Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

JOHANNES KEPLER UNIVERSITY LINZ

On a proper tensorial subgrid heat flux model

F.Xavier Trias[1], Firas Dabbagh[2], Daniel Santos[1], Andrey Gorobets[3], Assensi Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia
[2]Christian-Doppler Laboratory for Multi-scale Modelling of Multiphase Processes, Johannes Kepler University, Linz, Austria
[3]Keldysh Institute of Applied Mathematics of RAS, Russia

IACM and ECCOMAS are pleased to announce the joint organization of

14th. World Congress on Computational Mechanics (WCCM XIV) and 8th. European Congress on Computational Methods in Applie Sciences and Engineering (ECCOMAS 2020)

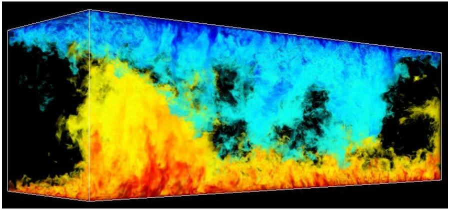
Virtual Congress 11-15 January, 2021

Contents

Motivation

Research question:

- Can we find a nonlinear SGS heat flux model with **good physical** and **numerical properties**, such that we can obtain satisfactory predictions for a turbulent Rayleigh-Bénard convection?



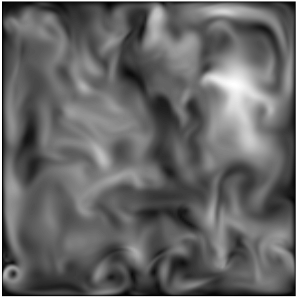
DNS of an air-filled Rayleigh-Bénard convection at $Ra = 10^{10}$

[1] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *On the evolution of flow topology in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 28:115105, 2016.

Motivation

Air-filled RB: $Pr = 0.7$

$Ra = 10^8$

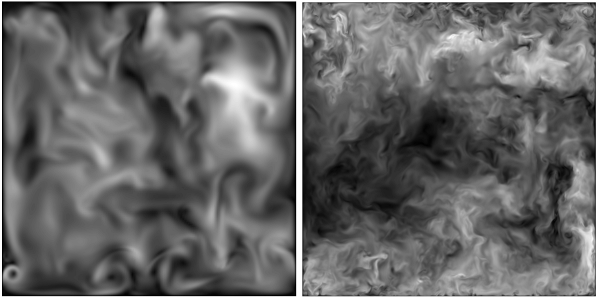


[2] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **Phys.Rev.Fluids**, 5:024603, 2020.

Motivation

Air-filled RB: $Pr = 0.7$

$Ra = 10^8$ $Ra = 10^{10}$

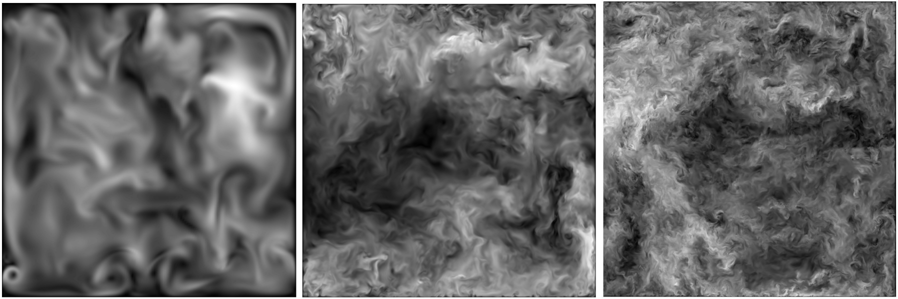


[2] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **Phys.Rev.Fluids**, 5:024603, 2020.

Motivation

Air-filled RB: $Pr = 0.7$

$Ra = 10^8$ $Ra = 10^{10}$ $Ra = 10^{11}$

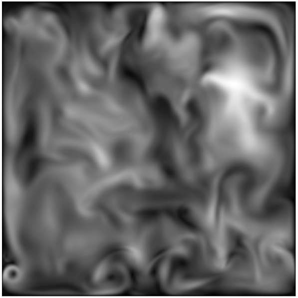


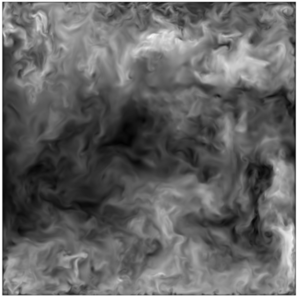
[2] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **Phys.Rev.Fluids**, 5:024603, 2020.

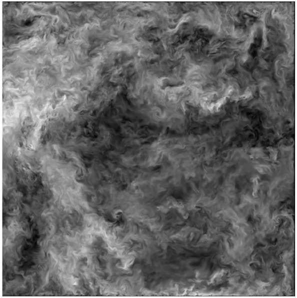
Motivation

Air-filled RB: $Pr = 0.7$

$Ra = 10^8$ | $Ra = 10^{10}$ | $Ra = 10^{11}$





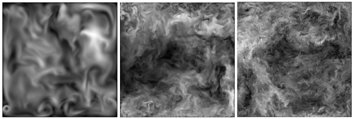


$208 \times 208 \times 400$ **17.5M** | $768 \times 768 \times 1024$ **607M** | $1662 \times 1662 \times 2048$ **5600M**

[2] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **Phys.Rev.Fluids**, 5:024603, 2020.

Motivation

DNS: $208 \times 208 \times 400$

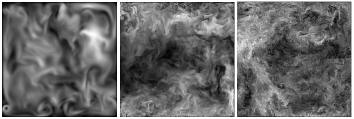


Motivation

DNS: $208 \times 208 \times 400$

$\boxed{\text{LES:} \quad 80 \times \quad 80 \times 120}$

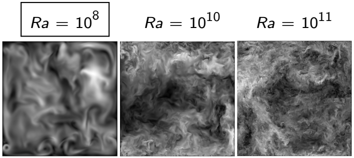
$\boxed{Ra = 10^8}$ $\quad Ra = 10^{10} \quad Ra = 10^{11}$

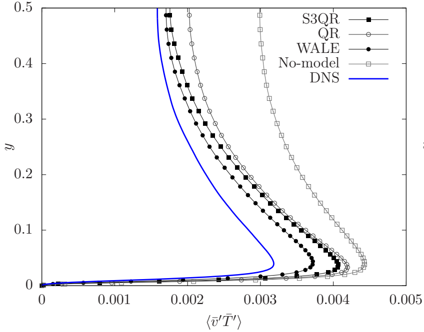


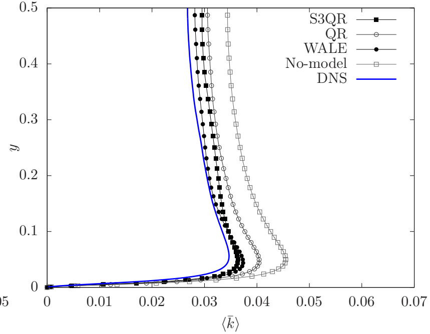
Motivation

DNS: 208 × 208 × 400

$\boxed{\text{LES: } 80 \times 80 \times 120}$



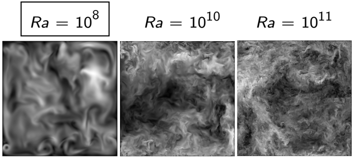


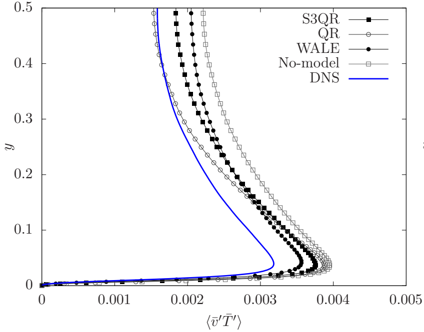


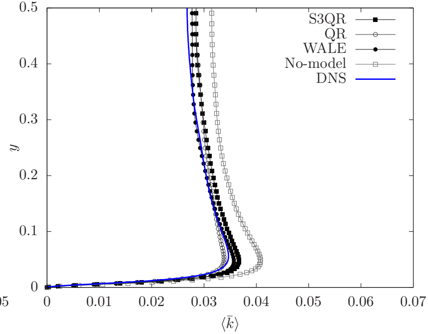
Motivation

DNS: $208 \times 208 \times 400$

$\boxed{\text{LES: } 110 \times 110 \times 168}$







How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} \qquad - \nabla \cdot \tau(\overline{u}) \;\; ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \;\longrightarrow\; \tau\;(\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u}\cdot\nabla)\overline{u} = \nu\nabla^2\overline{u} - \nabla\overline{p} + \overline{f} - \nabla\cdot\tau(\overline{u}) \quad ; \qquad \nabla\cdot\overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m\delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u}\cdot\nabla)\overline{T} = \alpha\nabla^2\overline{T} - \nabla\cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{eddy-diffusivity} \quad \longrightarrow \quad q \approx -\alpha_t\nabla\overline{T}$$

