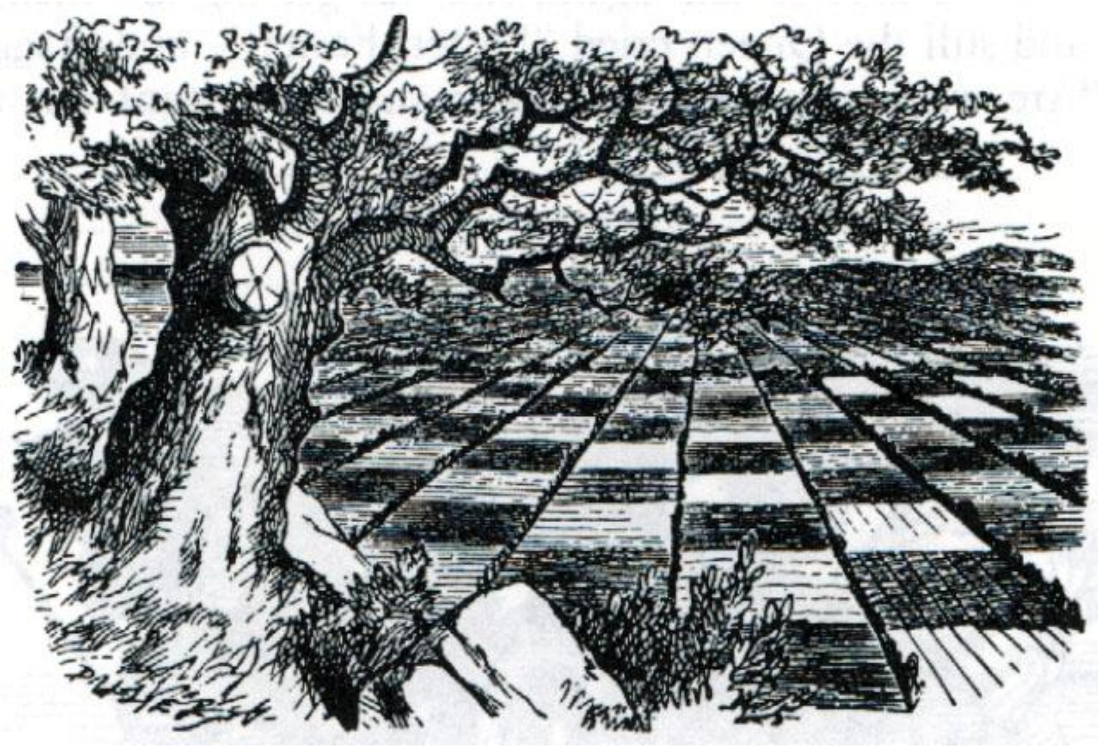
On a Conservative Solution to Checkerboarding:

Allowing Numerical Dissipation
Only When and Where Necessary

J.A. Hopman, L.P.J. Beerten, D. Santos, F.X. Trias, J. Rigola



DLES 14
Ercoftac Workshop
Direct & Large Eddy Simulation

Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Incompressible flow:
Pressure-Velocity coupling
Projection method
p-Poisson

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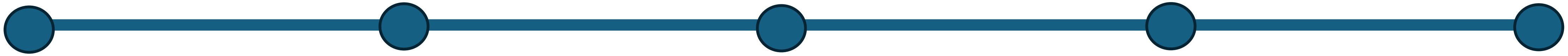
Collocated grid:
Grad(p) $\not\leftrightarrow$ U
Wide-stencil Laplacian
→ $L_c = M_c G_c$



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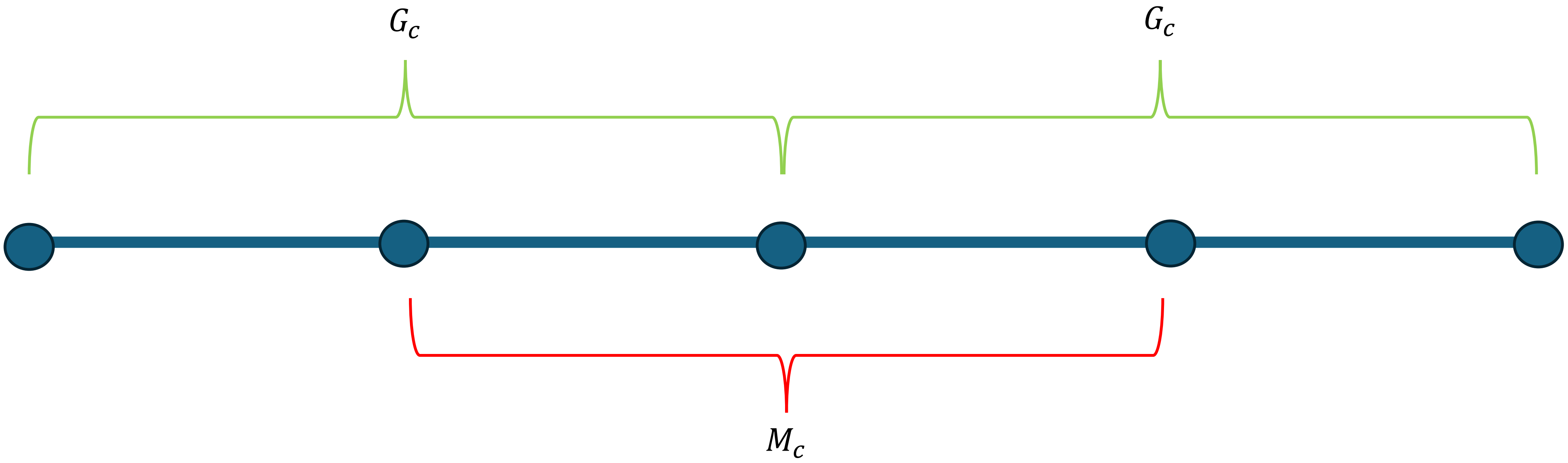

Wide stencil

DIM

$$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$$

$$L_c \tilde{\mathbf{p}}_c^{n+1} = M_c \mathbf{u}_c^p$$

$$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c \tilde{\mathbf{p}}_c^{n+1}$$

$$\mathbf{u}_s^{n+1} = \Gamma_{cs} \mathbf{u}_c^{n+1}$$

Wide stencil	
DIM	WIM
$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$	
$L_c \tilde{\mathbf{p}}_c^{n+1} = M_c \mathbf{u}_c^p$	
$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c \tilde{\mathbf{p}}_c^{n+1}$	
$\mathbf{u}_s^{n+1} = \Gamma_{cs} \mathbf{u}_c^{n+1}$	$\mathbf{u}_s^{n+1} = \Gamma_{cs} \mathbf{u}_c^p - G \tilde{\mathbf{p}}_c^{n+1}$

Wide stencil		Compact stencil	
DIM	WIM	DIM	
$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$			
		$\mathbf{u}_c^{p*} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c^p$	
$L_c\tilde{\mathbf{p}}_c^{n+1} = M_c\mathbf{u}_c^p$		$L\tilde{\mathbf{p}}'_c = M_c\mathbf{u}_c^{p*}$	
		$\tilde{\mathbf{p}}_c^{n+1} = \tilde{\mathbf{p}}_c^p + \tilde{\mathbf{p}}'_c$	
$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c^{n+1}$		$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^p - G\tilde{\mathbf{p}}_c^{n+1}$	
$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^{n+1}$	$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^p - G\tilde{\mathbf{p}}_c^{n+1}$	$\mathbf{u}_c^{n+1} = \Gamma_{sc}\mathbf{u}_s^{n+1}$	

Wide stencil		Compact stencil	
DIM	WIM	DIM	WIM
$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$			
		$\mathbf{u}_c^{p*} = \mathbf{u}_c^p - G_c \tilde{\mathbf{p}}_c^p$	
$L_c \tilde{\mathbf{p}}_c^{n+1} = M_c \mathbf{u}_c^p$		$L \tilde{\mathbf{p}}'_c = M_c \mathbf{u}_c^{p*}$	
		$\tilde{\mathbf{p}}_c^{n+1} = \tilde{\mathbf{p}}_c^p + \tilde{\mathbf{p}}'_c$	
$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c \tilde{\mathbf{p}}_c^{n+1}$		$\mathbf{u}_s^{n+1} = \Gamma_{cs} \mathbf{u}_c^p - G \tilde{\mathbf{p}}_c^{n+1}$	
$\mathbf{u}_s^{n+1} = \Gamma_{cs} \mathbf{u}_c^{n+1}$	$\mathbf{u}_s^{n+1} = \Gamma_{cs} \mathbf{u}_c^p - G \tilde{\mathbf{p}}_c^{n+1}$	$\mathbf{u}_c^{n+1} = \Gamma_{sc} \mathbf{u}_s^{n+1}$	$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c \tilde{\mathbf{p}}_c^{n+1}$

