WHAT EXACTLY IS THE FILTER LENGTH IN A FINITE-VOLUME BASED LES ?

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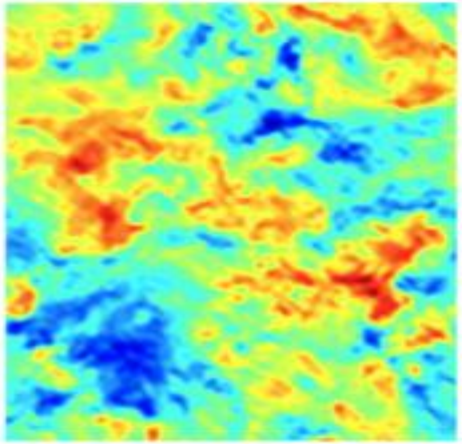
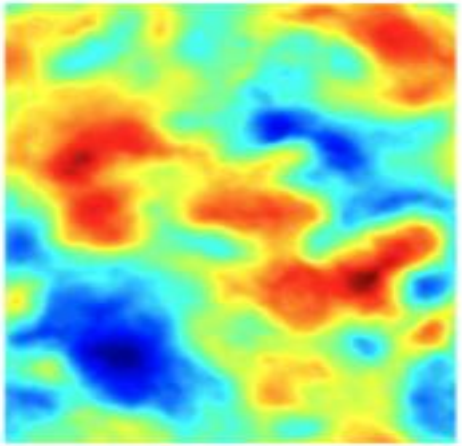




FILTERING

Full-scale model

$$\partial_t u + \partial_x(uu - \nu\partial_x u) = f$$

Spatially filtered

$$\overline{\partial_t u} + \overline{\partial_x(uu - \nu\partial_x u)} = \overline{f}$$

Assumption: filter commutes with differentiation

$$\partial_t \overline{u} + \partial_x (\overline{uu} - \nu \partial_x \overline{u}) = \overline{f}$$

Assumption: filter commutes with differentiation

$$\partial_t \overline{u} + \partial_x(\overline{uu} - \nu \partial_x \overline{u}) = \overline{f}$$

Closure model

$$\overline{uu} \approx \overline{u}\,\overline{u} + \tau(\overline{u})$$

LARGE EDDY SIMULATION

Full-scale

$$\partial_t u + \partial_x (u\,u - \nu \partial_x u) = f$$

LES model

$$\partial_t \overline{u} + \partial_x (\overline{u}\,\overline{u} - \nu \partial_x \overline{u}) = \overline{f} - \partial_x \tau(\overline{u})$$

