(5) proposed by Mockett *et al.* [5] it is $\mathcal{O}(\max\{\Delta x, \Delta y\})$ instead of $\delta_{\boldsymbol{\omega}} = \sqrt{\Delta x \Delta y}$; therefore, it also avoids a strong effect of the smallest grid spacing.

CONCLUSIONS AND PRELIMINARY RESULTS

In this work, a novel definition of subgrid characteristic length, δ, is proposed with the aim to answer the following research question: *can we find a simple and robust definition of δ that minimizes the effect of mesh anisotropies on the performance of SGS models?* In this regard, we consider the novel definition of δ_{lsq} proposed in Eq.(10) as a very good candidate. Preliminary results displayed in Figure 1 correspond to the classical experimental results obtained by Comte-Bellot and Corrsin [8]. LES results have been obtained using the Smagorinsky model, for a set of (artificially) stretched meshes. Results for pancake-like meshes with $32 \times 32 \times N_z$ and $N_z = \{32, 64, 128, 256, 512, 1024, 2048\}$ are displayed in Figure 1 (top): for increasing values of N_z the results obtained using the Deardorff definition, given in Eq.(3), diverge. This is because the value of δ tends to vanish and, therefore, the subgrid-scale models switch off. This is not the case for the definition of δ proposed in this work. Instead, results rapidly converge for increasing values of N_z. A similar behavior is observed in Figure 1 (bottom) for pencil-like meshes with $32 \times N_z \times N_z$ and $N_z = \{32, 64, 128, 256, 512, 768\}$. Therefore, the proposed definition of the subgrid characteristic length, δ_{lsq}, seems to minimize the effect of mesh anisotropies on the performance of subgrid-scale models. To test the performance of the new definition of δ for wall-bounded flows (*e.g.* turbulent channel flow), also for unstructured meshes, is part of our research plans. Detailed results will be presented in the final paper and in the conference.

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