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ON THE INTERPOLATION PROBLEM FOR THE POISSON EQUATION ON COLLOCATED MESHES

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Finite-volume collocated discretizations on unstructured meshes is the solution adopted for most of the general-purpose CFD codes such as ANSYS-FLUENT, OpenFOAM, etc. Despite the intrinsic errors due to the improper pressure gradient formulation, this approach is usually preferred over a staggered one due to its simple form. In this context, a fully-conservative discretization method for general unstructured grids was proposed in [1]: it exactly preserves the symmetries of the underlying differential operators on a collocated mesh. Likewise other collocated codes, to suppress the well-known checkerboard problem the Poisson equation is solved using a compact stencil. Using the same notation than in [1], this reads

$$\mathsf{L} p_c = \mathsf{M} u_s^p \tag{1}$$

where $\mathsf{L} = -\mathsf{M}\Omega_s^{-1}\mathsf{M}^T$ is the Laplacian operator, p_c is the cell-centered pressure field, u_s^p is a face-normal velocity and Ω_s is a diagonal matrix that contains the staggered control volumes. For staggered velocity fields, the projection onto a divergence-free space is a well-posed problem. This is not the case for collocated velocity fields. Namely, cell-centered velocity field, u_c^p, needs to be interpolated to the faces, $u_s^p = \Gamma_{c\to s} u_c^p$ using a cell-to-face interpolation, $\Gamma_{s\to c}$. Then, the staggered gradient, $\mathsf{G} p_c = -\Omega_s^{-1}\mathsf{M}^T$, of the pressure field obtained by solving Eq.(1) must be interpolated back to the cells. Namely, the overall procedure can be compactly written as follows

$$u_c^{n+1} = (\mathsf{I} + \Omega_c^{-1}\Gamma_{c\to s}^T \mathsf{M}^T \mathsf{L}^{-1} \mathsf{M} \Gamma_{c\to s}) u_c^p. \tag{2}$$

The new cell-centered velocity field will not be exactly incompressible, $\mathsf{M}\Gamma_{c\to s} u_c^{n+1} =\approx 0_c$, and the overall procedure will inevitable introduce some artificial dissipation. Apart from this well-known drawbacks of using collocated formulations, instability issues may also appear for highly distorted meshes. Namely, let us consider that we recursively apply the pseudo-projection given in Eq.(2). Then, we obtain

$$\mathsf{L} p_c^{n+1} = \mathsf{M} u_s^p + (\mathsf{L} - \mathsf{L}_c) p_c^n, \tag{3}$$

where $\mathsf{L}_c \equiv -\mathsf{M}\Gamma_{c\to s}\Omega_c^{-1}\Gamma_{c\to s}^T \mathsf{M}^T$ is the non-compact Laplacian operator. This can be viewed like a stationary iterative solver. The stability of this process will depend on the eigenvalues of $(\mathsf{L} - \mathsf{L}_c)$, which subsequently depend on the interpolation operators. This will be carefully analysed and results for general unstructured grids will be presented.

REFERENCES

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