14th World Congress on Computational Mechanics (WCCM)
ECCOMAS Congress 2020
19–24 July 2020, Paris, France

A GENERAL METHOD TO COMPUTE NUMERICAL DISPERSION ERROR

Jesus Ruano[1]*, Aleix Baez Vidal[1], Joaquim Rigola[1] and F.X. Trias[1]

[1] Heat and Mass Transfer Technological Center, Technical University of Catalonia
c/Colom 11, 08222 Terrassa (Barcelona)
jesusr@cttc.upc.edu, aleix@cttc.upc.edu, quim@cttc.upc.edu, xavi@cttc.upc.edu

Key Words: Dispersion error, Diffusion error, Structured/Unstructured grids, High-order methods

Numerical errors in finite differences (FD) of finite volumes (FV) are inherent to the discretization process. Diffusion error appears when non-linear or upwinded schemes are used. Aliasing error appears due to the fact the resulting non-linearity of multiplying modes, which produces modes higher than the cut-off frequency. Finally, dispersion error appears due to the truncation of the numerical derivative when a discrete scheme is used, independently of the differential scheme. Regarding this last error, the classical approach of Tam and Webb [1] and Lele [2] has been used to employ high-order schemes to reduce its effects. However, this approach cannot handle neither non-linear schemes nor unstructured meshes or non-uniform ones. More recent approaches, such as Pirozzoli [3] or Fauconnier and Dick [4], have been able to study the dispersive characteristics of non-linear schemes; however, meshes remained uniform. Thus, a more general approach that can handle both schemes non-linearity as well as non-regular meshes would be interesting to study how the combination of differential scheme and mesh affects dispersion.

In this context, this article presents a new methodology able to compute dispersion characteristics. This method does not require neither uniform mesh spacing nor linear discrete operators. From a practical point of view, the methodology allows the numerical study of dispersion errors in all kind of meshes, either non-uniform or unstructured. When evenly spaced one-dimensional meshes are used, the results obtained with this method coincide with the results when a classical approach is used. In this work, we present both the theoretical derivation as well as the details of the numerical methodology. We include a set of results with different meshes in combination with different numerical schemes, considering both linear and non-linear schemes and low and high-order schemes.

REFERENCES

[1] Tam, C. K. W. and Webb, J. C. *Dispersion-relation-preserving finite difference schemes for Computational acoustics*. Journal of Computational Physics (1993) 262–281.

[2] Lele, S. *Compact finite difference schemes with spectral-like resolution*. Journal of Computational Physics (1992).

[3] Pirozzoli, S. *On the spectral properties of shock-capturing schemes*. Journal of Computational Physics (2006) **219**(2):489–497.

[4] Fauconnier, D. and Dick, E. *On the spectral and conservation properties of nonlinear discretization operators*. Journal of Computational Physics (2011) **230**(12):4488–4518.