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

eddy-viscosity $\longrightarrow$ $\tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

eddy-diffusivity $\longrightarrow$ $q \approx -\alpha_t \nabla \overline{T}$

$$\boxed{\alpha_t = \frac{\nu_t}{Pr_t}}$$

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u}\cdot\nabla)\overline{u} = \nu\nabla^2\overline{u} - \nabla\overline{p} + \overline{f} - \nabla\cdot\tau(\overline{u}) \quad ; \qquad \nabla\cdot\overline{u} = 0$$

eddy-viscosity $\longrightarrow$ $\tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m\delta)^2 D_m(\overline{u})}$$

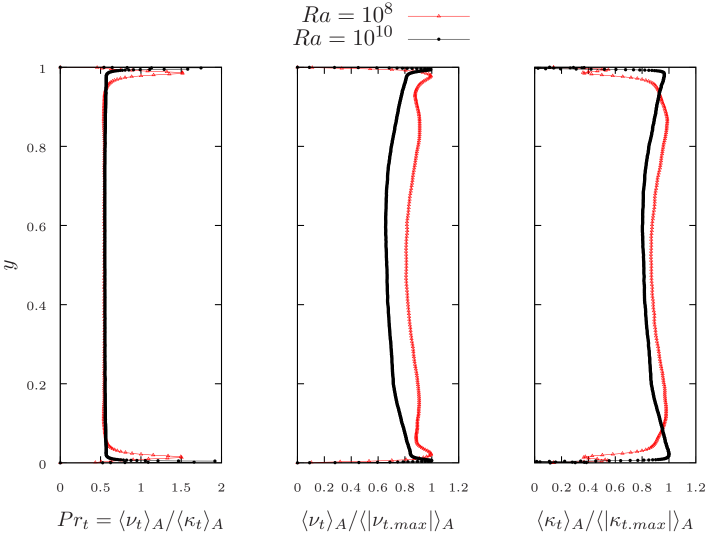
$$\partial_t \overline{T} + (\overline{u}\cdot\nabla)\overline{T} = \alpha\nabla^2\overline{T} - \nabla\cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

eddy-diffusivity $\longrightarrow$ $q \approx -\alpha_t \nabla\overline{T}$

$$\boxed{\alpha_t = \frac{\nu_t}{Pr_t}}$$

Pr_t?

How to model the subgrid heat flux in LES?



How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

eddy-viscosity $\longrightarrow$ $\tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

eddy-diffusivity $\longrightarrow$ $q \approx -\alpha_t \nabla \overline{T}$

$$\boxed{\xcancel{\alpha_t = \frac{\nu_t}{Pr_t}}}$$

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{eddy-diffusivity} \quad \longrightarrow \quad q \approx -\alpha_t \nabla \overline{T} \qquad (\equiv q^{eddy})$$

$$\text{gradient model} \quad \longrightarrow \quad q \approx -\frac{\delta^2}{12} G \nabla \overline{T} \qquad (\equiv q^{nl})$$

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u}\cdot\nabla)\overline{u} = \nu\nabla^2\overline{u} - \nabla\overline{p} + \overline{f} - \nabla\cdot\tau(\overline{u}) \quad ; \qquad \nabla\cdot\overline{u} = 0$$

eddy-viscosity $\longrightarrow$ $\tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m\delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u}\cdot\nabla)\overline{T} = \alpha\nabla^2\overline{T} - \nabla\cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

eddy-diffusivity $\longrightarrow$ $q \approx -\alpha_t\nabla\overline{T} \qquad (\equiv q^{eddy})$

gradient model $\longrightarrow$ $q \approx -\frac{\delta^2}{12}G\nabla\overline{T} \qquad (\equiv q^{nl})$

$$G \equiv \nabla\overline{u} \qquad\qquad q = -\frac{\delta^2}{12}G\nabla\overline{T} + \mathcal{O}(\delta^4)$$

A priori alignment trends[3]

$$\text{eddy-diffusivity} \longrightarrow \quad q \approx -\alpha_t \nabla \overline{T} \qquad (\equiv q^{eddy})$$

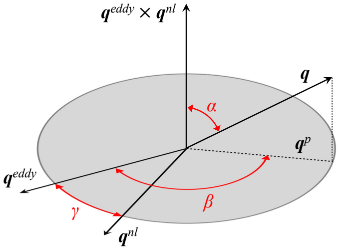
$$\text{gradient model} \longrightarrow \quad q \approx -\frac{\delta^2}{12} G \nabla \overline{T} \qquad (\equiv q^{nl})$$

[3] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *A priori study of subgrid-scale features in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 29:105103, 2017.

A priori alignment trends[3]

$$\text{eddy-diffusivity} \longrightarrow q \approx -\alpha_t \nabla \overline{T} \qquad (\equiv q^{eddy})$$

$$\text{gradient model} \longrightarrow q \approx -\frac{\delta^2}{12} G \nabla \overline{T} \qquad (\equiv q^{nl})$$

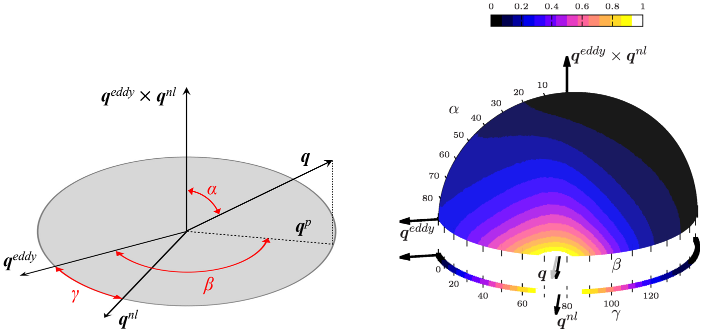


[3] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *A priori study of subgrid-scale features in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 29:105103, 2017.

A priori alignment trends[3]

eddy-diffusivity $\longrightarrow$ $q \approx -\alpha_t \nabla \overline{T}$ $(\equiv q^{eddy})$

gradient model $\longrightarrow$ $q \approx -\dfrac{\delta^2}{12} G \nabla \overline{T}$ $(\equiv q^{nl})$



[3] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *A priori study of subgrid-scale features in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 29:105103, 2017.

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u}\cdot\nabla)\overline{u} = \nu\nabla^2\overline{u} - \nabla\overline{p} + \overline{f} - \nabla\cdot\tau(\overline{u}) \quad ; \qquad \nabla\cdot\overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\;(\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m\delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u}\cdot\nabla)\overline{T} = \alpha\nabla^2\overline{T} - \nabla\cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{eddy-diffusivity} \quad \longrightarrow \quad \cancel{q \approx -\alpha_t\nabla\overline{T}} \qquad (\equiv q^{eddy})$$

$$\text{gradient model} \quad \longrightarrow \quad q \approx -\frac{\delta^2}{12}G\nabla\overline{T} \qquad (\equiv q^{nl})$$

[4] S.Peng and L.Davidson. **Int.J.Heat Mass Transfer**, 45:1393-1405, 2002.

