

Efficient strategies for solving the variable Poisson equation with large contrasts in the coefficients

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Discrete versions of Poisson's equation with high contrasts in the coefficients result in very ill-conditioned systems. Thus, its iterative solution represents a major challenge, for instance, in porous media and multiphase flow simulations, where considerable permeability and density ratios are usually found. The existing strategies trying to remedy this are highly dependant on whether the coefficients matrix remains constant at each time iteration [1] or not [2]. In this regard, incompressible multiphase flows with high-density ratios are particularly demanding as their resulting Poisson equation,

$$\nabla \cdot (\rho^{-1} \nabla p) = \nabla \cdot \mathbf{v}^p,$$

varies along with $\rho(\mathbf{x}, t)$, making the reconstruction of complex preconditioners generally impractical. This work presents a strategy for solving such versions of the variable Poisson equation. Roughly, we first transform it into its constant counterpart through the approximation proposed by Dodd and Ferrante [2]. Then, we take advantage of mesh reflection symmetries, which are common in multiphase flows, to block-diagonalise it by means of an inexpensive change of basis. Finally, we solve concurrently the resulting set of fully decoupled subsystems with virtually any solver. Such a strategy allows splitting the submatrices of the aforementioned subsystems into a common part plus another subsystem-dependent whose size is, in practice, negligible. As a result, we can reduce substantially the memory footprint of the solvers and replace the standard sparse matrix-vector products with the higher arithmetic intensity sparse matrix-matrix product. On the one hand, the subsystems' better-conditioning makes iterative solvers like Krylov subspace methods converge significantly faster. On the other, the use of sparse matrix-matrix product accelerates their (fewer) iterations considerably. The method and numerical results are going to be presented at the conference.

REFERENCES

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