Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

JOHANNES KEPLER
UNIVERSITY LINZ

On a proper tensor-diffusivity model for large-eddy simulations of buoyancy driven flows

F.Xavier Trias[1], Firas Dabbagh[1,2], Andrey Gorobets[1,3], Assensi Oliva[1]

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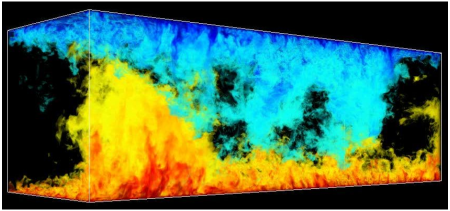
DLES12
ERCOFTAC WORKSHOP DIRECT AND LARGE EDDY SIMULATION 12
MADRID June 5-7 2019

Contents

Motivation

Research question:

- Can we find a nonlinear SGS heat flux model with **good physical** and **numerical properties**, such that we can obtain satisfactory predictions for a turbulent Rayleigh-Bénard convection?



DNS of an air-filled Rayleigh-Bénard convection at $Ra = 10^{10}$

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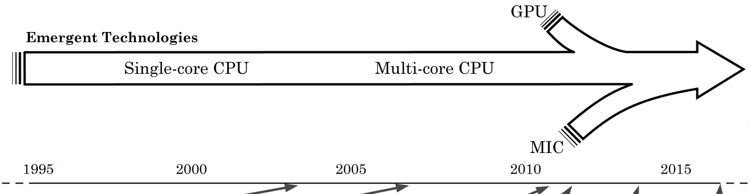
[1] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *On the evolution of flow topology in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 28:115105, 2016.



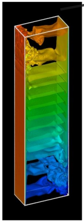
Emergent Technologies

Single-core CPU

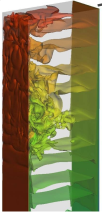
Multi-core CPU

GPU

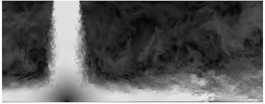
MIC



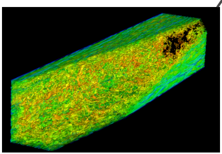
DHC
$Ra=10^9$
(3.2M)



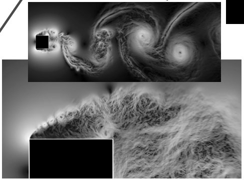
DHC
$Ra=10^{11}$
(111M)



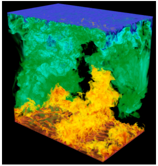
Impinging jet
Re=20000
(102M)



DUCT Re_t=1200 **(172M)**



SqCyl Re=22000 **(324M)**



RB
$Ra=10^{10}$ **(607M)**
$Ra=10^{11}$ **(5600M)**

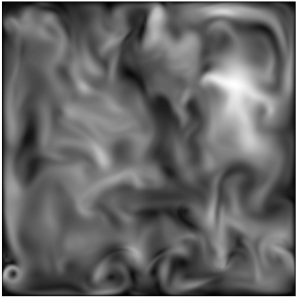
RES
RED ESPAÑOLA DE SUPERCOMPUTACIÓN

PRACE

Motivation

Air-filled RB: $Pr = 0.7$

$Ra = 10^8$



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[2] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **J.Fluid Mechanics**, (submitted).

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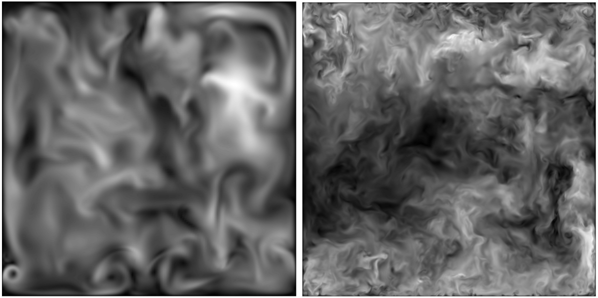

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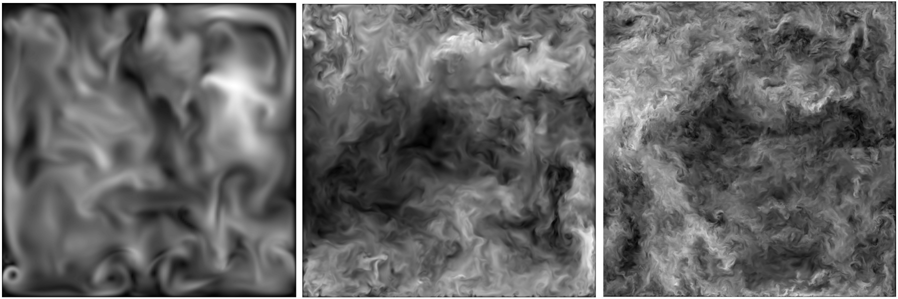

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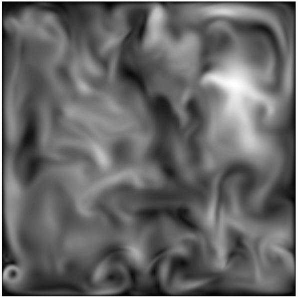
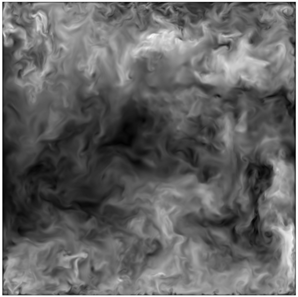