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u}\cdot\nabla)\overline{u} = \nu\nabla^2\overline{u} - \nabla\overline{p} + \overline{f} - \nabla\cdot\tau(\overline{u}) \quad ; \qquad \nabla\cdot\overline{u} = 0$$

$$\text{eddy-viscosity} \longrightarrow \tau(\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m\delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u}\cdot\nabla)\overline{T} = \alpha\nabla^2\overline{T} - \nabla\cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{eddy-diffusivity} \longrightarrow \cancel{q \approx -\alpha_t\nabla\overline{T}} \qquad (\equiv q^{eddy})$$

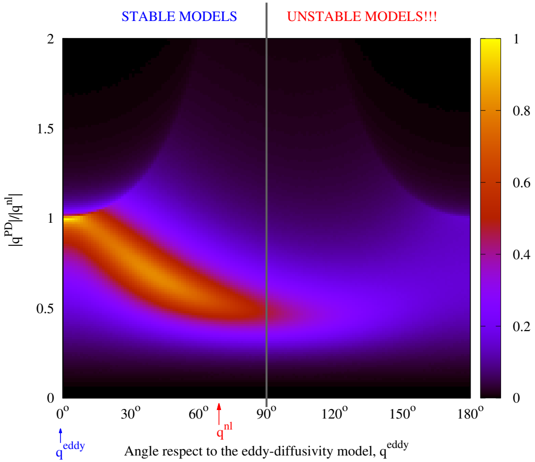
$$\text{gradient model} \longrightarrow q \approx -\frac{\delta^2}{12}G\nabla\overline{T} \qquad (\equiv q^{nl})$$

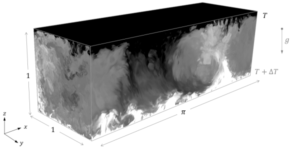
$$\text{Peng\&Davidson}^4 \longrightarrow q \approx -\frac{\delta^2}{12}S\nabla\overline{T} \qquad (\equiv q^{PD})$$

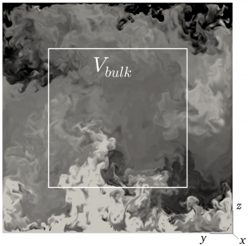
[4] S.Peng and L.Davidson. **Int.J.Heat Mass Transfer**, 45:1393-1405, 2002.

A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T} \qquad \boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$







How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{eddy-diffusivity} \quad \longrightarrow \quad \cancel{q \approx -\alpha_t \nabla \overline{T}} \qquad (\equiv q^{eddy})$$

$$\text{gradient model} \quad \longrightarrow \quad \cancel{q \approx -\frac{\delta^2}{12} G \nabla \overline{T}} \qquad (\equiv q^{nl})$$

$$\text{Peng\&Davidson} \quad \longrightarrow \quad \cancel{q \approx -\frac{\delta^2}{12} S \nabla \overline{T}} \qquad (\equiv q^{PD})$$

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

eddy-viscosity $\longrightarrow$ $\tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

mixed model $\longrightarrow$ $q \approx q^{nl} + \sigma q^{eddy} \qquad (\equiv q^{mix})$

[5] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{mixed model} \quad \longrightarrow \quad q \approx q^{nl} + \sigma q^{eddy} \qquad (\equiv q^{mix})$$

$$\text{Daly\&Harlow}^5 \quad \longrightarrow \quad q \approx -\mathcal{T}_{SGS} \frac{\delta^2}{12} G G^T \nabla \overline{T} \qquad (\equiv q^{DH})$$

[5] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u}\cdot\nabla)\overline{u} = \nu\nabla^2\overline{u} - \nabla\overline{p} + \overline{f} - \nabla\cdot\tau(\overline{u}) \quad ; \qquad \nabla\cdot\overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m\delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u}\cdot\nabla)\overline{T} = \alpha\nabla^2\overline{T} - \nabla\cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{mixed model} \quad \longrightarrow \quad q \approx q^{nl} + \sigma q^{eddy} \qquad (\equiv q^{mix})$$

$$\text{Daly\&Harlow}^5 \quad \longrightarrow \quad q \approx -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T} \qquad (\equiv q^{DH})$$

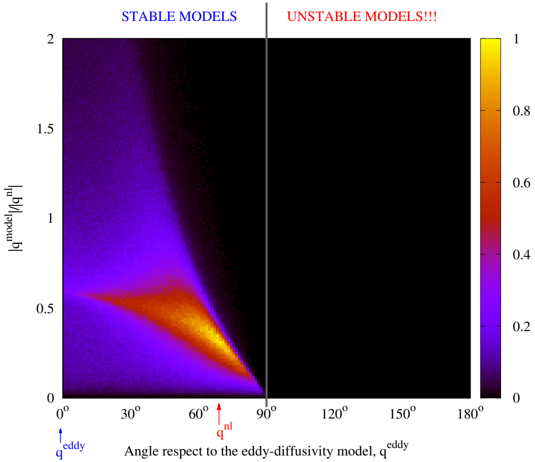
$$\mathcal{T}_{SGS} = 1/|S|$$

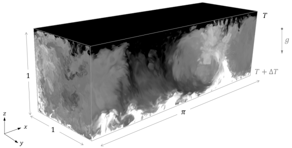
[5] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

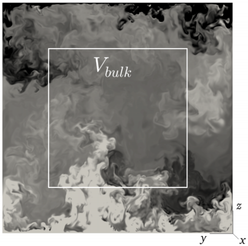
A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS} \frac{\delta^2}{12} G G^T \nabla \overline{T}}$$

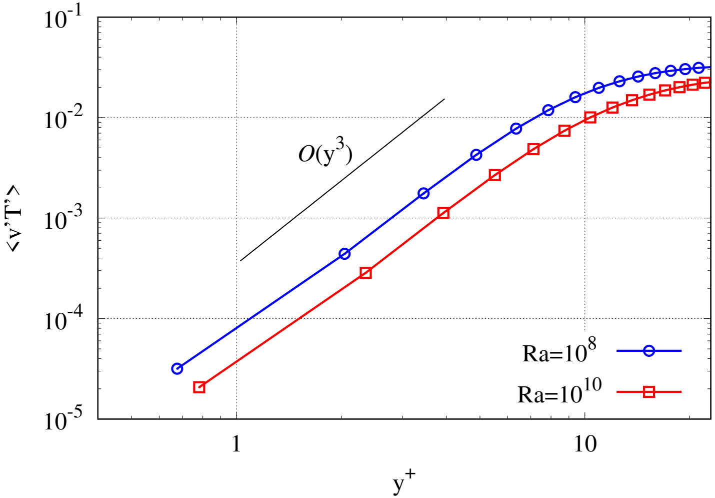






What about near-wall scaling?

What about near-wall scaling?
$\Longrightarrow$ Answer: it should be $\mathcal{O}(y^3)$



Near-wall scaling for DH model?

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\,; \quad \mathcal{T}_{SGS} = 1/|S|}$$

Near-wall scaling for DH model? $\Longrightarrow$ Answer: it is $\mathcal{O}(y^1)$ instead of $\mathcal{O}(y^3)$

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\;; \qquad \mathcal{T}_{SGS} = 1/|S|}$$

$$u = ay + \mathcal{O}(y^2);\;\; v = by^2 + \mathcal{O}(y^3);\;\; w = cy + \mathcal{O}(y^2);\;\; T = dy + \mathcal{O}(y^2)$$

$$G = \begin{pmatrix} y & 1 & y \\ y^2 & y & y^2 \\ y & 1 & y \end{pmatrix};\; \nabla\overline{T} = \begin{pmatrix} y \\ 1 \\ y \end{pmatrix} \Longrightarrow \boxed{GG^T\nabla\overline{T} = \begin{pmatrix} y \\ y^2 \\ y \end{pmatrix} = \mathcal{O}(y^1)}$$

$$\boxed{\mathcal{T}_{SGS} = 1/|S| = \mathcal{O}(y^0)}$$

Near-wall scaling for DH model?

$\Longrightarrow$ Answer: it is $\mathcal{O}(y^1)$ instead of $\mathcal{O}(y^3)$

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\ ; \qquad \mathcal{T}_{SGS} = 1/|S|}$$

$$u = ay + \mathcal{O}(y^2);\quad v = by^2 + \mathcal{O}(y^3);\quad w = cy + \mathcal{O}(y^2);\quad T = dy + \mathcal{O}(y^2)$$

$$G = \begin{pmatrix} y & 1 & y \\ y^2 & y & y^2 \\ y & 1 & y \end{pmatrix};\ \nabla\overline{T} = \begin{pmatrix} y \\ 1 \\ y \end{pmatrix} \Longrightarrow \boxed{GG^T\nabla\overline{T} = \begin{pmatrix} y \\ y^2 \\ y \end{pmatrix} = \mathcal{O}(y^1)}$$

$$\boxed{\mathcal{T}_{SGS} = 1/|S| = \mathcal{O}(y^0)}$$

Idea: build a $\mathcal{T}_{SGS}$ with the proper $\mathcal{O}(y^2)$ scaling!!!