BOX FILTER

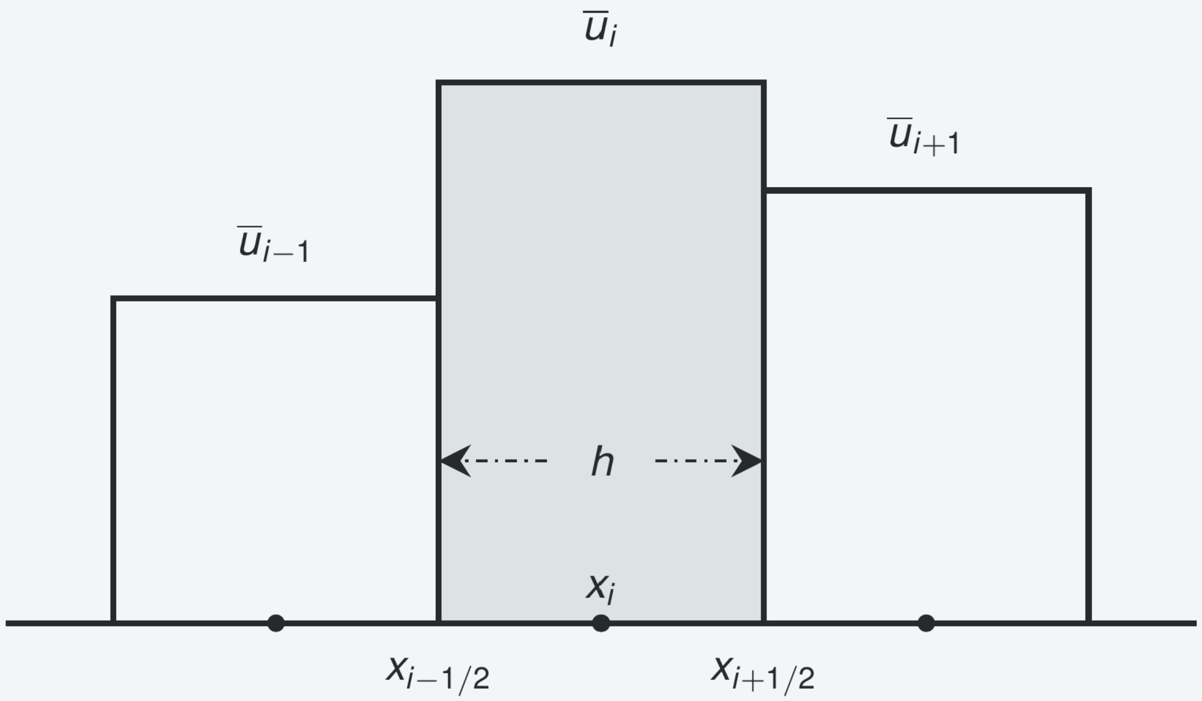
Box filter with filterlength h

$$\overline{u}(x,t) = \frac{1}{h}\int_{x-h/2}^{x+h/2} u(\xi,t)\,d\xi$$

Commutes with differentiation

$$\partial_x \overline{u} = \frac{u(x+h/2) - u(x-h/2)}{h} = \overline{\partial_x u}$$