Wide stencil		Compact stencil	
DIM	WIM	DIM	WIM
$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$			
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$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c^{n+1}$		$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^p - G\tilde{\mathbf{p}}_c^{n+1}$	
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Checkerboarding

Wide stencil		Compact stencil	
DIM	WIM	DIM	WIM
$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$			
		$\mathbf{u}_c^{p*} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c^p$	
$L_c\tilde{\mathbf{p}}_c^{n+1} = M_c\mathbf{u}_c^p$		$L\tilde{\mathbf{p}}_c' = M_c\mathbf{u}_c^{p*}$	
		$\tilde{\mathbf{p}}_c^{n+1} = \tilde{\mathbf{p}}_c^p + \tilde{\mathbf{p}}_c'$	
$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c^{n+1}$		$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^p - G\tilde{\mathbf{p}}_c^{n+1}$	
$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^{n+1}$	$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^p - G\tilde{\mathbf{p}}_c^{n+1}$	$\mathbf{u}_c^{n+1} = \Gamma_{sc}\mathbf{u}_s^{n+1}$	$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c^{n+1}$
Checkerboarding	Convective error Not strictly dissipative		

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DIM	WIM	DIM	WIM
$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$			
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Checkerboarding	Convective error Not strictly dissipative		

$$\Gamma_{cs}\Gamma_{sc}G \longrightarrow G$$

Wide stencil		Compact stencil	
DIM	WIM	DIM	WIM
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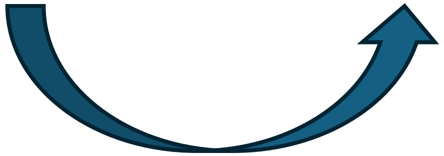
Checkerboarding	Convective error Not strictly dissipative	Smoothing	



$$\Gamma_{cs}\Gamma_{sc}G \longrightarrow G$$

Wide stencil		Compact stencil	
DIM	WIM	DIM	WIM
$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$			
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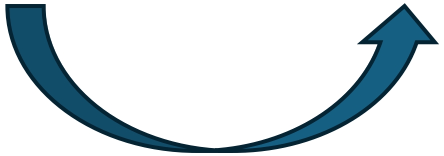
Checkerboarding	Convective error Not strictly dissipative	Smoothing	Pressure error -Strictly dissipative* -L computationally favourable -Allows error order reduction



$$\Gamma_{cs}\Gamma_{sc}G \longrightarrow G$$

Wide stencil		Compact stencil	
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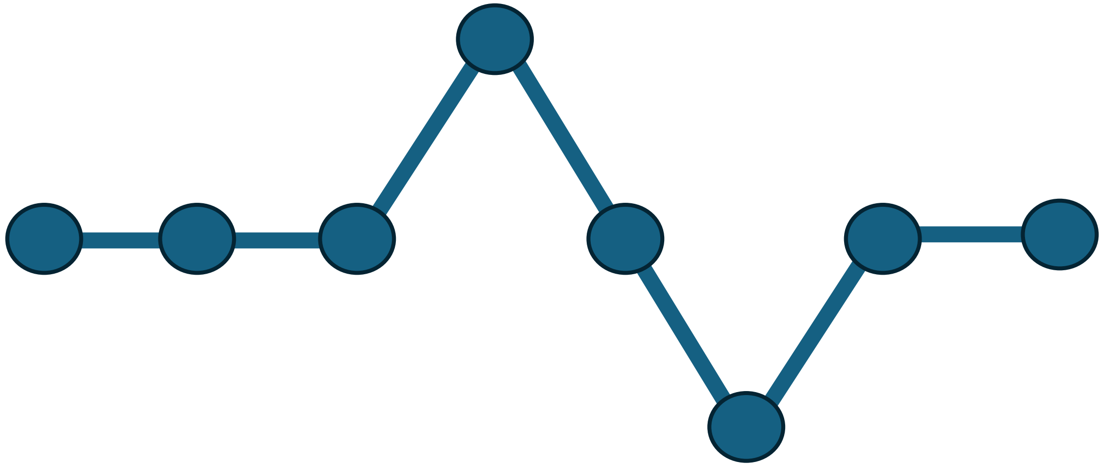
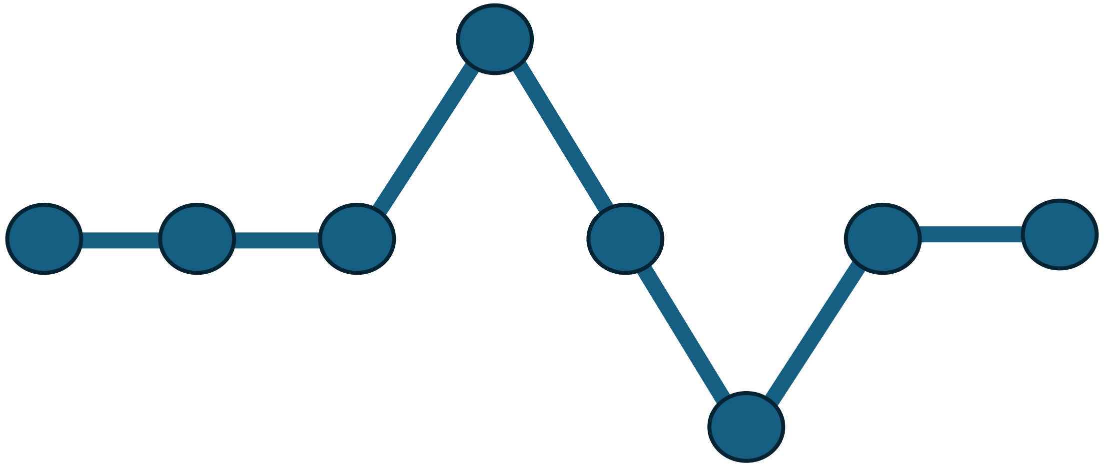
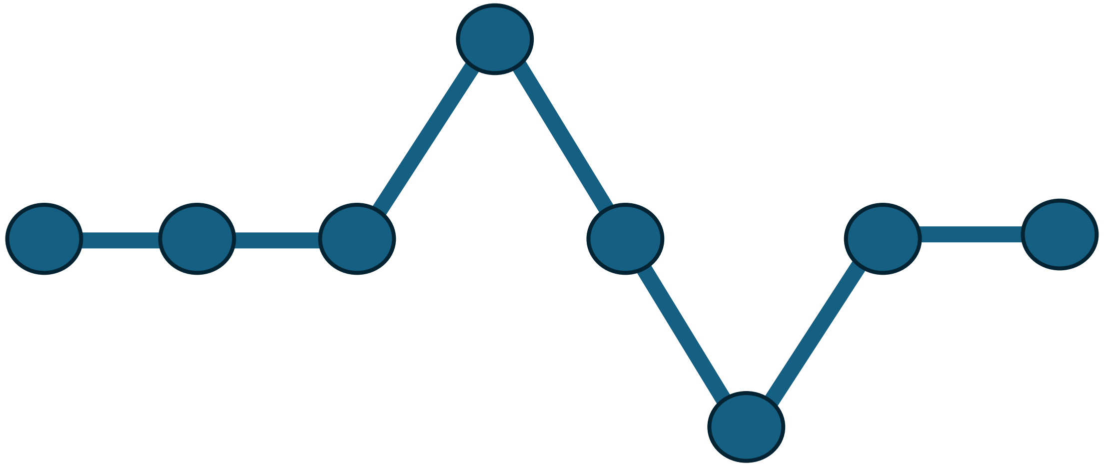
$$\Gamma_{cs}\Gamma_{sc}G \longrightarrow G$$

$$\Gamma_{sc}\Gamma_{cs}\mathbf{u}_c^p \longrightarrow \mathbf{u}_c^p$$





$$\begin{aligned}\mathbf{p}_c &= \begin{bmatrix}0 & 0 & 1 & 0 & \text{-}1 & 0\end{bmatrix}\\ \mathbf{p}_c^- &= \begin{bmatrix}1 & \text{-}1 & 1 & \text{-}1 & 1 & \text{-}1\end{bmatrix}\end{aligned}$$



