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ON A CONSERVATIVE SOLUTION TO CHECKERBOARDING: ALLOWING NUMERICAL DISSIPATION ONLY WHEN AND WHERE NECESSARY

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INTRODUCTION

CFD codes with industrial applications commonly rely on a collocated grid arrangement, which handles complex geometries and unstructured meshes better than the staggered grid arrangement, with the added benefit of allowing a computationally more efficient data structure. If a central-differencing scheme is used to discretise the spatial differential operators, a set of wide-stencil operators is obtained. The gradient and divergence operator make use of a 3-wide stencil, creating a decoupling between the value of the central cell and its resulting differential value. The Laplacian operator in turn makes use of a 5-wide stencil, similarly disconnecting the central cell from its direct neighbours. In incompressible flows, this odd-even decoupling can lead to spurious pressure modes. These modes will persist because they are invisible to the gradient operator, offering no feedback onto the collocated velocity field. This problem is commonly known as the checkerboard problem.

The most widely used class of methods to solve this problem is through a weighted interpolation method (WIM), of which the pressure-weighted interpolation method, attributed to Rhie and Chow [1], is the most well-known example. This method establishes a coupling between directly neighbouring cells by adding a correction term which includes a cell-to-face pressure gradient. Usually this connection is constructed implicitly through a compact-stencil Laplacian operator, which additionally decreases computational complexity and cost. The application of these correction term leads to a non-zero discrete divergence at either the cell- or face-centered velocities. This, in turn, unavoidably introduces numerical dissipation to the evolution of kinetic energy, through the convective or pressure term [2].

When using symmetry-preserving methods for collocated grids, this numerical error remains as the largest source of numerical dissipation [3]. This error can at times be of the same magnitude as the applied LES models and therefore greatly interfere with turbulence modelling and high fidelity simulations [4]. To decrease the order of the pressure error introduced by the compact-stencil Laplacian, these works made use of a pressure predictor in the momentum prediction equation. Although greatly reducing the numerical dissipation, this adjustment makes the method more prone to checkerboarding, especially in case a small time-step is used or if the solution reaches a steady state.

The work of Hopman et al. [5] introduced a method that is able to dynamically change the pressure predictor, allowing more numerical dissipation if the solution starts showing oscillations. This was achieved by first answering a question that has been avoided in existing literature that discusses this topic: How should checkerboarding be quantified? By introducing a global, normalised, non-dimensional checkerboard coefficient, the pressure prediction could be regulated. This work explores other possible uses for this coefficient, whilst also introducing a local coefficient. By doing so, numerical dissipation can be limited, not only in when, but also in where it is allowed.

GLOBAL SCALAR ADJUSTMENT

In [5] the checkerboard coefficient was defined as:

$$C_{cb} = \frac{\mathbf{p}_c^T (L - L_c)\mathbf{p}_c}{\mathbf{p}_c^T L \mathbf{p}_c}, \tag{1}$$

where L_c and L denote the wide- and compact-stencil Laplacian, respectively. The coefficient ranges between 0 (smooth) and 1 (pure checkerboard). For this coefficient to regulate the strength of the coupling in the pressure field, there are two options (denoted with superscripts a and b) to apply C_{cb} to the projection method:

$$\mathbf{u}_c^p = \Delta t R\left(\mathbf{u}_c, \mathbf{u}_s\right) - \left(1 - C_{cb}^a\right) G_c \tilde{\mathbf{p}}_c^n, \tag{2}$$

$$L_c \tilde{\mathbf{p}}_c' - C_{cb}^b \left(L - L_c\right) \tilde{\mathbf{p}}_c' = M_c \mathbf{u}_c^p, \tag{3}$$

$$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c \tilde{\mathbf{p}}_c', \tag{4}$$

$$\mathbf{u}_s^{n+1} = \Gamma_{cs} \mathbf{u}_c^{n+1} - C_{cb}^b \left(I - \Gamma_{cs}\Gamma_{sc}\right) G \tilde{\mathbf{p}}_c', \tag{5}$$

where the notation of [3] has been used. Option (a) was tested in [5] and gave good results, in which the pressure predictor was closer to $\tilde{\mathbf{p}}_c^n$ in the absence of checkerboarding and started to decrease in cases oscillations were detected in the domain. Option (b) involves solving a Poisson equation with a denser Laplacian which is usually avoided, however, equation (3) can be rewritten as:

$$L \tilde{\mathbf{p}}_c'^{(k+1)} = \frac{1}{C_{cb}^b} M_c \mathbf{u}_c^p - \frac{C_{cb}^b - 1}{C_{cb}^b} L_c \tilde{\mathbf{p}}_c'^{(k)}, \tag{6}$$

which finds the same solution but treats the wide-stencil Laplacian explicitly.