Building proper models for the subgrid heat flux

Let us consider models that are based on the invariants of the tensor GG^T

$$q \approx -C_M \left(P_{GG^T}^{p} Q_{GG^T}^{q} R_{GG^T}^{r} \right) \frac{\delta^2}{12} GG^T \nabla \overline{T} \qquad (\equiv q^{S2})$$

Building proper models for the subgrid heat flux

Let us consider models that are based on the invariants of the tensor GG^T

$$q \approx -C_M \left(P_{GG^T}^p Q_{GG^T}^q R_{GG^T}^r \right) \frac{\delta^2}{12} GG^T \nabla \overline{T} \qquad (\equiv q^{S2})$$

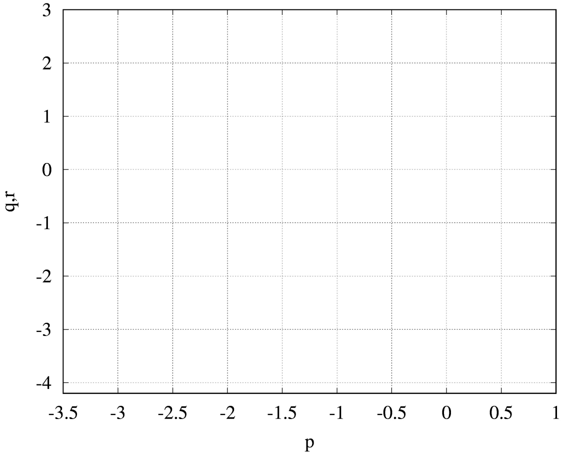
	P_{GG^T}	Q_{GG^T}	R_{GG^T}
Formula	$2(Q_\Omega - Q_S)$	$V^2 + Q_G^2$	R_G^2
Wall-behavior	$\mathcal{O}(y^0)$	$\mathcal{O}(y^2)$	$\mathcal{O}(y^6)$
Units	$[T^{-2}]$	$[T^{-4}]$	$[T^{-6}]$

$$-6r - 4q - 2p = 1 \ \ [T]; \qquad 6r + 2q = s,$$

where s is the slope for the asymptotic near-wall behavior, *i.e.* $\mathcal{O}(y^s)$.

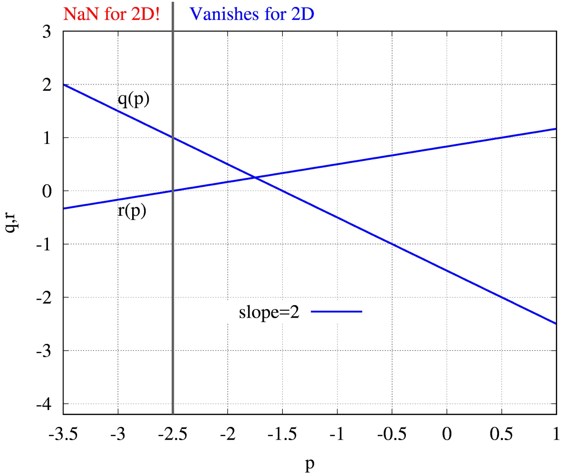
Building proper models for the subgrid heat flux

Solutions: $q(p, s) = -(1 + s)/2 - p$ and $r(p, s) = (2s + 1)/6 + p/3$



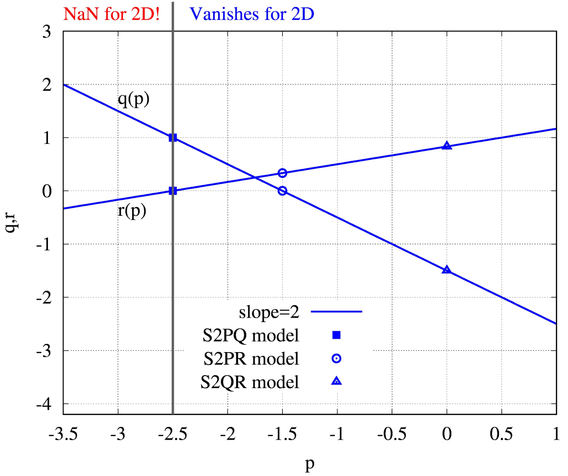
Building proper models for the subgrid heat flux

Solutions: $q(p,s) = -(1+s)/2 - p$ and $r(p,s) = (2s+1)/6 + p/3$



Building proper models for the subgrid heat flux

Solutions: $q(p,s) = -(1+s)/2 - p$ and $r(p,s) = (2s+1)/6 + p/3$

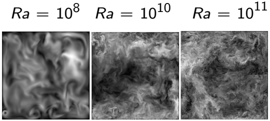


A priori analysis in the bulk

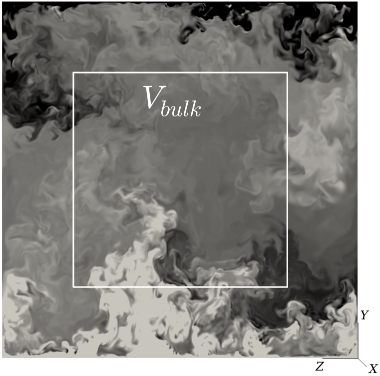
Estimation of the model constant, C_{s2pqr}

Playing with exponent p...

$$\boxed{q^{S2PQR} \approx -C_{s2pqr} \left(P^{p}_{GG^T} Q^{-(p+3/2)}_{GG^T} R^{(2p+5)/6}_{GG^T} \right) \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$



$$\frac{< |q^{S2PQR}| >_{bulk}}{< |q| >_{bulk}} = 1 \longrightarrow C_{s2pqr}$$



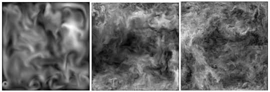
A priori analysis in the bulk

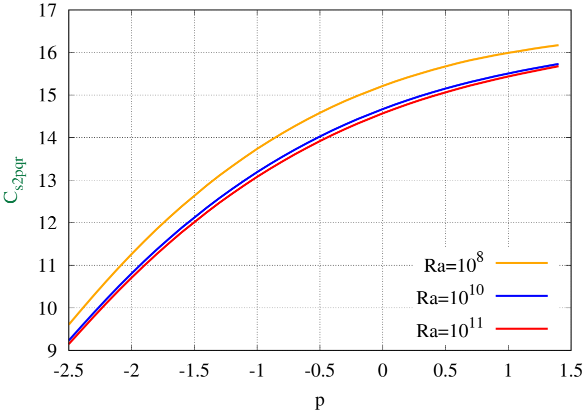
Estimation of the model constant, C_{s2pqr}

Playing with exponent p...

$$q^{S2PQR} \approx -C_{s2pqr} \left(P^{p}_{GG^T} Q^{-(p+3/2)}_{GG^T} R^{(2p+5)/6}_{GG^T} \right) \frac{\delta^2}{12} GG^T \nabla \overline{T}$$

$Ra = 10^8$ $Ra = 10^{10}$ $Ra = 10^{11}$





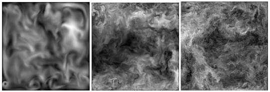
A priori analysis in the bulk

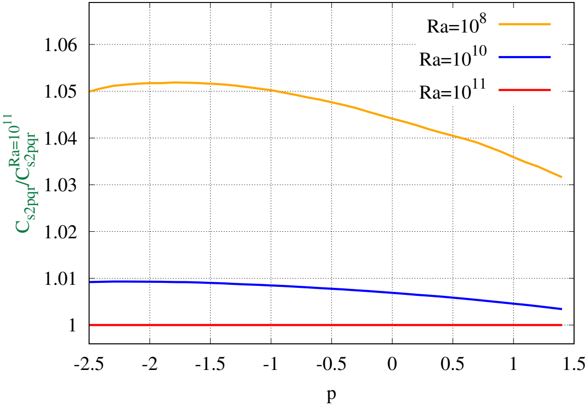
Estimation of the model constant, C_{s2pqr}

Playing with exponent p...

$$q^{S2PQR} \approx -C_{s2pqr} \left(P^{p}_{GG^T} Q^{-(p+3/2)}_{GG^T} R^{(2p+5)/6}_{GG^T} \right) \frac{\delta^2}{12} GG^T \nabla \overline{T}$$

$Ra = 10^8$ $\quad Ra = 10^{10}$ $\quad Ra = 10^{11}$





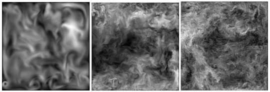
A priori analysis in the bulk

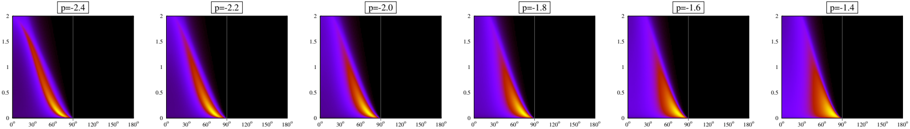
Alignment trends...

