Energy preserving multiphase flows: Application to falling films.

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14th World Congress Computational Mechanics
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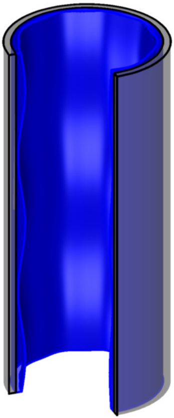
Overview

1. Introduction
2. Energy preserving
3. Results

Falling films in industry



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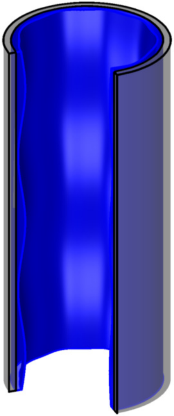
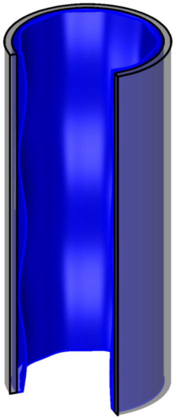


Advantages

- Short residence time

Falling films in industry



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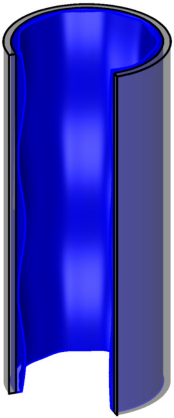

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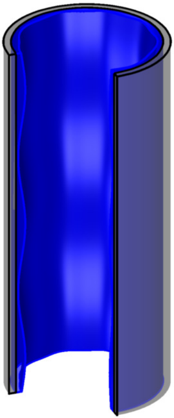

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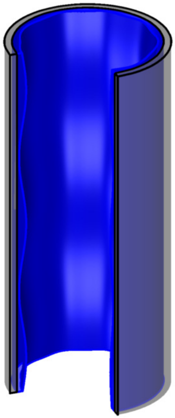

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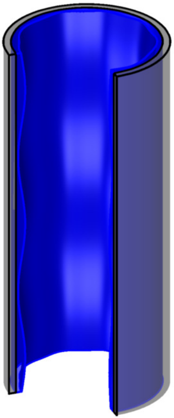

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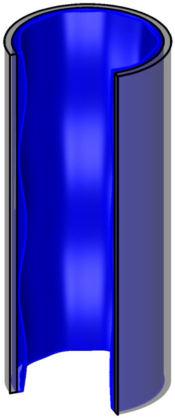

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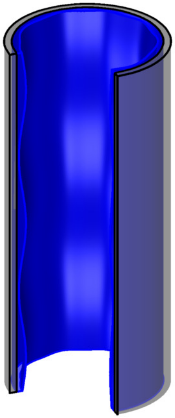

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- Extreme density ratio
- Hydrodynamic instability at $Re = 0$

Governing equations

Incompressible

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Navier-Stokes

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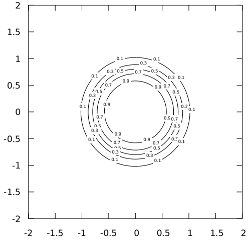

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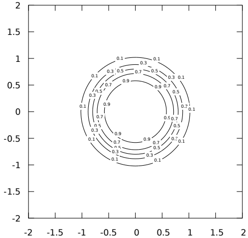

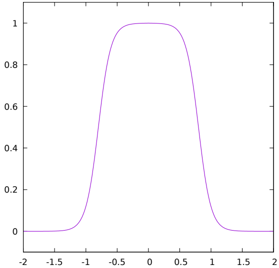

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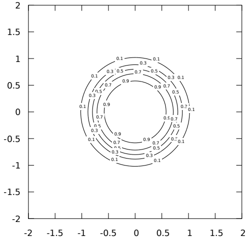

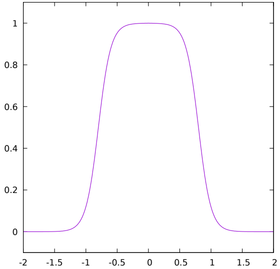

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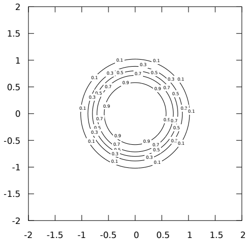

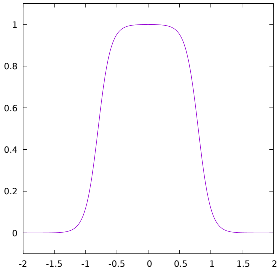

$$\rho = 1 + \left(\frac{\rho_1}{\rho_0} - 1\right)\theta \qquad \mu = 1 + \left(\frac{\mu_1}{\mu_0} - 1\right)\theta$$