SCHUMANN'S FILTER



$$\overline{u}_i(t) = \frac{1}{h}\int_{x_i-h/2}^{x_i+h/2} u(\xi, t)\, d\xi$$

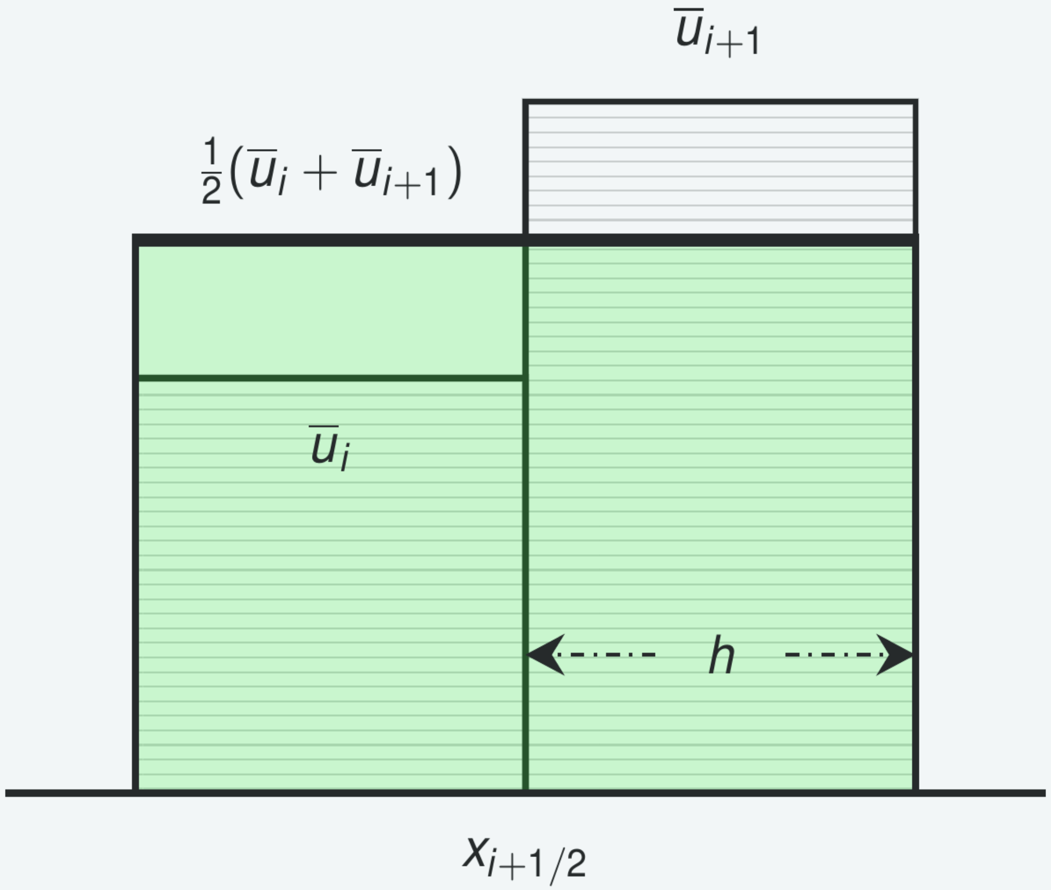
Conservation

$$h\frac{d\overline{u}_i}{dt} + u^2_{i+1/2} - u^2_{i-1/2} = \cdots$$

Cell-to-face interpolation

$$u_{i+1/2} \approx \tfrac{1}{2}(\overline{u}_i + \overline{u}_{i+1})$$

CELL-TO-FACE INTERPOLATION



$$\frac{1}{2}(\overline{u}_i + \overline{u}_{i+1}) =$$

$$\frac{1}{2h}\int_{x_{i+1/2}-h}^{x_{i+1/2}+h} u(\xi, t)\, d\xi = \widetilde{u}_{i+1/2}$$

Box filter with length 2h

Conservation

$$h\frac{d\overline{u}_i}{dt} + \widetilde{u}^2_{i+1/2} - \widetilde{u}^2_{i-1/2} = -\sigma_{i+1/2} + \sigma_{i-1/2} + \cdots$$