$208 \times 208 \times 400$

17.5M

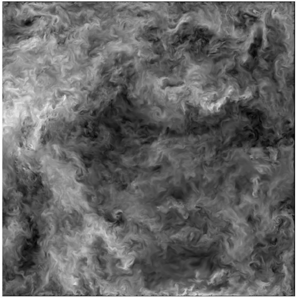
$Ra = 10^{10}$



$768 \times 768 \times 1024$

607M

$Ra = 10^{11}$



$1662 \times 1662 \times 2048$

5600M

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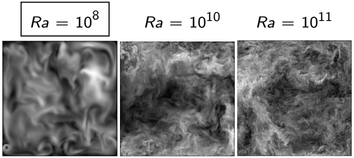
Motivation

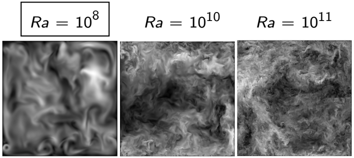
DNS: 208 × 208 × 400



Motivation

DNS: $208 \times 208 \times 400$

$\boxed{\text{LES:} \quad 80 \times \quad 80 \times 120}$



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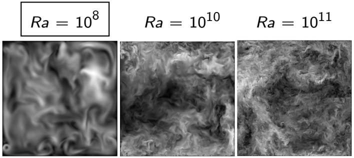
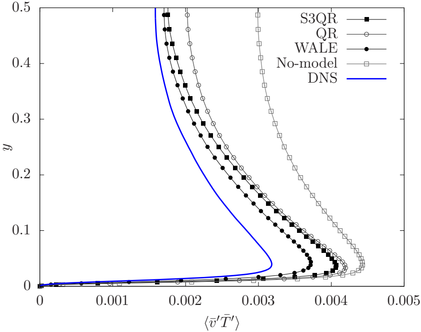
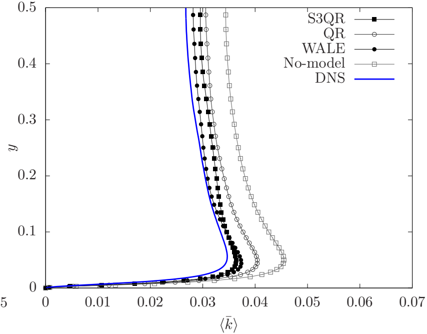


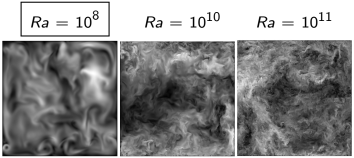


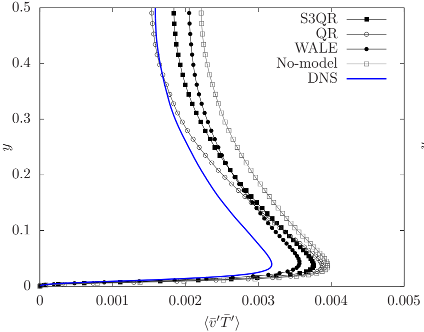


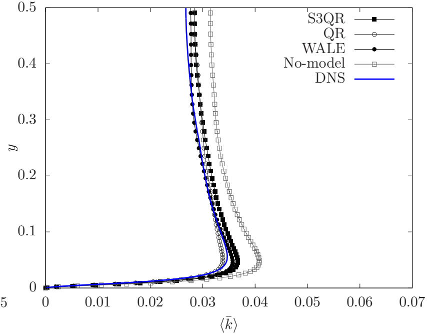
Motivation

DNS: $208 \times 208 \times 400$

LES: $110 \times 110 \times 168$







How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} \qquad - \nabla \cdot \tau(\overline{u}) \;\; ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \;\; \longrightarrow \;\; \tau \; (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

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$$Pr_t?$$

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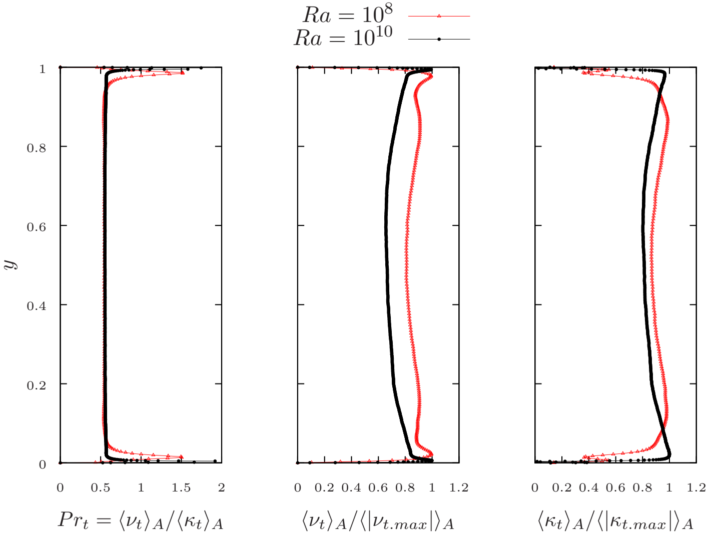