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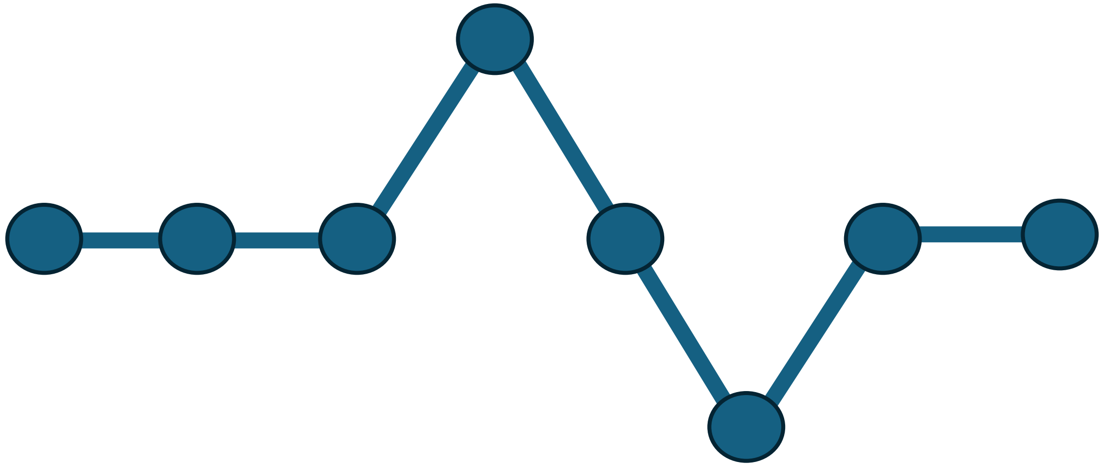
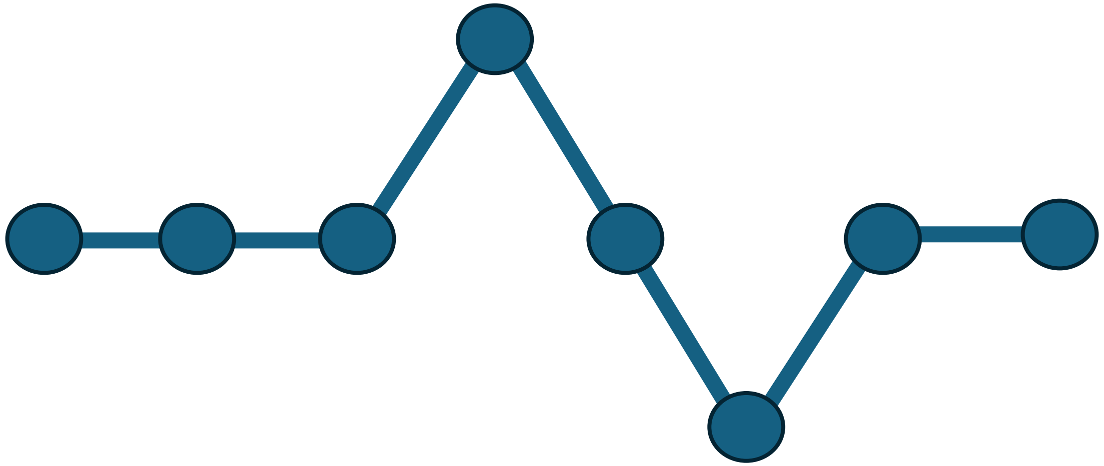
$$\mathbf{p}_c^T \mathbf{p}_c^{-} = 0$$



$$\begin{aligned} \mathbf{p}_c &= [0 \quad 0 \quad 1 \quad 0 \quad -1 \quad 0] \\ \mathbf{p}_c^- &= [1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1] \\ \mathbf{p}_c^T \mathbf{p}_c^- &= 0 \end{aligned}$$

$$\begin{aligned} C^{cb} &= 1 - \frac{\|G_c \mathbf{p}_c\|}{\|G \mathbf{p}_c\|} \\ &= 1 - \frac{\mathbf{p}_c^T G_c^T \Omega G_c \mathbf{p}_c}{\mathbf{p}_c^T G^T \Omega_s G \mathbf{p}_c} \\ &= \frac{\mathbf{p}_c^T \left(L - L_c\right) \mathbf{p}_c}{\mathbf{p}_c^T L \mathbf{p}_c} \end{aligned}$$



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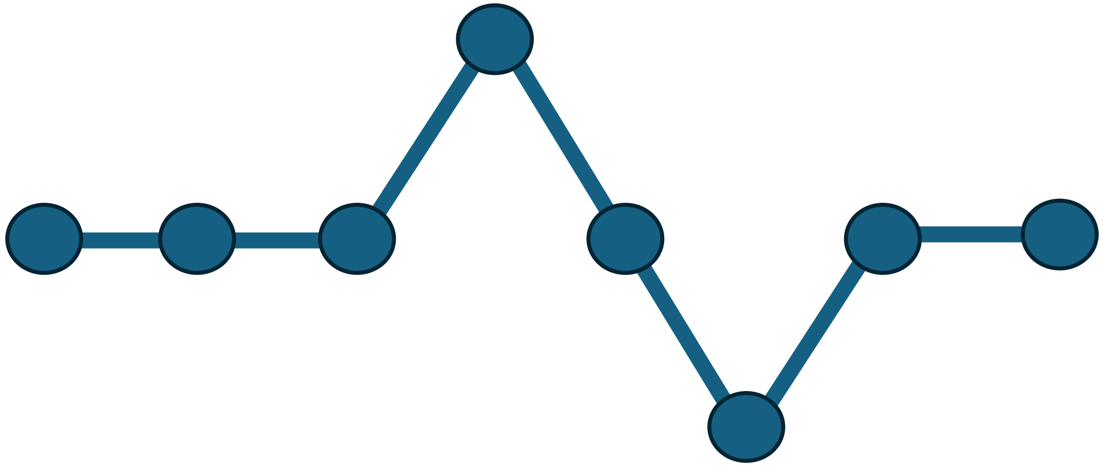
$$\|\alpha_d\| = \alpha_d^T \Omega_d \alpha_d$$



$$
\begin{aligned}
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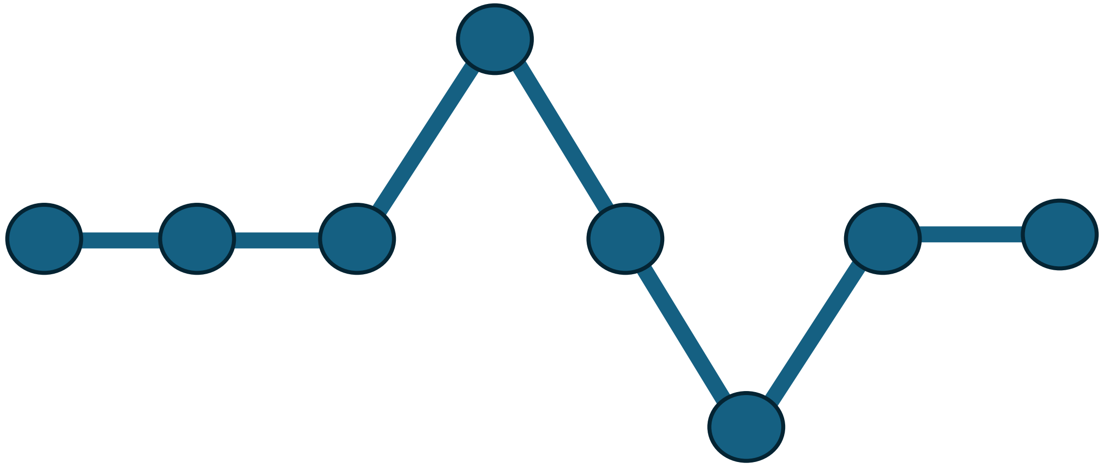
\end{aligned}
$$