Closure

$$\sigma = u^2 - \widetilde{u}^2 \approx \sigma(\widetilde{u})$$

ADDING 2 NEIGHBORS

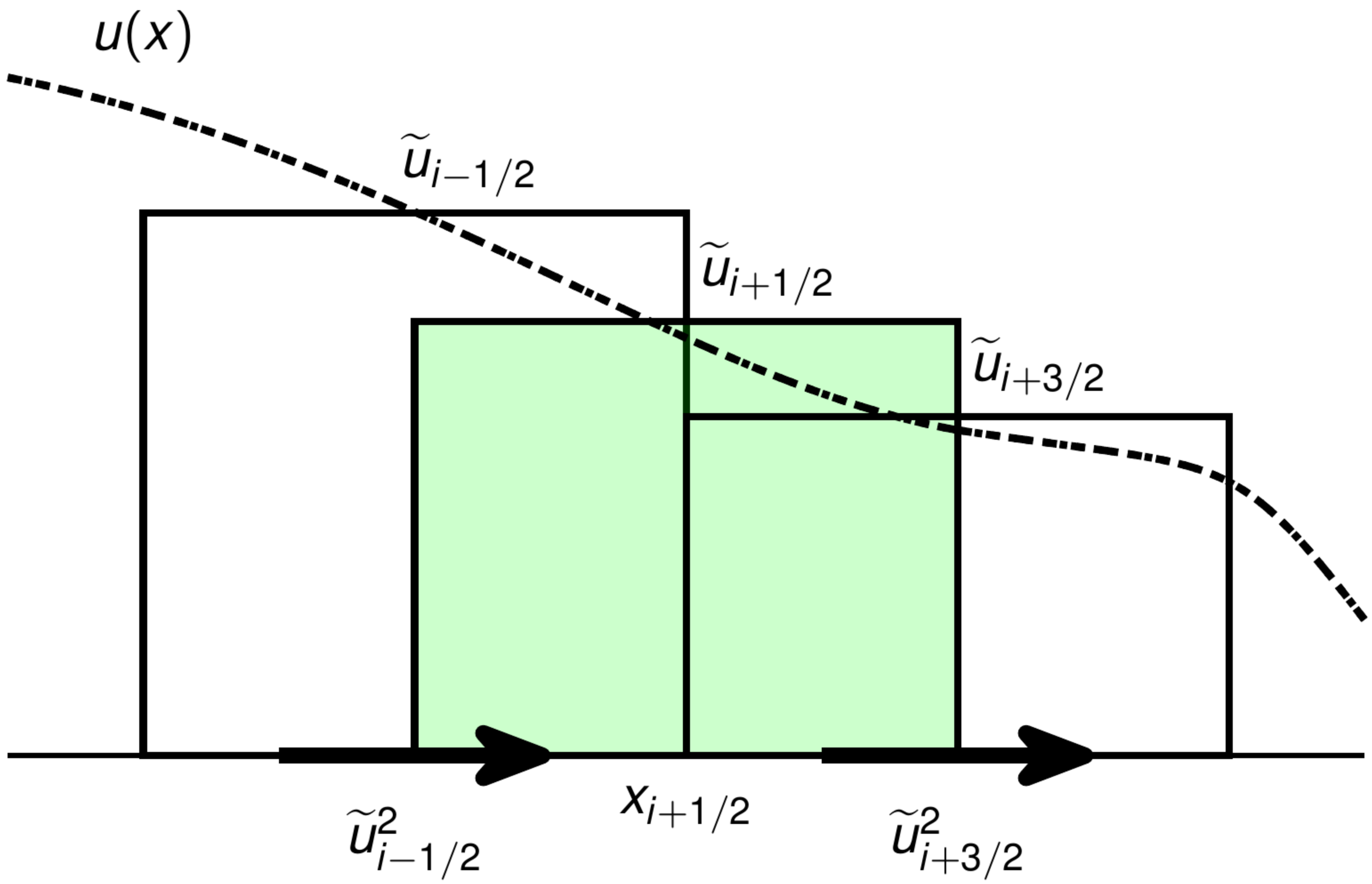
$$h\frac{d\overline{u}_i}{dt} + \widetilde{u}^2_{i+1/2} - \widetilde{u}^2_{i-1/2} = -\sigma_{i+1/2} + \sigma_{i-1/2} + \cdots$$

$$h\frac{d\overline{u}_{i+1}}{dt} + \widetilde{u}^2_{i+3/2} - \widetilde{u}^2_{i+1/2} = -\sigma_{i+3/2} + \sigma_{i+1/2} + \cdots$$

$+$

$$2h\frac{d\widetilde{u}_{i+1/2}}{dt} + \widetilde{u}^2_{i+3/2} - \widetilde{u}^2_{i-1/2} = -\sigma_{i+3/2} + \sigma_{i-1/2} + \cdots$$

OVERLAPPING FILTERS



FILTERED DYNAMCS

$$2h\frac{d\widetilde{u}_{i+1/2}}{dt} + \widetilde{u}^2_{i+3/2} - \widetilde{u}^2_{i-1/2} = -\sigma_{i+3/2} + \sigma_{i-1/2} + \cdots$$

Finite-difference filter

$$\phi_{i+3/2} - \phi_{i-1/2} = \int_{x_{i-1/2}}^{x_{i+3/2}} \partial_x \phi(\xi)\, d\xi = 2h\,\widetilde{\partial_x \phi}_{i+1/2}$$

$$\partial_t \widetilde{u} + \widetilde{\partial_x \widetilde{u}^2} = -\partial_x \widetilde{(u^2 - \widetilde{u}^2)} + \cdots$$