LOCAL SCALAR ADJUSTMENT

To apply a local adjustment to the projection method, which only acts in areas of the domain where oscillations are detected, a new definition for the checkerboard coefficient has to be used. Similar to equation (1), the local checkerboard coefficient is a vector with an entry per cell i given by:

$$[\gamma_{cb}]_i = \begin{cases} \frac{[(L-L_c)\mathbf{p}_c]_i}{[L\mathbf{p}_c]_i} & \text{if } [L\mathbf{p}_c]_i \neq 0, \\ 0 & \text{if } [L\mathbf{p}_c]_i = 0. \end{cases} \quad (7)$$

As before, this opens up two options (denoted with superscripts α and β) to adjust the projection method, which are less obvious than before. Option (α) regulates the inclusion of a pressure gradient in the momentum prediction equation:

$$\mathbf{u}_c^p = \Delta t R(\mathbf{u}_c, \mathbf{u}_s) - (I - \gamma_{cb}^{\alpha}) G_c \tilde{\mathbf{p}}_c^n. \quad (8)$$

Applying this coefficient directly to the pressure field instead, as $G_c \left(I - \gamma_{cb}^{\alpha}\right) \tilde{\mathbf{p}}_c^n$, will have the effect of an uneven predictor pressure with false gradients, decreasing the overall accuracy of the predictor velocity. However, applying the coefficient as done in equation (8) makes it difficult to calculate the instantaneous pressure field at time-step $n+1$, and a new variable ϕ_c is introduced which can be calculated as:

$$\phi_c^{n+1} = G_c \tilde{\mathbf{p}}_c^{n+1} = (I - \gamma_{cb}^{\alpha}) G_c \tilde{\mathbf{p}}_c^n + G_c \tilde{\mathbf{p}}_c'. \quad (9)$$

Some applications require the calculation of the instantaneous pressure field, in which case every few time-steps the algorithm has to update the fields without using option (α). Option (β), again, involves an adjustment through combining the compact- and wide-stencil Laplacian operators. This combination is not straight-forward, and to maintain the symmetry of the operator the adjustment has to be made as follows:

$$\mathcal{L} = \overline{L}_c + \overline{L} = -M \left(\overline{\Omega}_s + \Gamma_{cs} \overline{\Omega} \Gamma_{cs}^T \right) M^T \quad (10)$$

with:

$$\overline{\Omega}_s = diag(\Gamma_{cs} \gamma_{cb}^{\beta}) \Omega_s, \quad (11)$$

$$\overline{\Omega} = \Omega - I_3 \otimes \left(diag(\gamma_{cb}^{\beta}) \Omega_c \right). \quad (12)$$

Again, to avoid solving a Poisson equation with a wide-stencil Laplacian, the equation can also be solved as:

$$\overline{L} \tilde{\mathbf{p}}_c'^{(k+1)} = M_c \mathbf{u}_c^p - \overline{L}_c \tilde{\mathbf{p}}_c'^{(k)}, \quad (13)$$

reaching a solution to $\tilde{\mathbf{p}}_c'$ iteratively.

NUMERICAL TESTS

All together, four methods are described in this work, method (a), (b), (α) and (β). The latter three methods are newly introduced and are tested and compared to the results form method (a) given in [5]. To do so, a two-dimensional Taylor-Green vortex was used to test the numerical dissipation of each method, whereas a lid-driven cavity is used to test the checkerboard suppressing qualities of the solvers. Finally, a turbulent channel flow is used to test the solver in unsteady conditions by measuring the kinetic energy budgets. The results will be compared to method (a) to show if additional gains can be made in conservational properties, by dynamically regulating checkerboarding not only in time, but also in space.

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REFERENCES

[1] Rhie, C. M., & Chow, W. L. (1983). Numerical study of the turbulent flow past an airfoil with trailing edge separation. AIAA journal, 21(11), 1525-1532.

[2] Felten, F. N., & Lund, T. S. (2006). Kinetic energy conservation issues associated with the collocated mesh scheme for incompressible flow. Journal of Computational Physics, 215(2), 465-484.

[3] Trias, F. X., Lehmkuhl, O., Oliva, A., Pérez-Segarra, C. D., & Verstappen, R. W. C. P. (2014). Symmetry-preserving discretization of Navier–Stokes equations on collocated unstructured grids. Journal of Computational Physics, 258, 246-267.

[4] Komen, E. M. J., Hopman, J. A., Frederix, E. M. A, Trias, F. X., Verstappen, R. W. C. P. (2021). A symmetry-preserving second-order time-accurate PISO-based method. Computers & Fluids, 225, 104979.

[5] Hopman, J. A., Alsalti-Baldellou, À., Trias, F. X. & Rigola, J. (2023). On a checkerboard-free, conservative method for turbulent flows. Proceedings of the Tenth International Symposium on Turbulence, Heat and Mass Transfer.