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gradient model $\longrightarrow$ $q \approx -\frac{\delta^2}{12} G \nabla \overline{T}$ $\qquad (\equiv q^{nl})$

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$$G \equiv \nabla \overline{u} \qquad\qquad q = -\frac{\delta^2}{12} G \nabla \overline{T} + \mathcal{O}(\delta^4)$$

A priori alignment trends[3]

$$\text{eddy-diffusivity} \longrightarrow \quad {\color{red}q} \approx -{\color{red}\alpha_t} \nabla \overline{T} \qquad (\equiv q^{eddy})$$

$$\text{gradient model} \longrightarrow \quad {\color{red}q} \approx -\frac{\delta^2}{12} G \nabla \overline{T} \qquad (\equiv q^{nl})$$

---

[3] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *A priori study of subgrid-scale features in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 29:105103, 2017.

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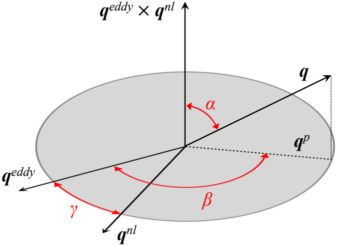

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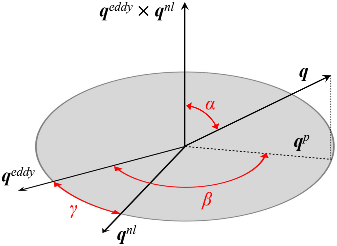

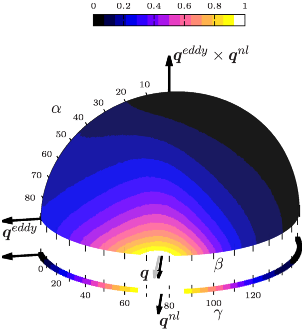

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[4] S.Peng and L.Davidson. **Int.J.Heat Mass Transfer**, 45:1393-1405, 2002.

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$$\text{Peng\&Davidson}^4 \quad \longrightarrow \quad q \approx -\frac{\delta^2}{12} S \nabla \overline{T} \qquad (\equiv q^{PD})$$

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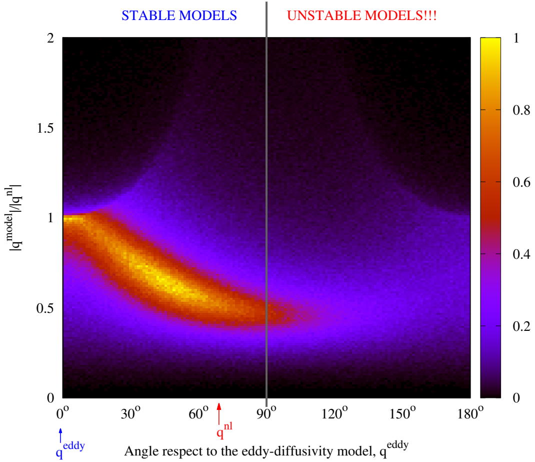
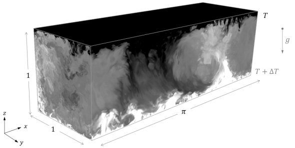
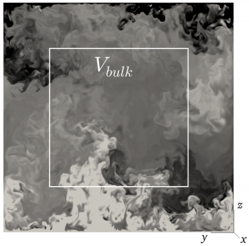
[4] S.Peng and L.Davidson. **Int.J.Heat Mass Transfer**, 45:1393-1405, 2002.

A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T} \qquad \boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$







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$$\text{Peng\&Davidson}^5 \quad \longrightarrow \quad \xcancel{q \approx -\frac{\delta^2}{12} S \nabla \overline{T}} \qquad (\equiv q^{PD})$$

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[5] S.Peng and L.Davidson. **Int.J.Heat Mass Transfer**, 45:1393-1405, 2002.

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$$\text{mixed model} \quad \longrightarrow \quad q \approx q^{nl} + \sigma q^{eddy} \qquad\qquad (\equiv q^{mix})$$

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[6] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

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$$\text{Daly\&Harlow}^6 \quad \longrightarrow \quad q \approx -\mathcal{T}_{SGS} \frac{\delta^2}{12} G G^T \nabla \overline{T} \qquad (\equiv q^{DH})$$

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[6] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

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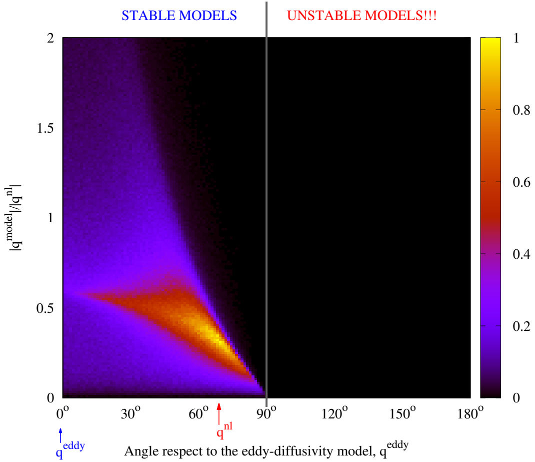
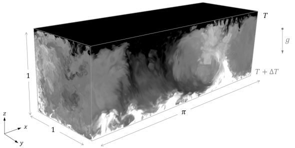
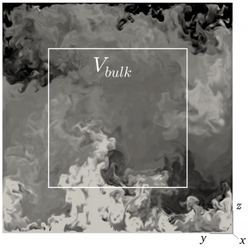
A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