$$\|\alpha_d\| = \alpha_d^T \Omega_d \alpha_d$$

$$L = MG = G^T \Omega_s G, \qquad L_c = M_c G_c = G_c^T \Omega G_c$$



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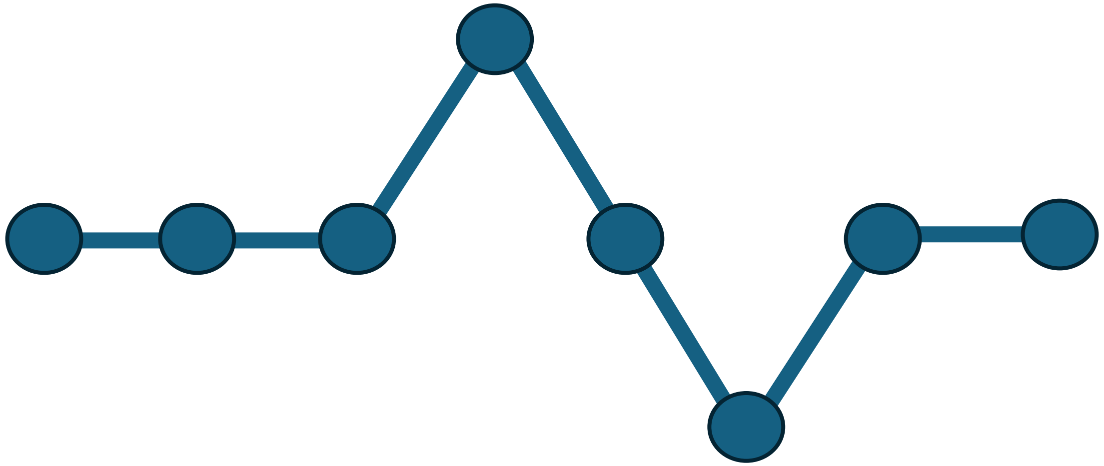
\end{aligned}
$$

$$
\begin{aligned}
G\mathbf{p}_c &= \begin{bmatrix}0 & 1 & \text{-}1 & \text{-}1 & 1 & 0\end{bmatrix}\\
G_c\mathbf{p}_c &= \begin{bmatrix}0 & \frac{1}{2} & 0 & \text{-}1 & 0 & \frac{1}{2}\end{bmatrix}
\end{aligned}
$$



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$$G\mathbf{p}_c = \begin{bmatrix} 0 & 1 & \text{-}1 & \text{-}1 & 1 & 0 \end{bmatrix}$$

$$G_c\mathbf{p}_c = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \text{-}1 & 0 & \frac{1}{2} \end{bmatrix}$$

$$C^{cb}\left(\mathbf{p}_c\right) = 1 - \frac{\|G_c \mathbf{p}_c\|}{\|G \mathbf{p}_c\|} = \frac{5}{8}$$

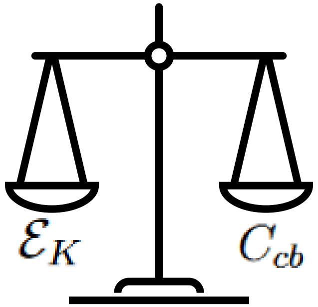
Global checkerboarding coefficient

Global scalar
Non-dimensionalised
Normalised [0, 1]
Able to detect local oscillations

Global checkerboarding coefficient

Global scalar
Non-dimensionalised
Normalised [0, 1]
Able to detect local oscillations

Usage:



$$\mathbf{p}_c^p = \theta_p \mathbf{p}_c^n$$

	θ_0	θ_1	θ_{dy}
θ_p	0	1	$1 - C^{cb}$

Global checkerboarding coefficient

Global scalar
Non-dimensionalised
Normalised [0, 1]
Able to detect local oscillations

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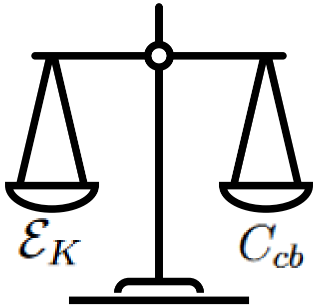

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	θ_0	θ_1	θ_{dy}
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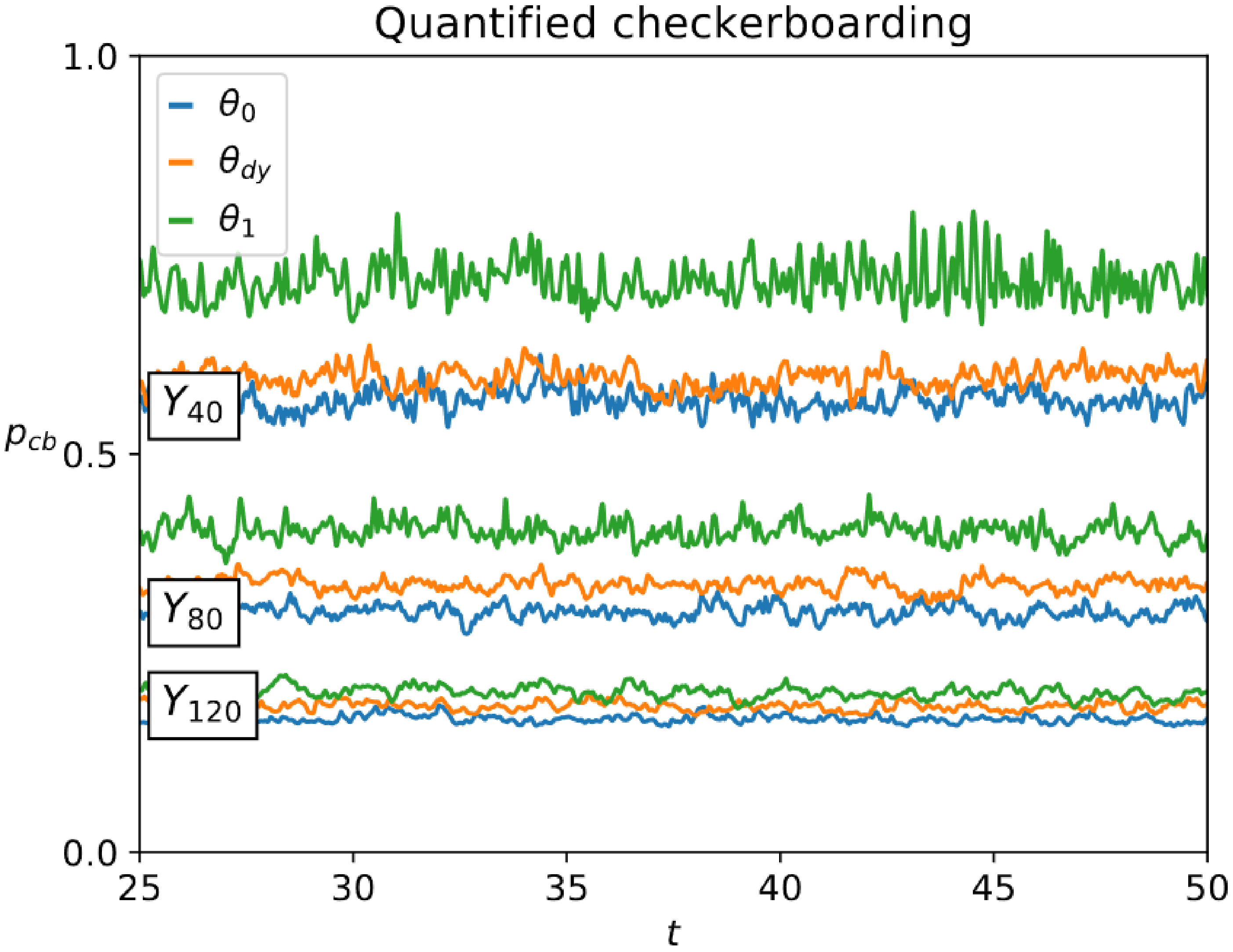
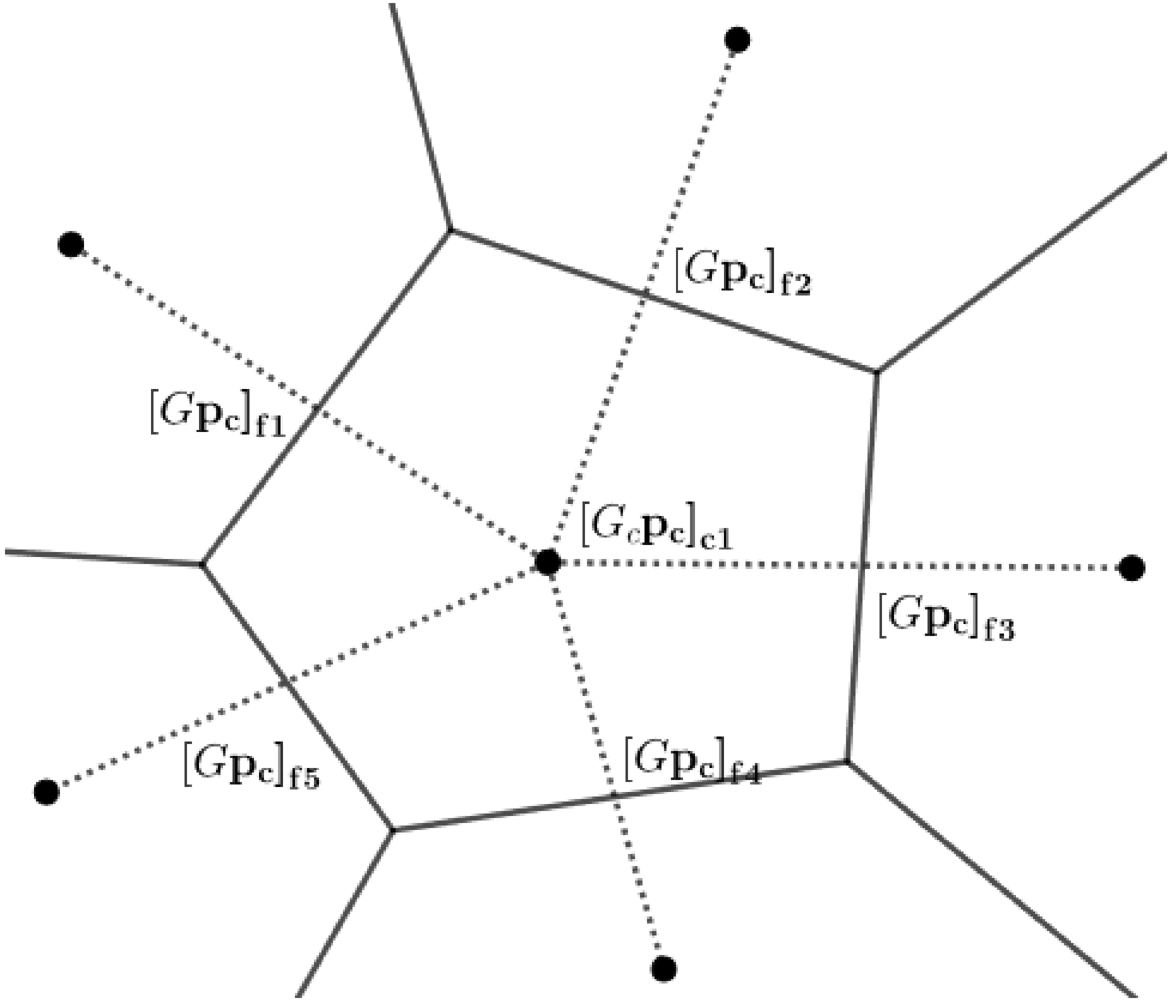


