The 9th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2024
3–7 June 2024, Lisbon, Portugal

ON THE CONDITIONS FOR A STABLE PROJECTION METHOD ON COLLOCATED UNSTRUCTURED GRIDS

D. Santos, J. A. Hopman, C.D. Pérez-Segarra, F.X. Trias

Heat and Mass Transfer Technological Center (CTTC), Technical University of Catalonia, Terrassa, Spain

{daniel.santos.serrano, jannes.hopman, cdavid.perez.segarra, francesc.xavier.trias}
@upc.edu

Keywords: *Finite volume method, Projection methods, Energy preserving, Symmetry-preserving, CFD, FSM, PISO*

Most popular CFD codes like OpenFOAM or ANSYS-FLUENT use finite-volume discretizations on collocated unstructured meshes. While these discretizations are preferred over staggered ones for their simplicity, some errors may arise due to this approach, such as the checkerboard problem. A possible solution to this problem would be using a compact Laplacian operator L instead of a wide-stencil Laplacian operator L_c. However, even for symmetry-preserving discretizations [1], this approach may lead to instabilities for highly distorted meshes due to the (artificial) kinetic energy added, which is given by:

$$\boldsymbol{p}_c^T(\mathsf{M}\tilde{\mathbf{A}}^{-1}\mathsf{G} - \mathsf{M}_c\mathbf{A}^{-1}\mathsf{G}_c)\boldsymbol{p}_c = \boldsymbol{p}_c^T(\mathsf{L} - \mathsf{L}_c)\boldsymbol{p}_c, \tag{1}$$

where $\mathbf{A} = I_3 \otimes \mathbf{A}_c$, $\tilde{\mathbf{A}}^{-1} = diag(\Pi_{c\to s} vec(\mathbf{A}_c^{-1}))$, and $\Pi_{c\to s}$ is the scalar cell-to-face interpolator. The necessary and sufficient conditions, for explicit ($\mathbf{A_c} = Id$) or implicit time integration ($\mathbf{A}_c$ diagonal with positive entries), to remove these instabilities by maintaining $\mathsf{L} - \mathsf{L}_c$ small and semi-negative definite are summarized in the following theorem:

Theorem Assume our projection method preserves the symmetry of the differential operators and ends up with a velocity correction that adds a kinetic energy error of the form $\boldsymbol{p}_c^T(\mathsf{L} - \mathsf{L}_c)\boldsymbol{p}_c$ (such as the Fractional Step Method or PISO); then, this contribution is negative if and only if the volume-weighted cell-to-face interpolator is used and these two geometrical conditions are satisfied by the mesh:

- **1.** $V_k = \sum_f \tilde{V}_{k,f} n_{i,f}^2, \forall k \in \{1, ..., n\}, i \in \{x, y, z\}$,
- **2.** $\sum_f \tilde{V}_{k,f} n_{i,f} n_{j,f} = 0, \forall k \in \{1, ..., n\}, i, j \in \{x, y, z\}, i \neq j$.

where $\tilde{V}_{k,f}$ is the quantity of staggered volume associated to control volume k, and $n_{i,f}$ are the components of the face-normal vector. This imposes geometrical constraints in the mesh that will be discussed during the conference.

REFERENCES

[1] F.X. Trias, O. Lehmkuhl, A. Oliva, C.D. Pérez-Segarra, R.W.C.P. Verstappen. Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured meshes. Journal of Computational Physics, 258:246-267, 2014.