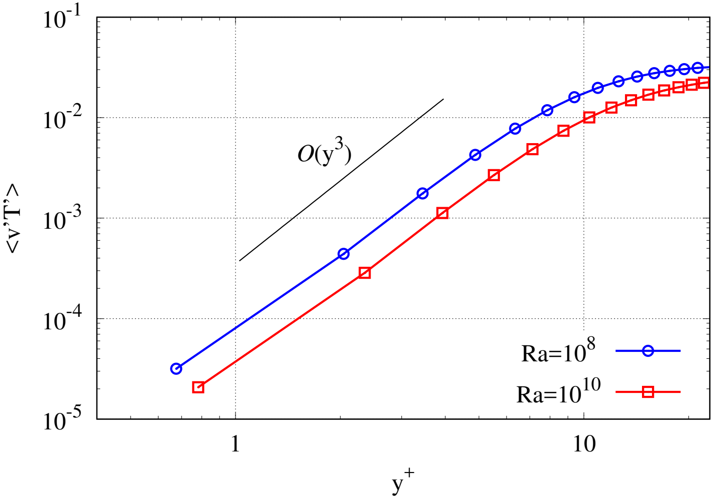
$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12} G G^T \nabla \overline{T}}$$







Near-wall scaling



Near-wall scaling

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\,; \qquad \mathcal{T}_{SGS} = 1/|S|}$$

$$u = ay + \mathcal{O}(y^2);\;\; v = by^2 + \mathcal{O}(y^3);\;\; w = cy + \mathcal{O}(y^2);\;\; T = dy + \mathcal{O}(y^2)$$

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$$G = \left(\begin{array}{ccc} y & 1 & y \\ y^2 & y & y^2 \\ y & 1 & y \end{array}\right); \;\; \nabla\overline{T} = \left(\begin{array}{c} y \\ 1 \\ y \end{array}\right)$$

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$$\boxed{\mathcal{T}_{SGS} = 1/|S| = \mathcal{O}(y^0)}$$

Idea: build a $\mathcal{T}_{SGS}$ with the proper $\mathcal{O}(y^2)$ scaling!!!

Building proper models for the subgrid heat flux

Let us consider models that are based on the invariants of the tensor GG^T

$$q \approx -C_M \left(P_{GG^T}^p Q_{GG^T}^q R_{GG^T}^r \right) \frac{\delta^2}{12} GG^T \nabla \overline{T} \qquad (\equiv q^{S2})$$

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Formula	$2(Q_\Omega - Q_S)$	$V^2 + Q_G^2$	R_G^2

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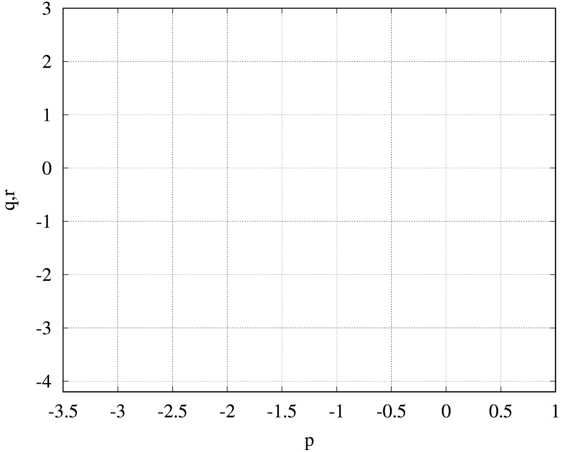
$$-6r - 4q - 2p = 1 \ \ [T]; \qquad 6r + 2q = s,$$

where s is the slope for the asymptotic near-wall behavior, *i.e.* $\mathcal{O}(y^s)$.

Building proper models for the subgrid heat flux

Solutions: $q(p, s) = -(1+s)/2 - p$ and $r(p, s) = (2s+1)/6 + p/3$



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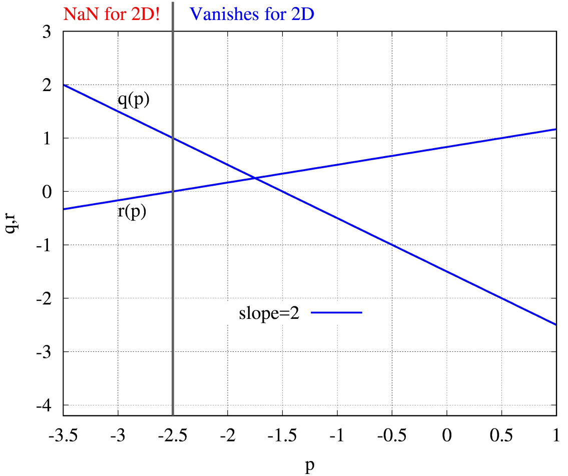