Local checkerboarding coefficient



$$[\gamma_c^{cb}]_i = \frac{[\Pi_{sc}||G\mathbf{p}_c||]_i}{[||G_c\mathbf{p}_c||]_i}$$

Hybrid Laplacian operator

$$\gamma_s^{cb} = \Pi_{cs}\gamma_c^{cb}$$
$$\gamma^{cb} = I_3 \otimes \gamma_c^{cb}$$

$$L_H = M_c(I - \gamma^{cb})G_c + M\gamma_s^{cb}G$$
$$G_H = \Gamma_{cs}(I - \gamma^{cb})G_c + \gamma_s^{cb}G$$
$$L_H = MG_H$$

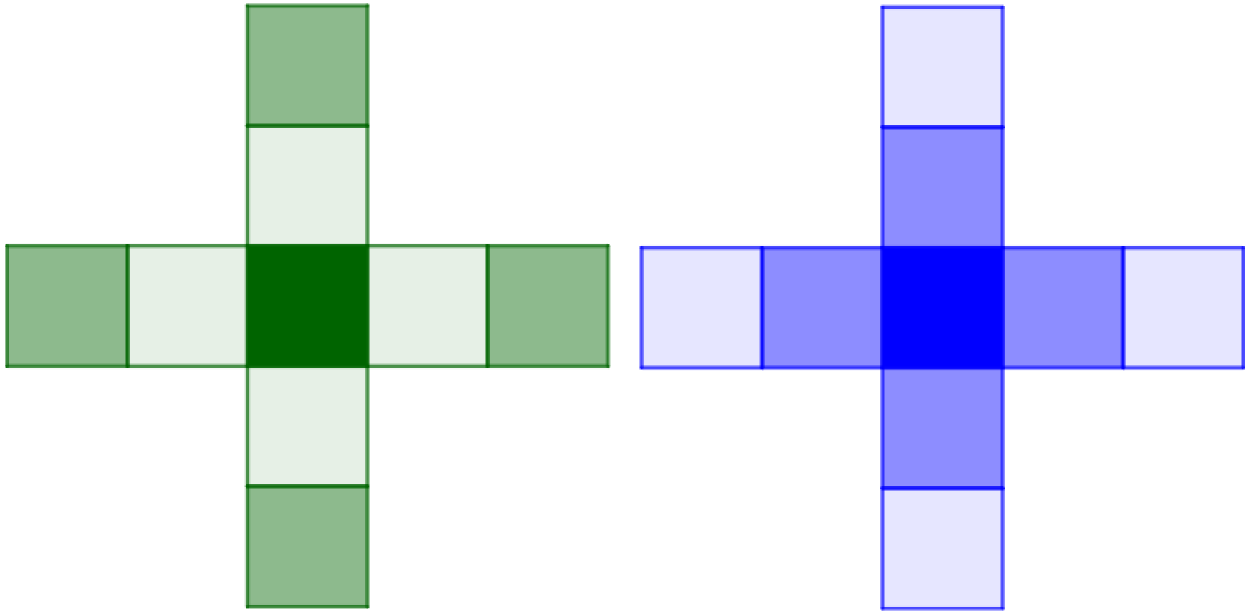
Global coefficient can be used similarly, by:

$$\gamma^{cb} = C^{cb}I$$

Hybrid Laplacian operator

$$\gamma_s^{cb} = \Pi_{cs}\gamma_c^{cb}$$
$$\gamma^{cb} = I_3 \otimes \gamma_c^{cb}$$

$$L_H = M_c(I - \gamma^{cb})G_c + M\gamma_s^{cb}G$$
$$G_H = \Gamma_{cs}(I - \gamma^{cb})G_c + \gamma_s^{cb}G$$
$$L_H = MG_H$$



$$[\gamma^{\mathbf{cb}}]_{\mathbf{i}} < [\gamma^{\mathbf{cb}}]_{\mathbf{j}}$$

Global coefficient can be used similarly, by:

$$\gamma^{cb} = C^{cb}I$$

Possible applications

$$
\begin{aligned}
\mathbf{u}_c^p &= \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right) \\
\mathbf{u}_c^{p*} &= \mathbf{u}_c^p - \gamma^{cb}[G_c\tilde{\mathbf{p}}_c]^n \\
L_H\tilde{\mathbf{p}}_c' &= M_c\mathbf{u}_c^{p*} \\
\mathbf{u}_s^{n+1} &= \Gamma_{cs}\mathbf{u}_c^{p*} - G_H\tilde{\mathbf{p}}_c' \\
\mathbf{u}_c^{n+1} &= \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c' \\
[G_c\tilde{\mathbf{p}}_c]^{n+1} &= \gamma^{cb}[G_c\tilde{\mathbf{p}}_c]^n + G_c\tilde{\mathbf{p}}_c'
\end{aligned}
$$