Playing with exponent p...

$$\boxed{q^{S2PQR} \approx -C_{s2pqr}\left(P^{p}_{GG^T}\, Q^{-(p+3/2)}_{GG^T}\, R^{(2p+5)/6}_{GG^T}\right)\frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

$Ra = 10^8$ $Ra = 10^{10}$ $\boxed{Ra = 10^{11}}$





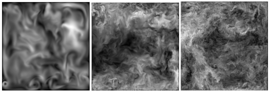
A priori analysis in the bulk

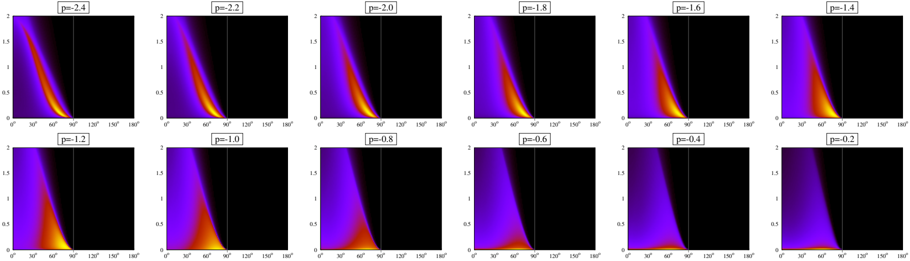
Alignment trends...

Playing with exponent p...

$$\boxed{q^{S2PQR} \approx -C_{s2pqr}\left(P^{p}_{GG^T} Q^{-(p+3/2)}_{GG^T} R^{(2p+5)/6}_{GG^T}\right)\frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

$Ra = 10^8$ $Ra = 10^{10}$ $\boxed{Ra = 10^{11}}$





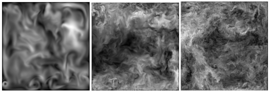
A priori analysis in the bulk

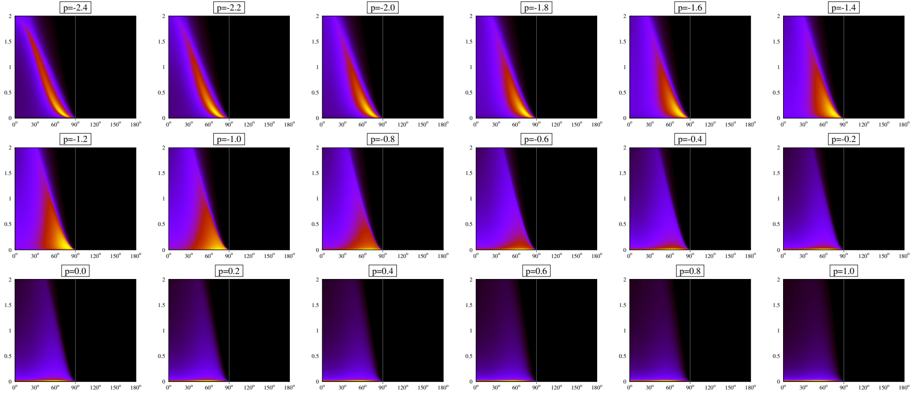
Alignment trends...

Playing with exponent p...

$$\boxed{q^{S2PQR} \approx -C_{s2pqr} \left(P_{GG^T}^{p} \, Q_{GG^T}^{-(p+3/2)} \, R_{GG^T}^{(2p+5)/6} \right) \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

$Ra = 10^8$ $Ra = 10^{10}$ $\boxed{Ra = 10^{11}}$

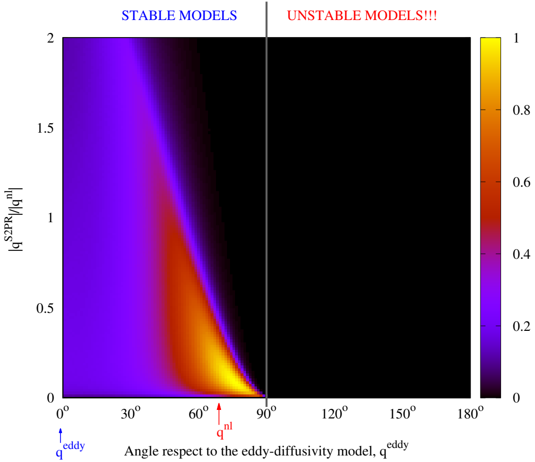


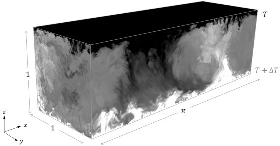


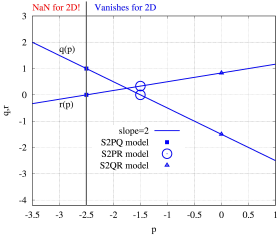
A priori alignment trends of S2PR in the bulk

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

$$\boxed{q^{s2PR} \equiv -C_{s2pr} P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

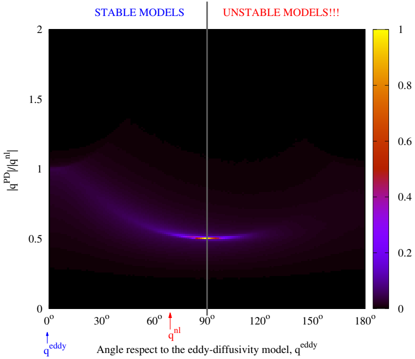






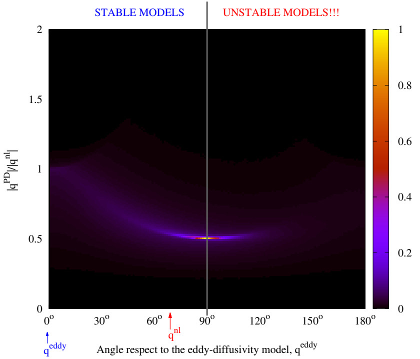
A priori alignment trends of S2PR in the near-wall region

$$\boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$



A priori alignment trends of S2PR in the near-wall region

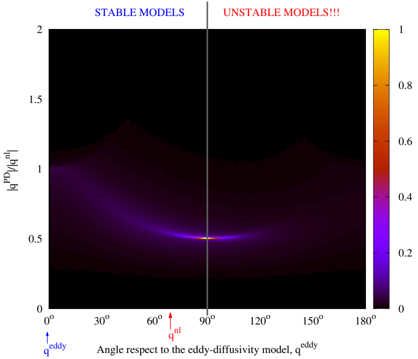
$$\boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$



$$\cos \beta^{model} = \frac{q^{eddy} \cdot q^{model}}{|q^{eddy}||q^{model}|}$$

A priori alignment trends of S2PR in the near-wall region

$$\boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$



$$\cos \beta^{PD} = \frac{\nabla \overline{T} \cdot S \nabla \overline{T}}{|\nabla \overline{T}| \cdot |S \nabla \overline{T}|} = \mathcal{O}(y^1)$$