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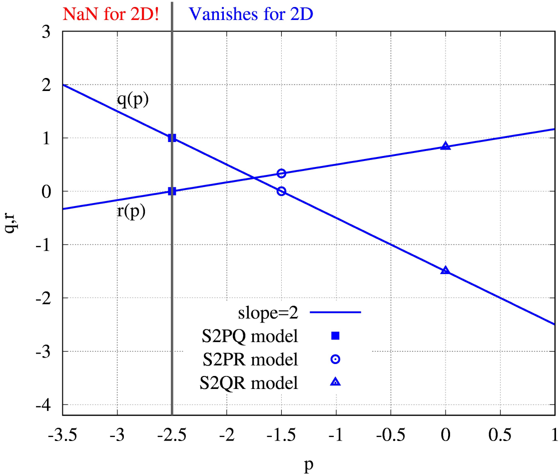

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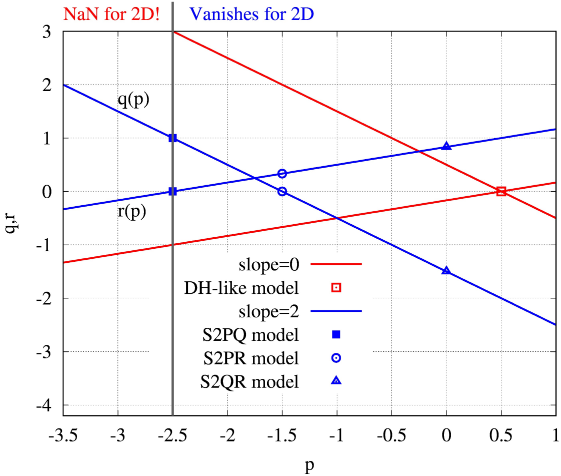
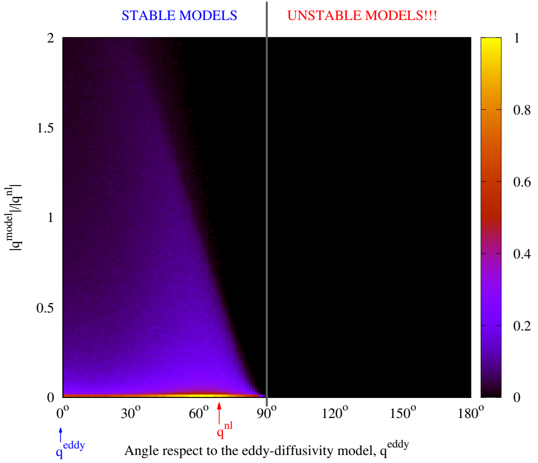
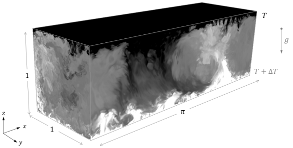


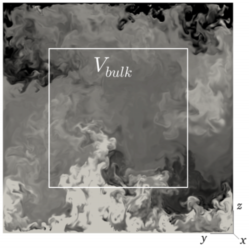
A priori alignment trends of S2QR

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

$$\boxed{q^{s2QR} \equiv -C_M Q_{GG^T}^{3/2} R_{GG^T}^{5/6} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$







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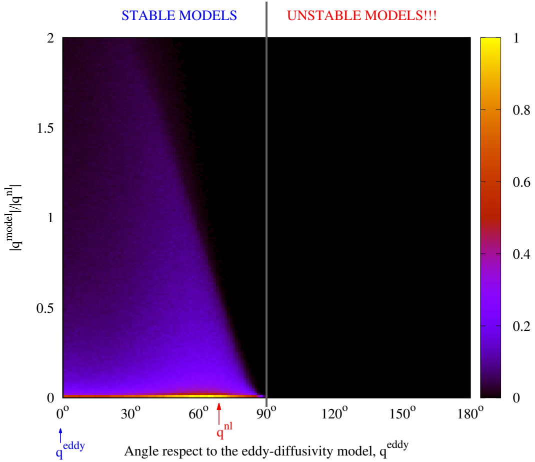

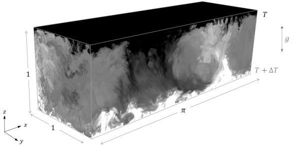

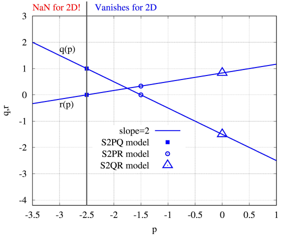

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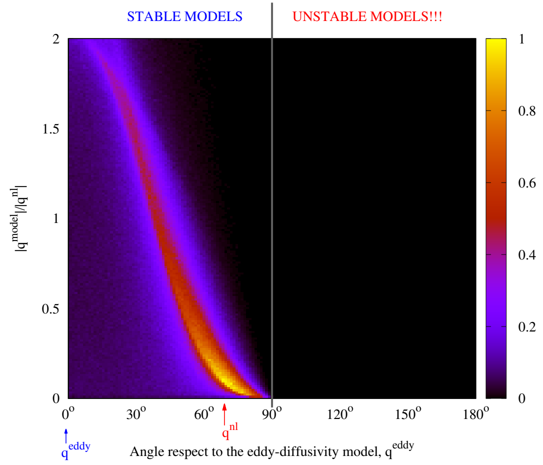