Possible applications

$$\mathbf{u}_c^p = \mathcal{F}\left(\mathbf{u}_c, \mathbf{u}_s\right)$$

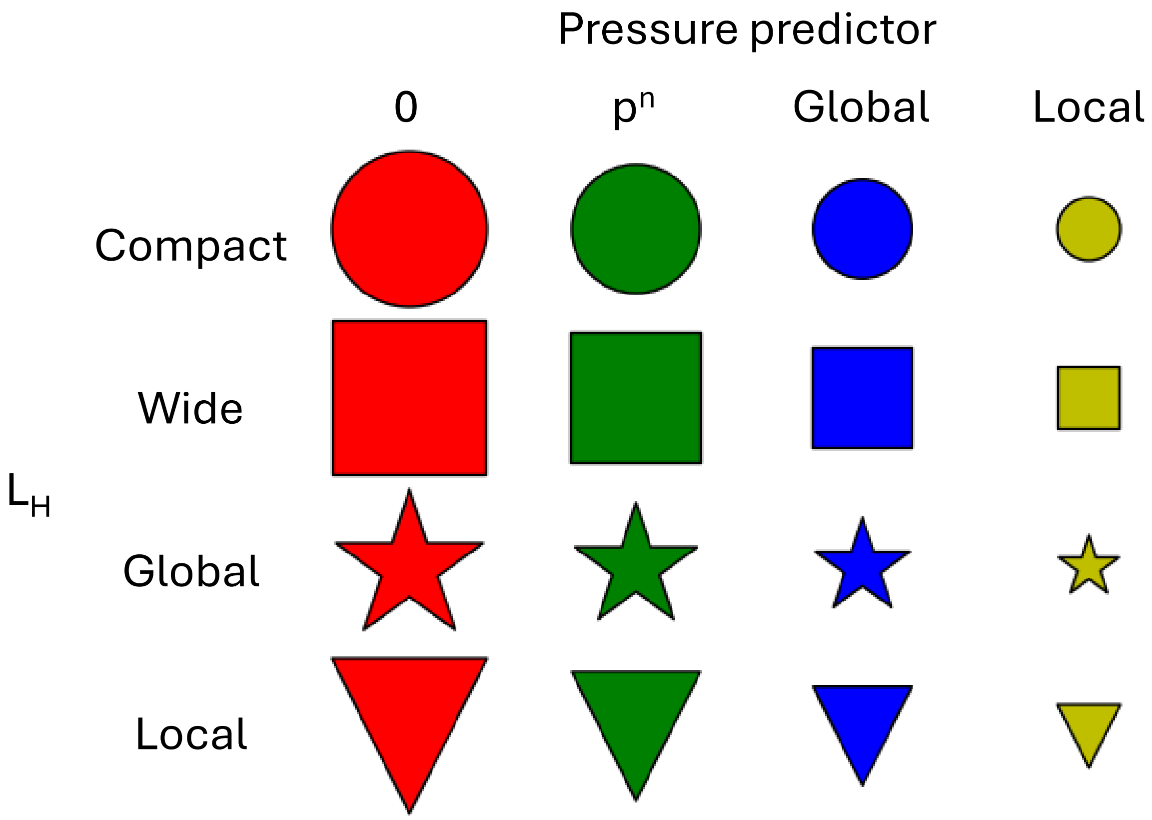
$$\mathbf{u}_c^{p*} = \mathbf{u}_c^p - \gamma^{cb}[G_c\tilde{\mathbf{p}}_c]^n$$

$$L_H\tilde{\mathbf{p}}_c' = M_c\mathbf{u}_c^{p*}$$

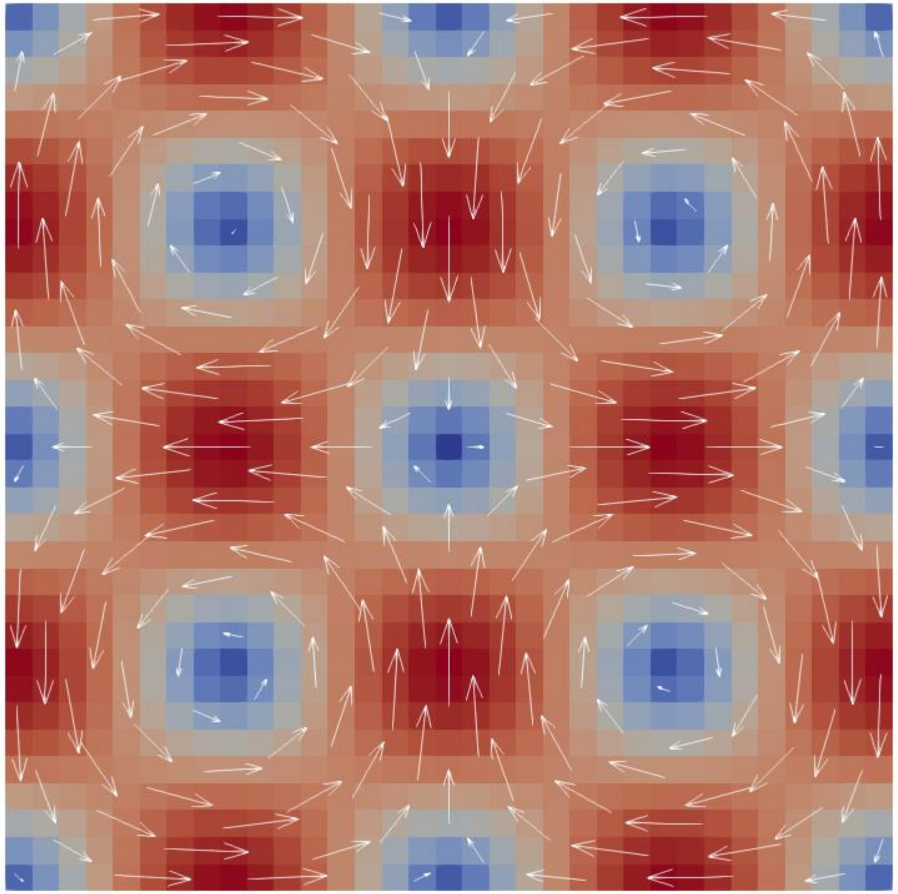
$$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^{p*} - G_H\tilde{\mathbf{p}}_c'$$

$$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c'$$

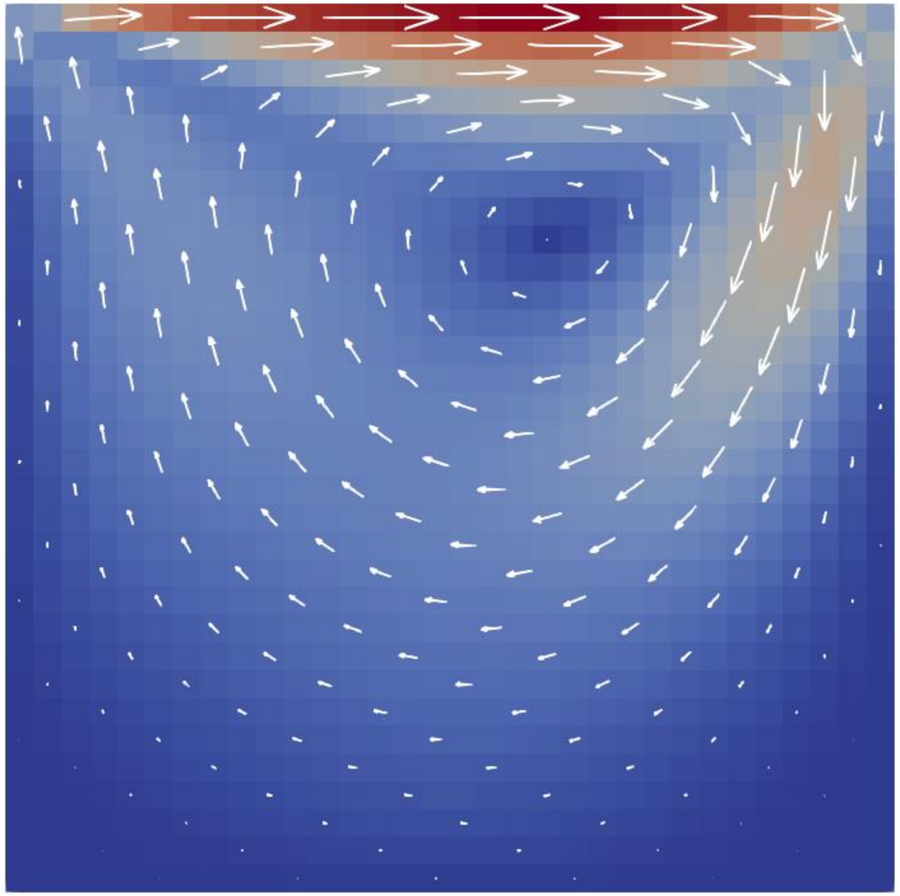
$$[G_c\tilde{\mathbf{p}}_c]^{n+1} = \gamma^{cb}[G_c\tilde{\mathbf{p}}_c]^n + G_c\tilde{\mathbf{p}}_c'$$