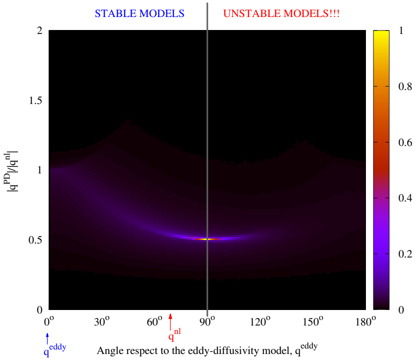
$$\lim_{y \to 0^+} \frac{|q^{PD}|}{|q^{nl}|} = \lim_{y \to 0^+} \frac{|S \nabla \overline{T}|}{|G \nabla \overline{T}|} = \frac{1}{2}$$

$$\cos \beta^{model} = \frac{q^{eddy} \cdot q^{model}}{|q^{eddy}||q^{model}|}$$

A priori alignment trends of S2PR in the near-wall region

$$\boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$

$$\boxed{q^{s2PR} \equiv -C_{s2pr} P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

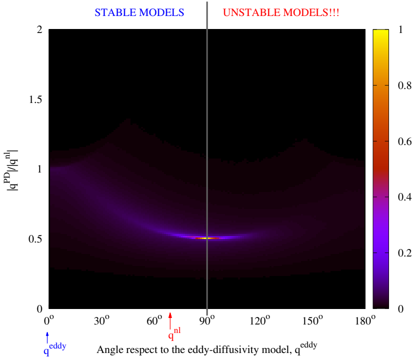


$$\cos \beta^{model} = \frac{q^{eddy} \cdot q^{model}}{|q^{eddy}||q^{model}|}$$

A priori alignment trends of S2PR in the near-wall region

$$\boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$

$$\boxed{q^{s2PR} \equiv -C_{s2pr} P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$



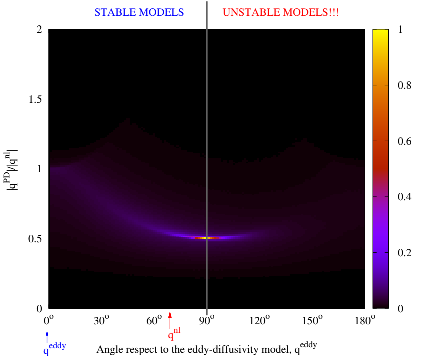
$$\cos \beta^{S2PQR} = \frac{\nabla \overline{T} \cdot GG^T \nabla \overline{T}}{|\nabla \overline{T}| \cdot |GG^T \nabla \overline{T}|} = \mathcal{O}(y^1)$$

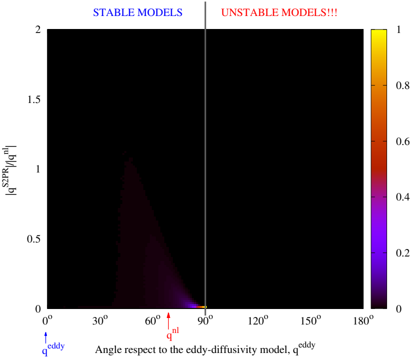
$$\cos \beta^{model} = \frac{q^{eddy} \cdot q^{model}}{|q^{eddy}||q^{model}|}$$

A priori alignment trends of S2PR in the near-wall region

$$\boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$

$$\boxed{q^{s2PR} \equiv -C_{s2pr} P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} G G^T \nabla \overline{T}}$$





$$\cos \beta^{model} = \frac{q^{eddy} \cdot q^{model}}{|q^{eddy}||q^{model}|}$$

A posteriori results?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

eddy-viscosity $\longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

A posteriori results?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

eddy-viscosity $\longrightarrow \quad \tau(\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

⚠ But first we need to answer the following **research question**:

- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

A posteriori results?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\textbf{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

⚠ But first we need to answer the following **research question**:

- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

Idea: let's do an LES for momentum and a DNS for temperature!

DNS at very low Pr number

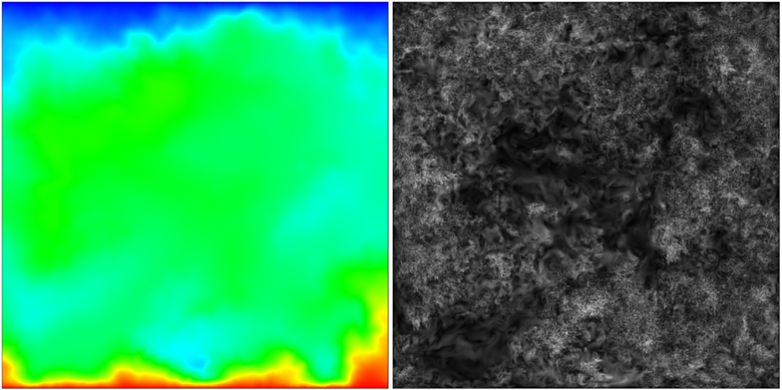
Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$.

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale

DNS at very low *Pr* number

Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$. $\boxed{\text{Here: } \eta_T \approx 53.2\eta_K}$

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale



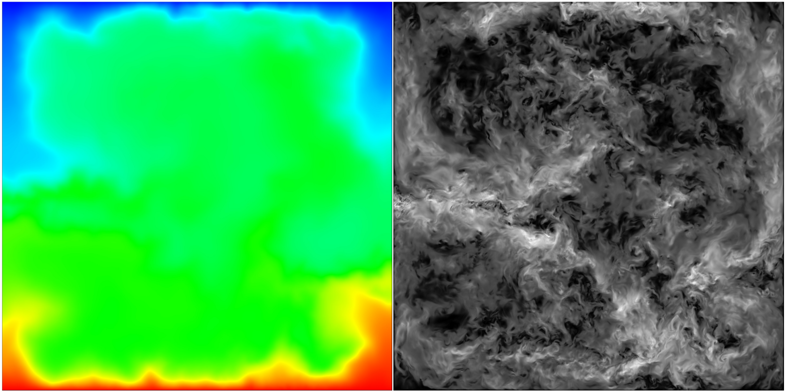
DNS of a RB at $Ra = 7.14 \times 10^6$ and $Pr = 0.005$ (liquid sodium)

$488 \times 488 \times 1280 \approx$ **305M**

DNS at very low Pr number

Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$. $\boxed{\text{Here: } \eta_T \approx 53.2\eta_K}$

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale



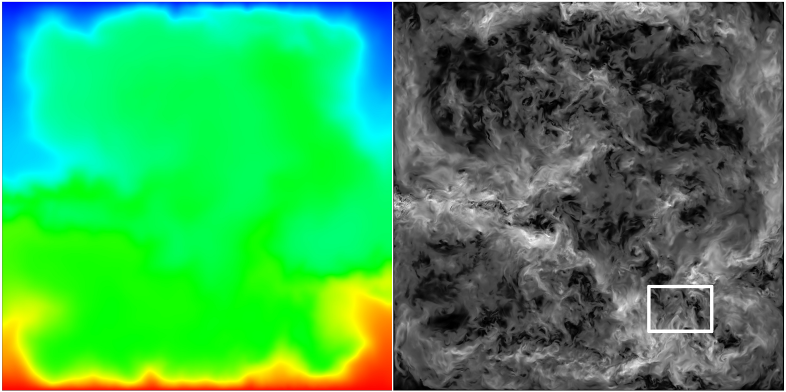
DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)

$966 \times 966 \times 2048 \approx$ **1911M**

DNS at very low *Pr* number

Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$. $\boxed{\text{Here: } \eta_T \approx 53.2\eta_K}$

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale



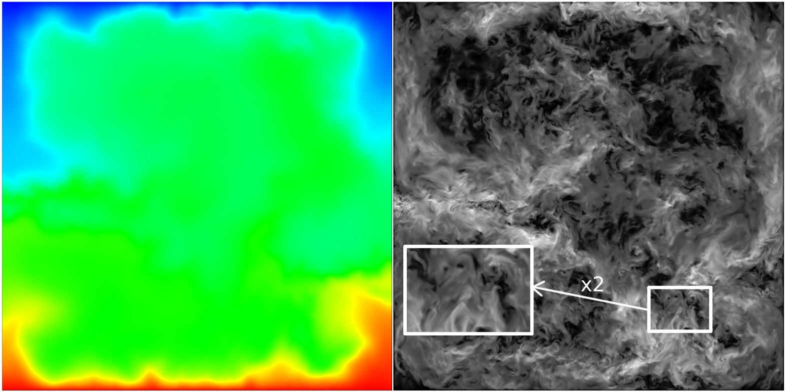
DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)