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$$\frac{dE_m}{dt} = \frac{dE_k}{dt} + \frac{dE_p}{dt} = -(S, 2\mu S) < 0$$

Discrete

Conservative Level-Set

$$\frac{d\theta_c}{dt} = C(u_f)_f \theta_c$$

Navier-Stokes - Continuum Surface Force[1]

$$\frac{dRu_f}{dt} = -C(Ru_f)_f u_f - Gp_c + Lu_f + We k_f G\theta_c + \rho_f \hat{g}$$

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Performing a discrete energy balance:

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$$\frac{dE_k}{dt} = -(\mathrm{u_f}, \mathrm{C}(\mathrm{R}\mathrm{u_f})_f \mathrm{u_f}) - \cancel{(\mathrm{u_f}, \mathrm{G}\mathrm{p_c})} + (\mathrm{u_f}, \mathrm{L}\mathrm{u_f}) + We\,(\mathrm{u_f}, \mathrm{k_f}\mathrm{G}\theta_c) + (\mathrm{u_f}, \rho_f \hat{g}) - \frac{1}{2}\left(\mathrm{u_f}, \mathrm{u_f}\Pi\frac{d\rho_c}{dt}\right)$$

$$\frac{dE_p}{dt} = We\,(\mathrm{G}\mathrm{C}(\mathrm{u_f})_c\theta_c, \hat{\mathrm{n}}_f) - (\mathrm{u_f}, \rho_f \hat{g})$$

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& + We\left(\mathrm{u_f}, \mathrm{k_f}\mathrm{G}\theta_\mathrm{c}\right) + \left(\mathrm{u_f}, \rho_\mathrm{f}\hat{g}\right) \\
& - \frac{1}{2}\left(\mathrm{u_f}, \mathrm{u_f}\Pi\frac{d\rho_\mathrm{c}}{dt}\right) \\
\frac{dE_p}{dt} =& We\left(\mathrm{G}\mathrm{C}(\mathrm{u_f})_\mathrm{c}\theta_\mathrm{c}, \hat{\mathrm{n}}_\mathrm{f}\right) - \left(\mathrm{u_f}, \rho_\mathrm{f}\hat{g}\right)
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&\quad - \frac{1}{2}\left(u_f, u_f \Pi \frac{d\rho_c}{dt}\right) \\
\frac{dE_p}{dt} &= \cancel{We\left(GC(u_f)_c\theta_c, \hat{n}_f\right)}{}^{[2]} - \cancel{\left(u_f, \rho_f \hat{g}\right)} \\
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Consistent mass and momentum[3]

$$\frac{dE_m}{dt} = -\left(\mathrm{u_f}, \mathrm{C}(\mathrm{R}\mathrm{u_f})_\mathrm{f}\mathrm{u_f}\right) + \left(\mathrm{u_f}, \mathrm{L}\mathrm{u_f}\right) - \frac{1}{2}\left(\mathrm{u_f}, \mathrm{U}\Pi\frac{d\rho_c}{dt}\right)$$

[3] Shahab Mirjalili and Ali Mani. "Consistent, energy-conserving momentum transport for simulations of two-phase flows using the phase field equations". In: *J. Comput. Phys.* 1 (2020), p. 109918. arXiv: 1912.10096.

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&= -\left(\mathsf{u_f}, \frac{1}{2}\mathsf{U}\Pi\mathsf{D}\mathsf{R}\mathsf{u_f}\right) - \cancel{\left(\mathsf{u_f}, \overline{\mathsf{C}(\mathsf{R}\mathsf{u_f})}\mathsf{u_f}\right)} \\
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$$= -\cancel{\left(u_f, \frac{1}{2} U\Pi DRu_f\right)} - \cancel{\left(u_f, \overline{C(Ru_f)} u_f\right)}$$

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$$\frac{d\rho_c}{dt} = C(u_f)_f \rho_c = DRu_f$$

Energy-preserving discretization

$$\frac{dE_m}{dt} = (u_f, Lu_f) < 0$$

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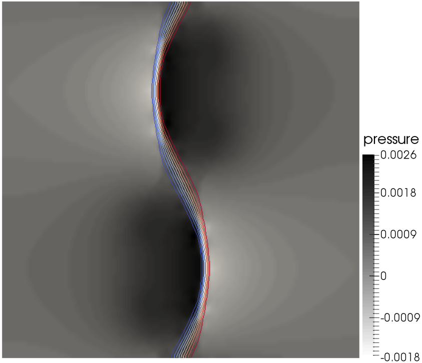
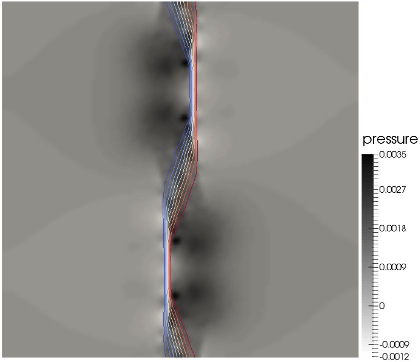
Falling Film