Test cases

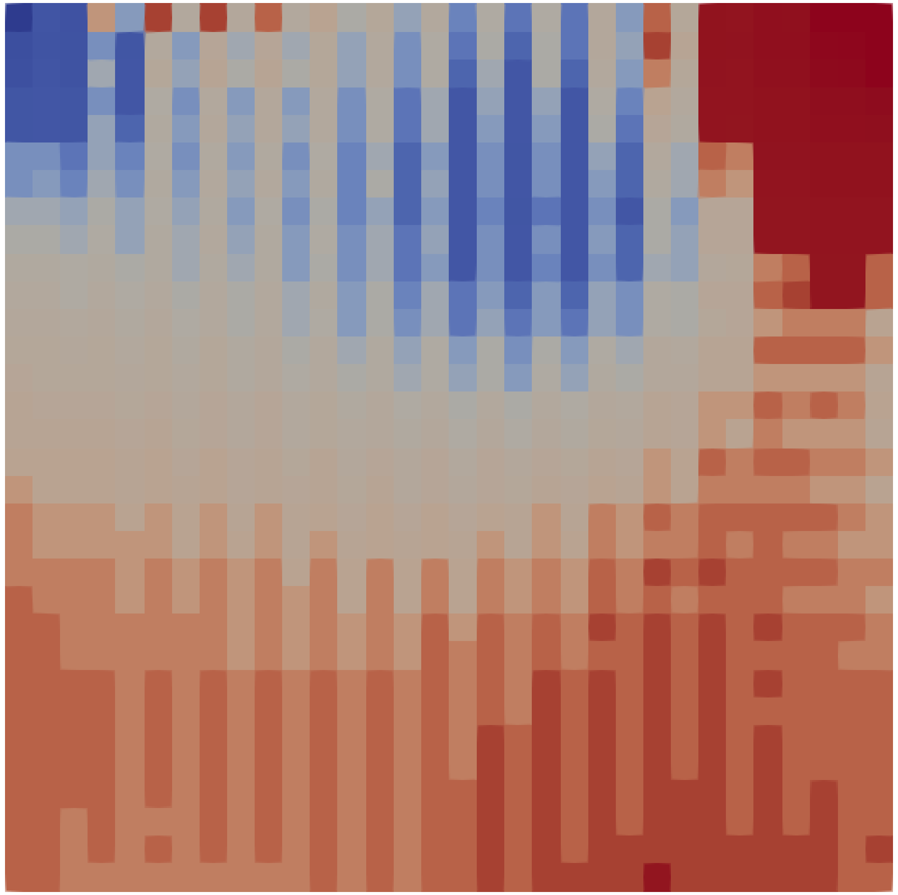


Taylor-Green vortex

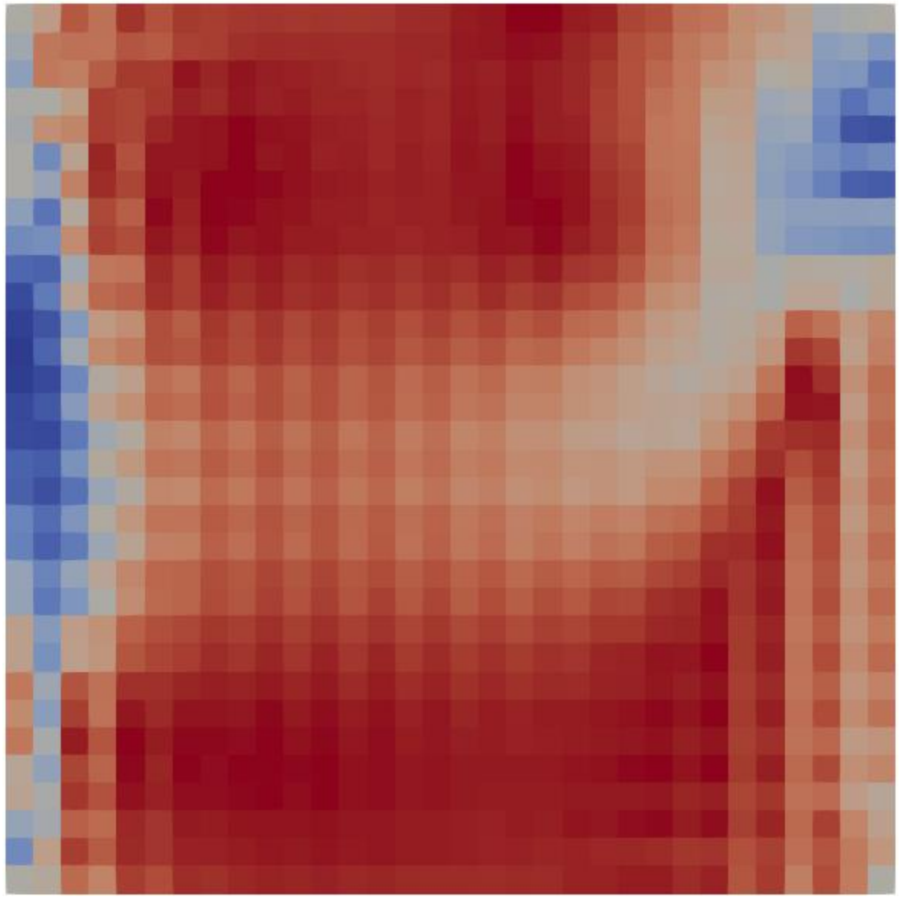


Lid-driven cavity

Local checkerboard coefficient

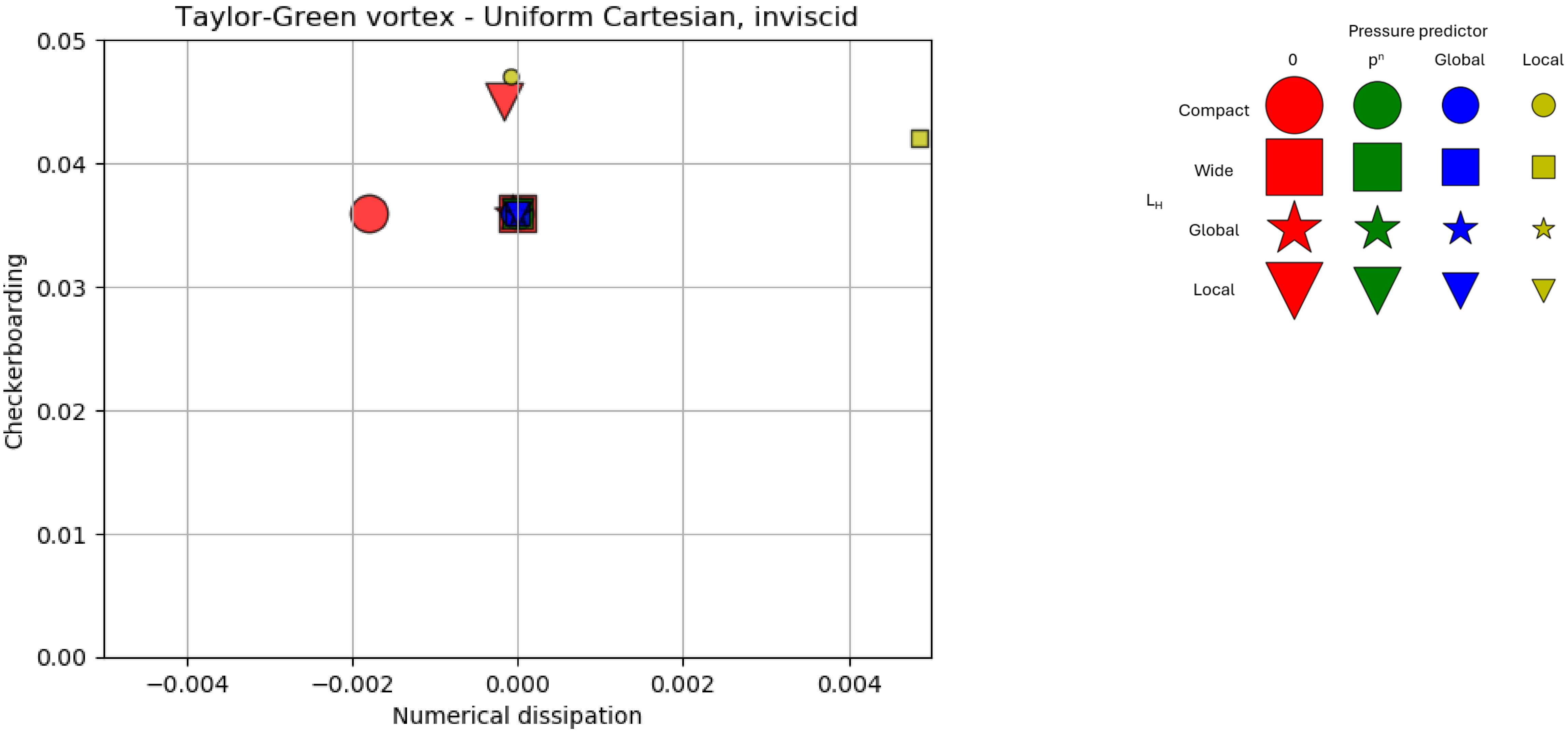


Lid-driven cavity pressure

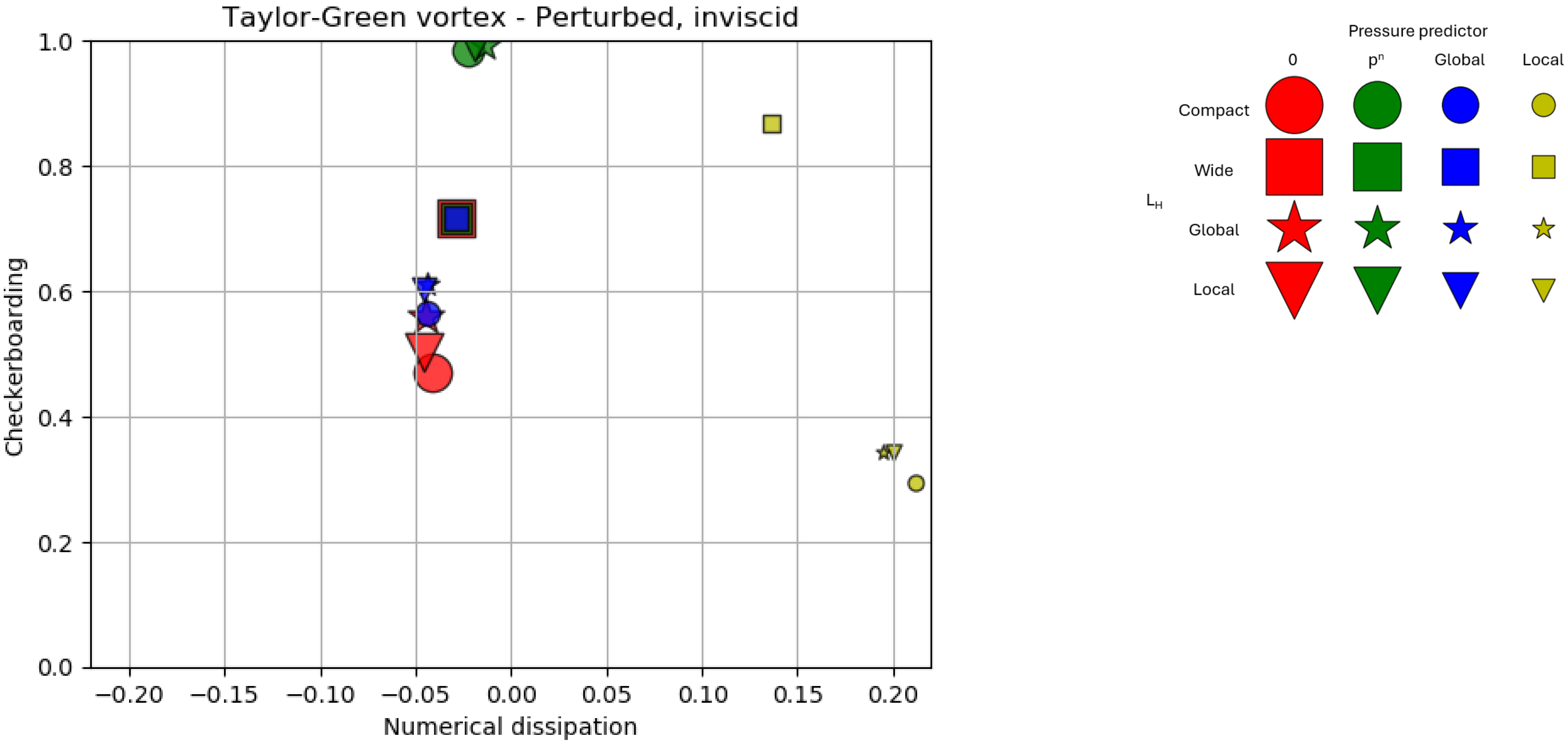


γ_c^{cb}

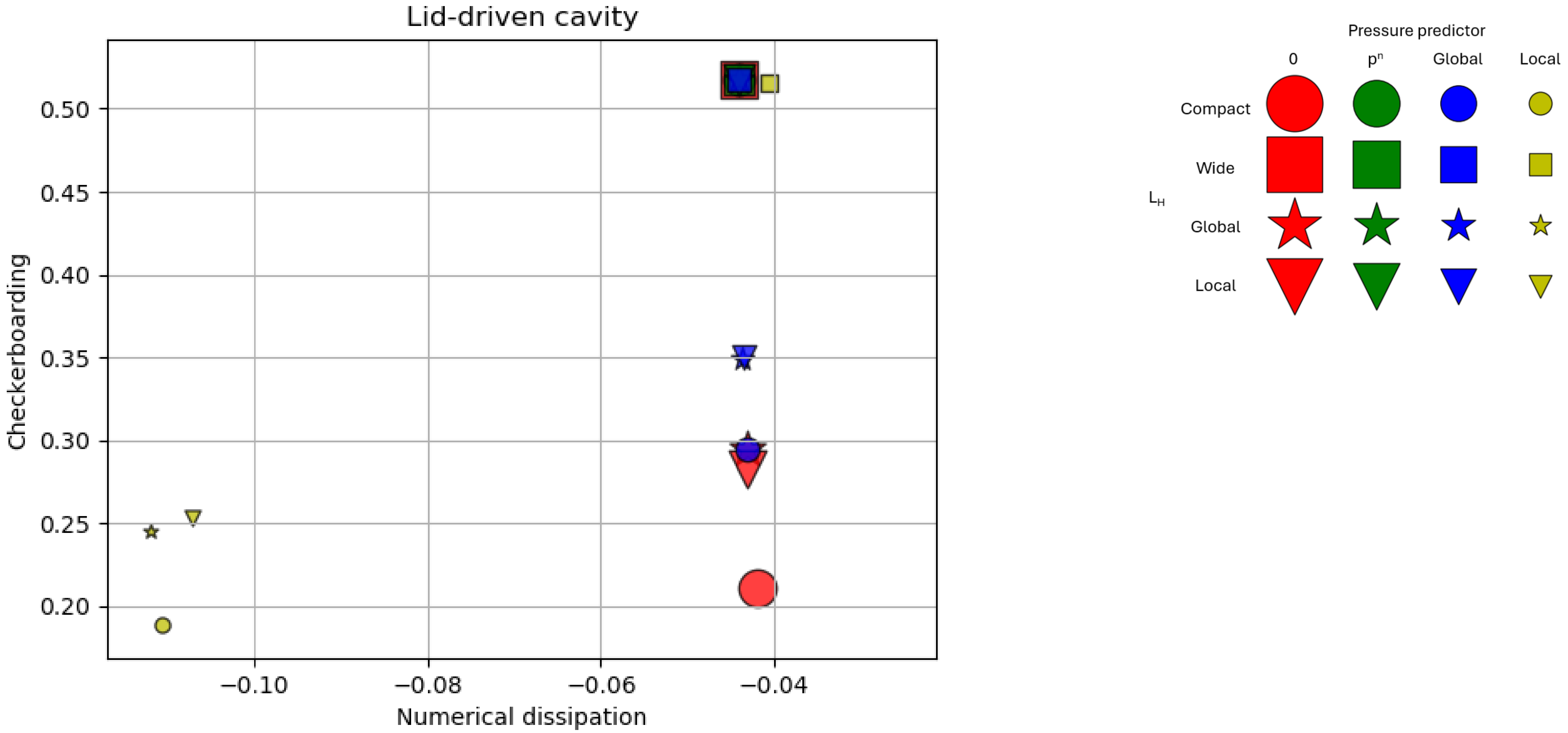
Results – Taylor-Green vortex



Results – Taylor-Green vortex



Results – Lid-driven cavity



Conclusions

Local quantification method developed
Different possible usages demonstrated
Practical applications seem limited

- Hybrid L is costly
- Local p^p seems erratic

Thanks for attending!

Any questions?