$966 \times 966 \times 2048 \approx$ **1911M**

DNS at very low Pr number

Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$. $\boxed{\text{Here: } \eta_T \approx 53.2\eta_K}$

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale



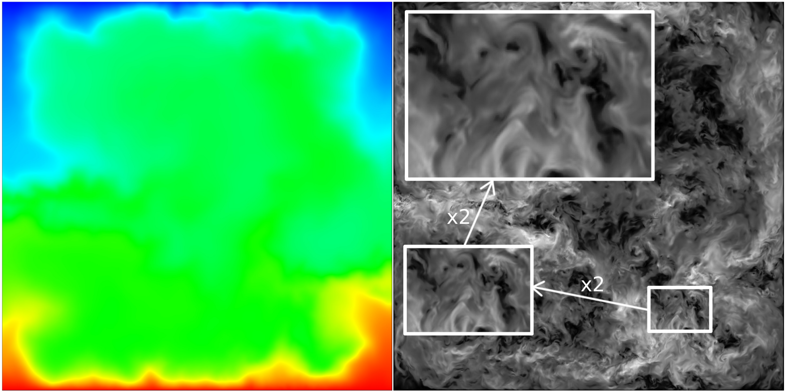
DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)

$966 \times 966 \times 2048 \approx$ **1911M**

DNS at very low *Pr* number

Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$. $\boxed{\text{Here: } \eta_T \approx 53.2\eta_K}$

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale



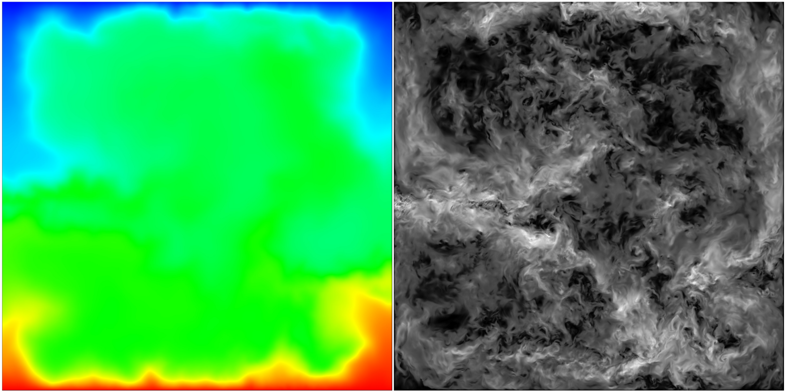
DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)

$966 \times 966 \times 2048 \approx$ **1911M**

DNS at very low Pr number

Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$. $\boxed{\text{Here: } \eta_T \approx 53.2\eta_K}$

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale

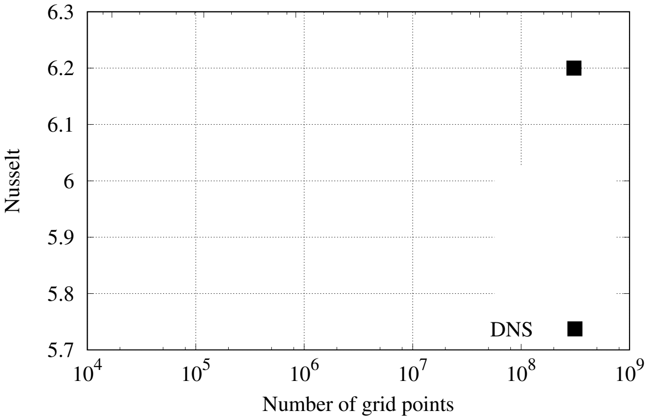


DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)

$966 \times 966 \times 2048 \approx$ **1911M**

LES[6] results at very low Pr number

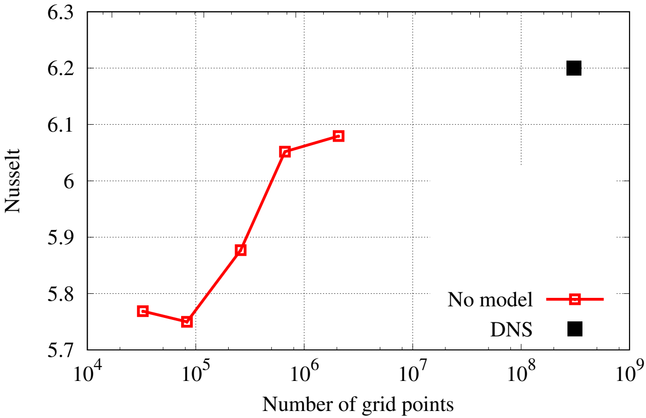
RB at $\boxed{Ra = 7.14 \times 10^6}$ and $Pr = 0.005$ (DNS $\rightarrow 488 \times 488 \times 1280 \approx$ **305M**)



[6] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

LES[6] results at very low Pr number

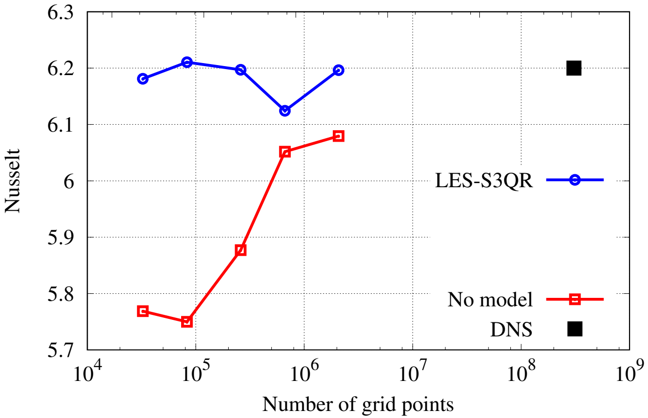
RB at $\boxed{Ra = 7.14 \times 10^6}$ and $Pr = 0.005$ (DNS $\rightarrow 488 \times 488 \times 1280 \approx$ **305M**)



[6] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

LES[6] results at very low Pr number

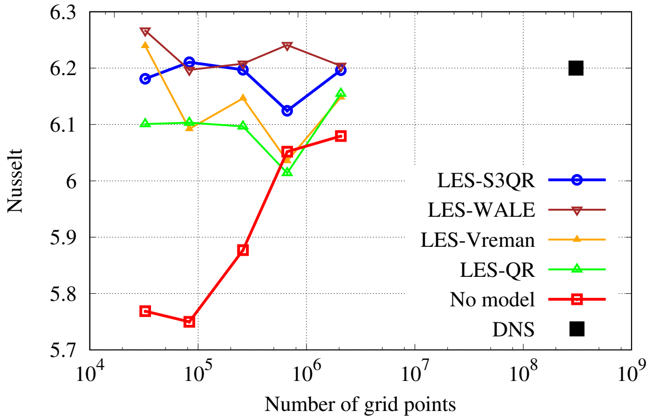
RB at $\boxed{Ra = 7.14 \times 10^6}$ and $Pr = 0.005$ (DNS $\rightarrow 488 \times 488 \times 1280 \approx$ **305M**)



[6] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

LES[6] results at very low Pr number

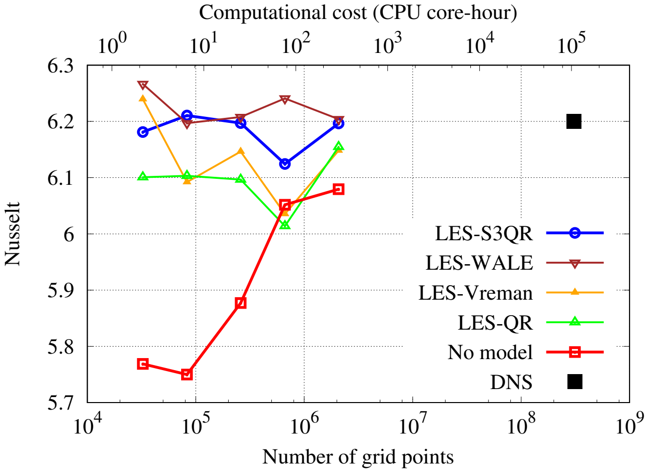
RB at $\boxed{Ra = 7.14 \times 10^6}$ and $Pr = 0.005$ (DNS $\rightarrow 488 \times 488 \times 1280 \approx$ **305M**)