Falling Film



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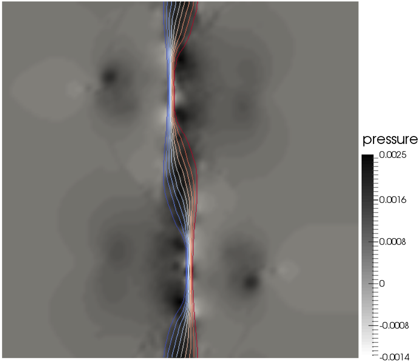

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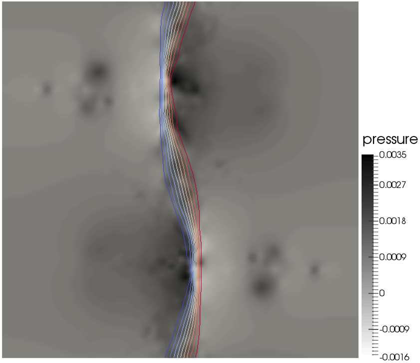

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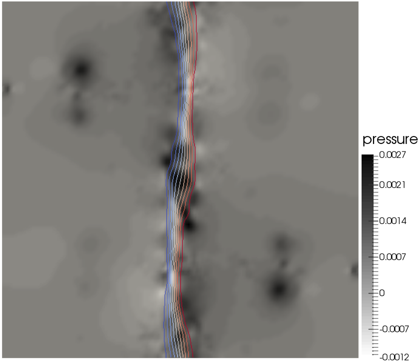

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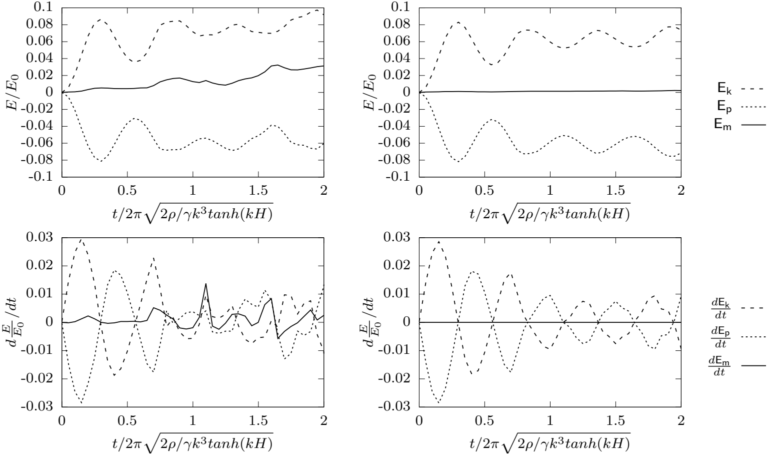

Figure: Energy fluctuation for the falling film

Simulation of industrial falling films

id	fluid	$T(^\circ C)$	X_{abs}	Ka	$\Pi_\rho(\times 10^{-3})$	$\Pi_\mu(\times 10^{-3})$
A	H_2O	9.9	0.00	2420	0.60	9.37
B	$H_2O/LiBr$	50.0	0.60	443	0.35	2.45
C	$H_2O/Carrol$	50.0	0.67	150	0.35	1.10

Table: Estimated fluid properties dimensionless groups. Because thermal properties are estimated at a characteristic state, values are rounded to ease its handling.

DNS of vertically falling films

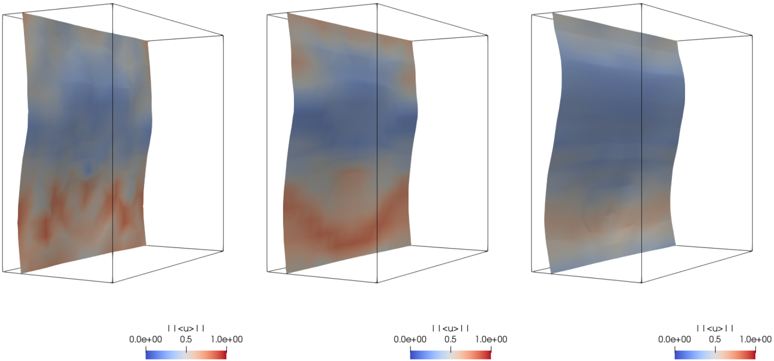

Figure: Velocity magnitude on the film surfaces for cases A, B and C at $Re = 150$ after $T = 100$.

Summary

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