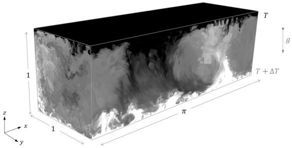

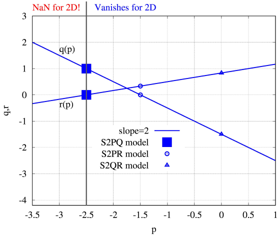

A priori alignment trends of S2PR

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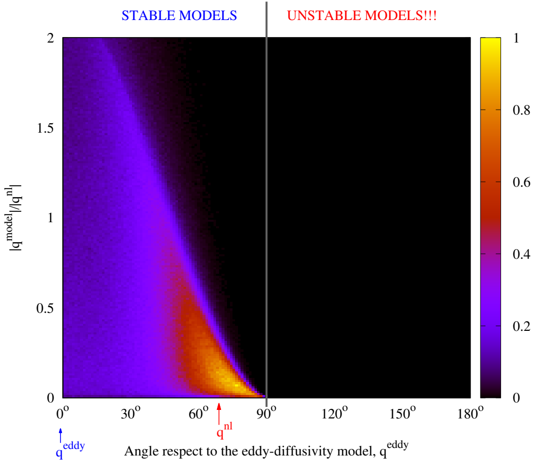

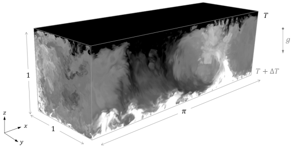

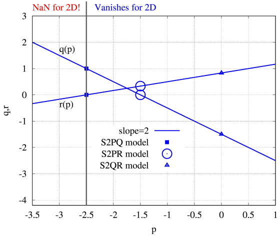

A priori alignment trends of S2PR in the near-wall region

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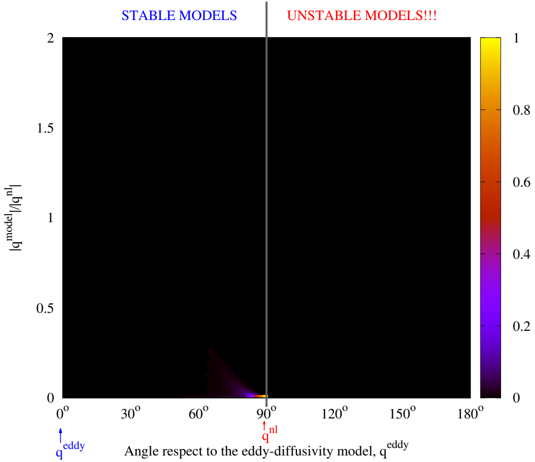

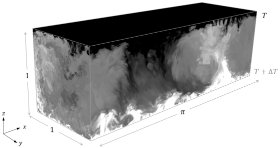

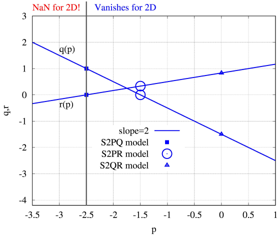

A priori alignment trends of DH in the near-wall region

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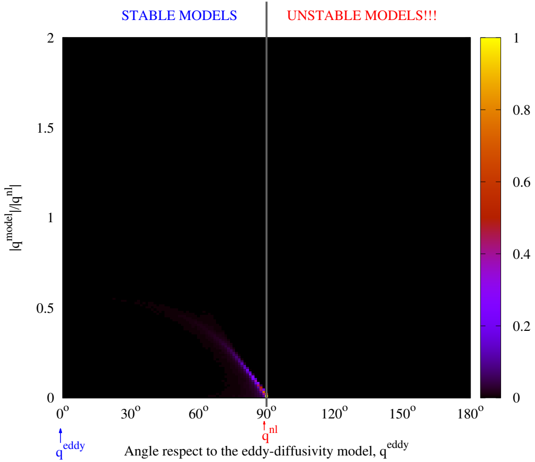

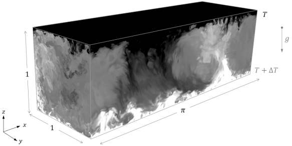

A posteriori results?

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⚠ But first we need to answer the following **research question**:

- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

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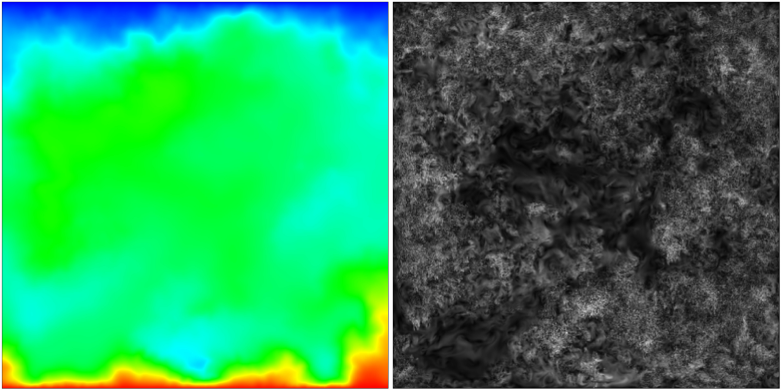
Idea: let's do an LES for momentum and a DNS for temperature!