[6] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

LES[6] results at very low Pr number

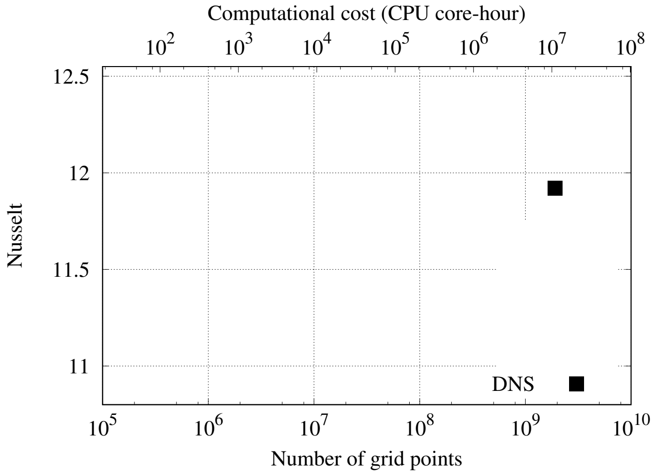
RB at $\boxed{Ra = 7.14 \times 10^6}$ and $Pr = 0.005$ (DNS $\rightarrow 488 \times 488 \times 1280 \approx$ **305M**)



[6]F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

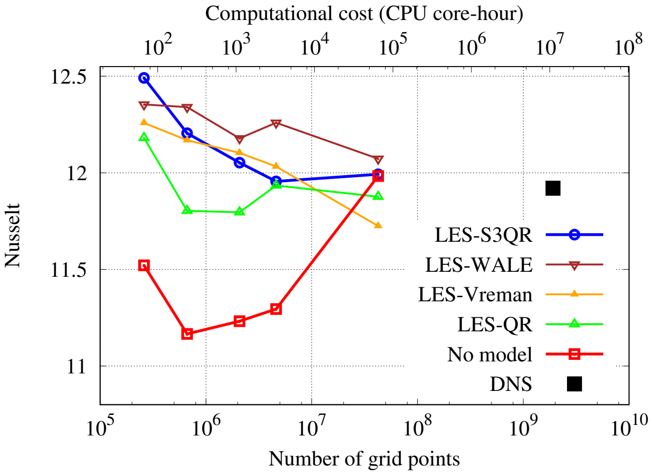
LES results at very low Pr number

RB at $\boxed{Ra = 7.14 \times 10^7}$ and $Pr = 0.005$ (DNS $\rightarrow 966 \times 966 \times 2048 \approx$ **1911M**)



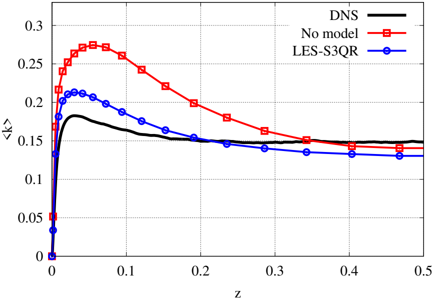
LES results at very low Pr number

RB at $\boxed{Ra = 7.14 \times 10^7}$ and $Pr = 0.005$ (DNS $\rightarrow 966 \times 966 \times 2048 \approx$ **1911M**)

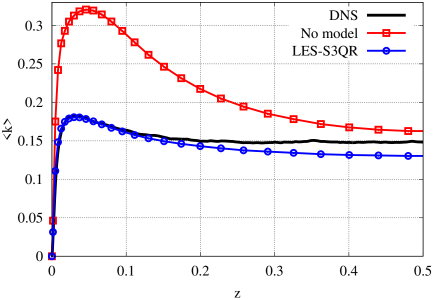


LES results at very low Pr number

RB at $\boxed{Ra = 7.14 \times 10^6}$ and $Pr = 0.005$ (DNS $\rightarrow 488 \times 488 \times 1280 \approx$ **305M**)



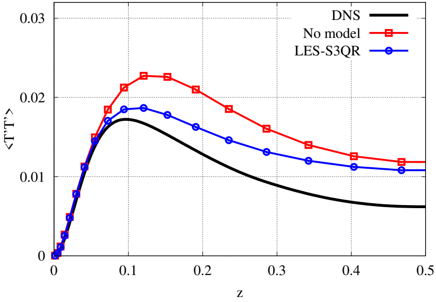
$64 \times 32 \times 32$



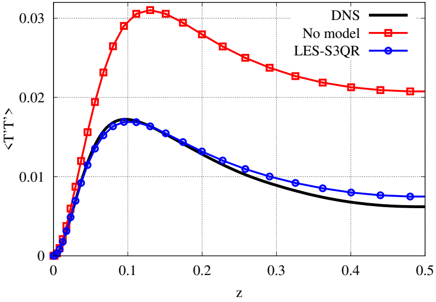
$96 \times 52 \times 52$

LES results at very low Pr number

RB at $\boxed{Ra = 7.14 \times 10^6}$ and $Pr = 0.005$ (DNS $\rightarrow 488 \times 488 \times 1280 \approx$ **305M**)



$64 \times 32 \times 32$



$96 \times 52 \times 52$

Concluding remarks

- A new tensor-diffusivity model has been proposed[7]

$$\boxed{q^{s2PR} \equiv -C_{s2pr} P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

- Locally defined, unconditionally stable and vanishes for 2D flows ✓
- Good *a priori* alignment trends and proper near-wall scaling ✓
- Eddy-viscosity models work for RB ✓

[7] F.X.Trias, F.Dabbagh, A.Gorobets, C.Oliet. *On a proper tensor-diffusivity model for LES of buoyancy-driven turbulence*, **Flow Turbul Combust**, 105:393-414, 2020.

[8] F.X.Trias, A.Gorobets, M.Silvis, R.Verstappen, A.Oliva. *A new subgrid characteristic length for turbulence simulations on anisotropic grids*, **Phys.Fluids**, 26:115109, 2017.

Concluding remarks

- A new tensor-diffusivity model has been proposed[7]

$$\boxed{q^{s2PR} \equiv -C_{s2pr} P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

- Locally defined, unconditionally stable and vanishes for 2D flows ✓
- Good *a priori* alignment trends and proper near-wall scaling ✓
- Eddy-viscosity models work for RB ✓

Future research:

- *A posteriori* tests using q^{s2PR} for RB

[7] F.X.Trias, F.Dabbagh, A.Gorobets, C.Oliet. *On a proper tensor-diffusivity model for LES of buoyancy-driven turbulence*, **Flow Turbul Combust**, 105:393-414, 2020.

[8] F.X.Trias, A.Gorobets, M.Silvis, R.Verstappen, A.Oliva. *A new subgrid characteristic length for turbulence simulations on anisotropic grids*, **Phys.Fluids**, 26:115109, 2017.

Concluding remarks

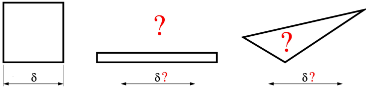
- A new tensor-diffusivity model has been proposed[7]

$$\boxed{q^{s2PR} \equiv -C_{s2pr} P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

- Locally defined, unconditionally stable and vanishes for 2D flows ✓
- Good *a priori* alignment trends and proper near-wall scaling ✓
- Eddy-viscosity models work for RB ✓

Future research:

- *A posteriori* tests using q^{s2PR} for RB
- How δ should be defined for highly anisotropic grids[8]?



[7] F.X.Trias, F.Dabbagh, A.Gorobets, C.Oliet. *On a proper tensor-diffusivity model for LES of buoyancy-driven turbulence*, **Flow Turbul Combust**, 105:393-414, 2020.
[8] F.X.Trias, A.Gorobets, M.Silvis, R.Verstappen, A.Oliva. *A new subgrid characteristic length for turbulence simulations on anisotropic grids*, **Phys.Fluids**, 26:115109, 2017.

Thank you for your virtual attendance