DNS at very low Pr number

Why? scale separation scales with $Pr^{0.5}$

DNS at very low *Pr* number

Why? scale separation scales with $Pr^{0.5}$ (≈ 0.07 is our case)



DNS of a RB at $Ra = 7.14 \times 10^6$ and $Pr = 0.005$ (liquid sodium)
$488 \times 488 \times 1280 \approx$ **305M**

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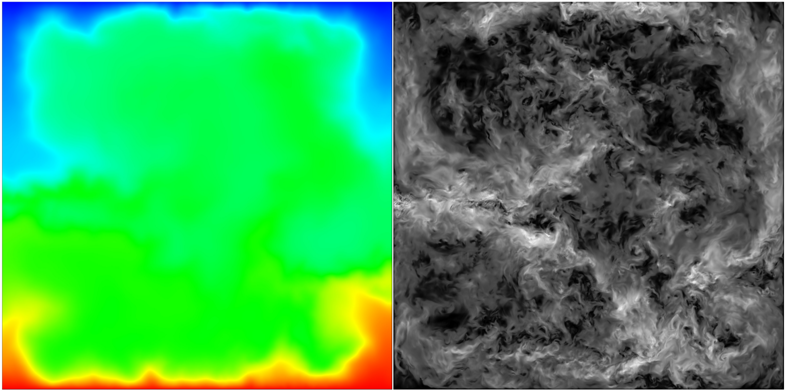

DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)
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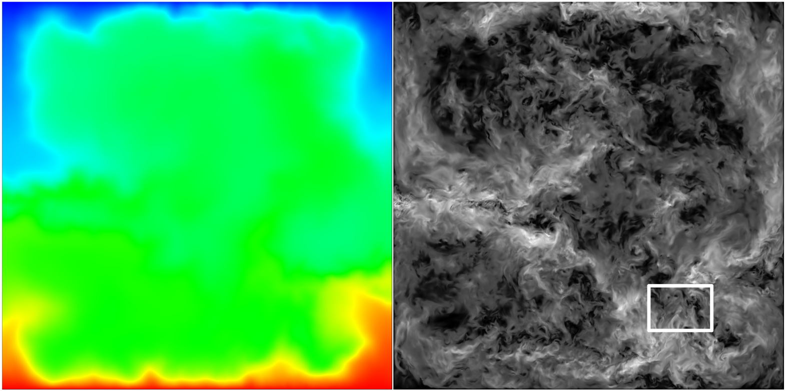

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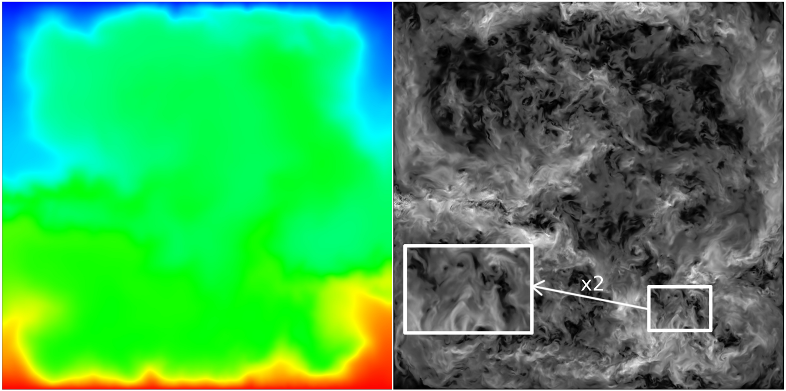

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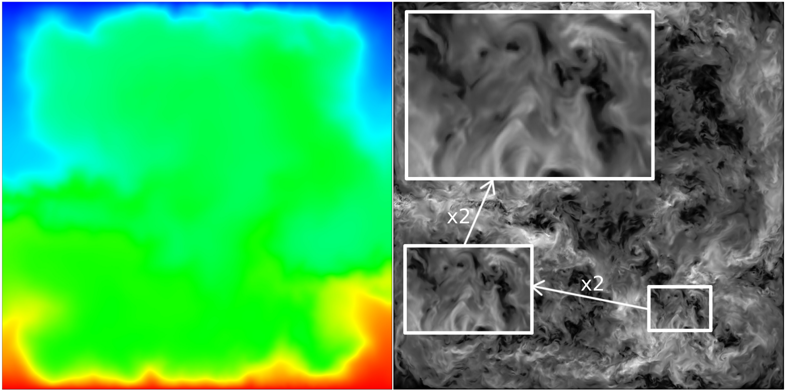

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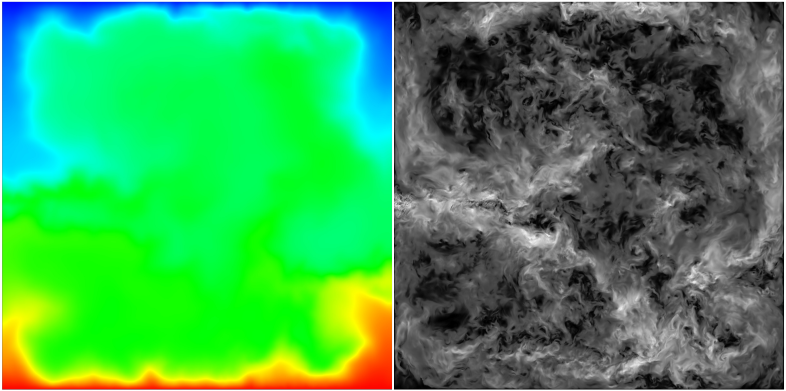
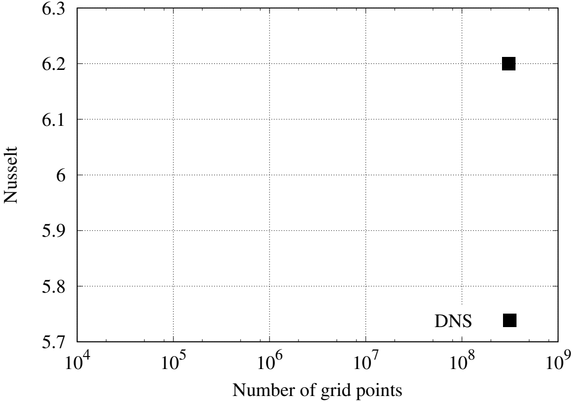


DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)
$966 \times 966 \times 2048 \approx$ **1911M**

LES[7] results at very low *Pr* number



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[7] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

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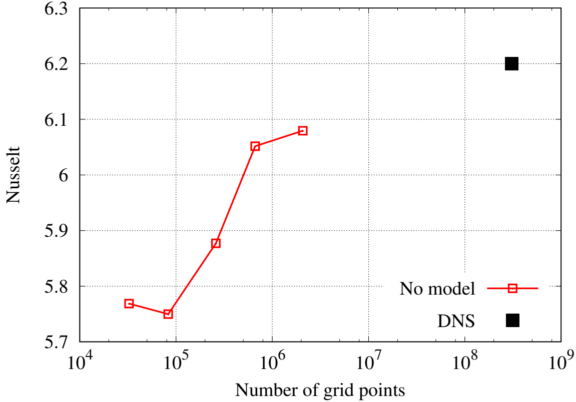

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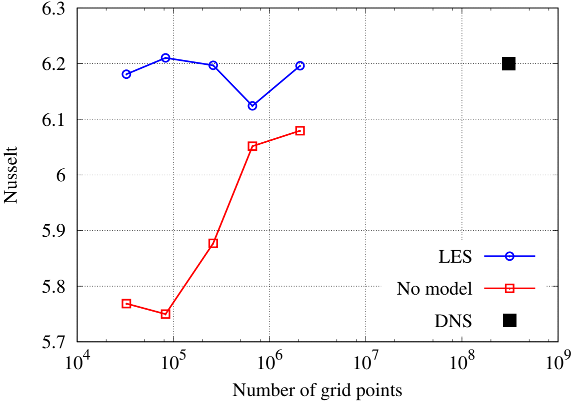

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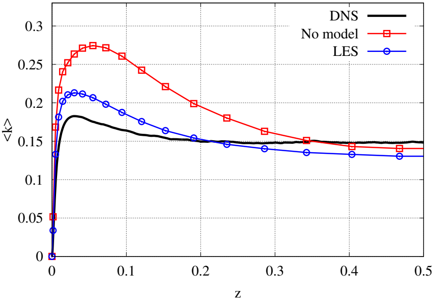
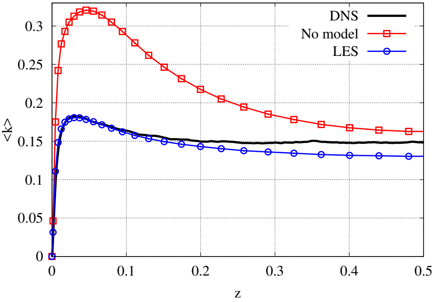
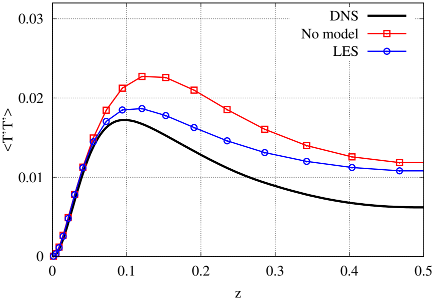


$64 \times 32 \times 32$



$96 \times 52 \times 52$

LES results at very low *Pr* number



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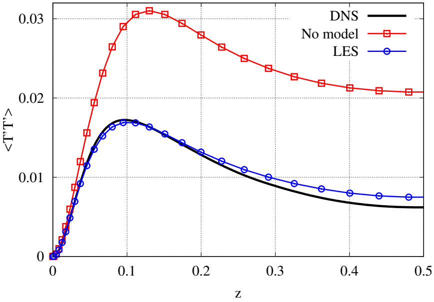

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Concluding remarks

- A new tensor-diffusivity model has been proposed

$$\boxed{q^{s2PR} \equiv -C_M P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

- Locally defined, unconditionally stable and vanishes for 2D flows ✓
- Good *a priori* alignment trends and proper near-wall scaling ✓
- Eddy-viscosity models work for RB ✓

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[8] F.X.Trias, A.Gorobets, M.Silvis, R.Verstappen, A.Oliva. *A new subgrid characteristic length for turbulence simulations on anisotropic grids*, **Phys.Fluids**, 26:115109, 2017.

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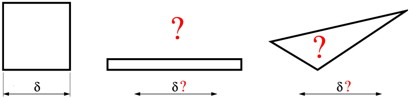
Near future:

- *A posteriori* tests using q^{s2PR} for Rayleigh-Bénard convection.
- How δ should be defined for highly anisotropic grids[8]?



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Thank you for your attention