Exact at x $_{i+1/2}$, not yet closed

Closure problem for 2h-filter

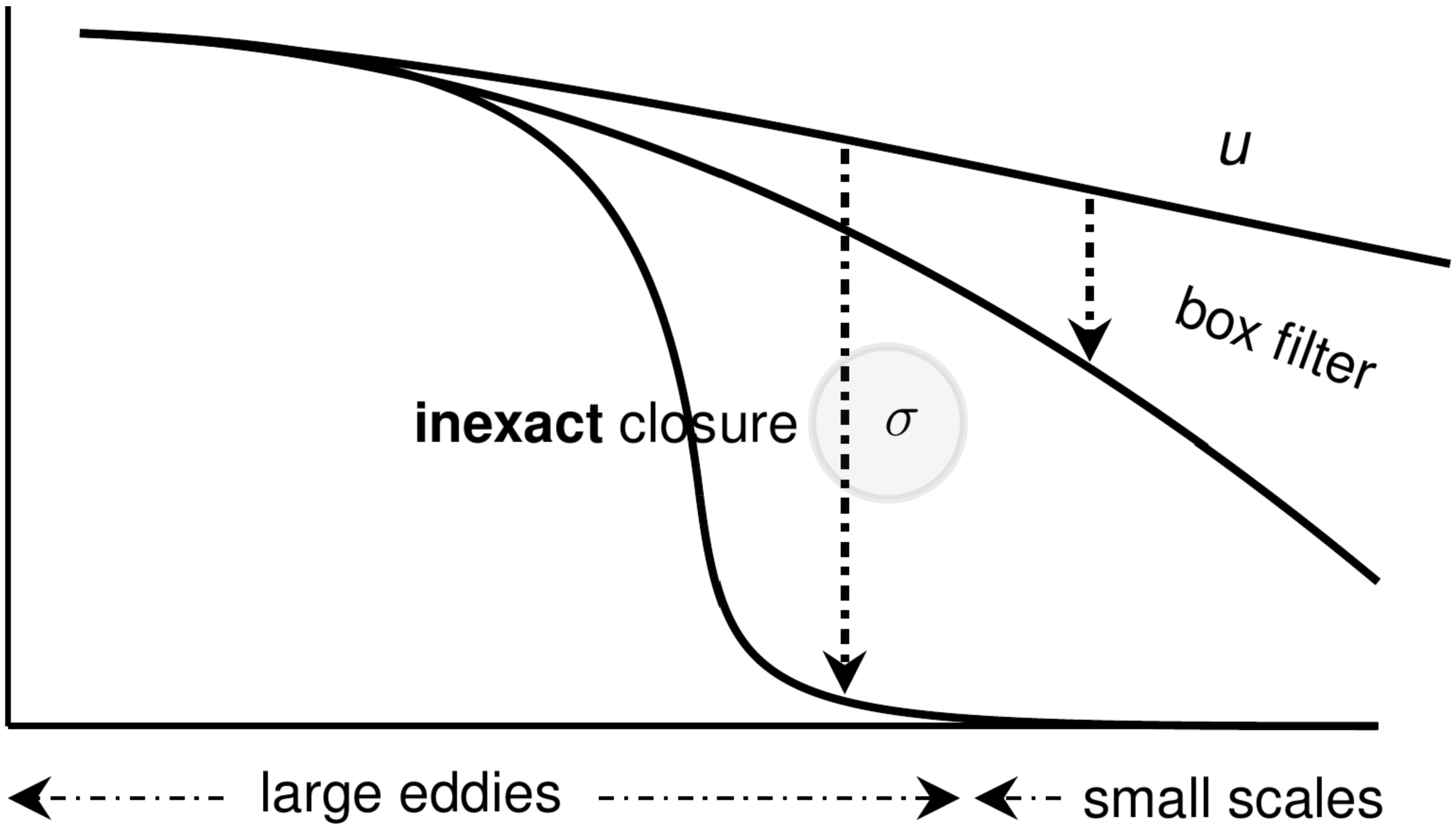
$$\partial_t \widetilde{u} + \widetilde{\partial_x \widetilde{u}^2} = -\widetilde{\partial_x \sigma(\widetilde{u})} + \cdots$$

$$\sigma(\widetilde{u}) \approx u^2 - \widetilde{u}^2$$

Note: 1) this equation is Galilean invariant
2) summation over all cells yields:

$$\frac{d}{dt}\int_{\text{entire domain}} u(x,t)\,dx = \textbf{boundary term}$$

SCALE SEPARATION



2 FILTERS

FVM filter

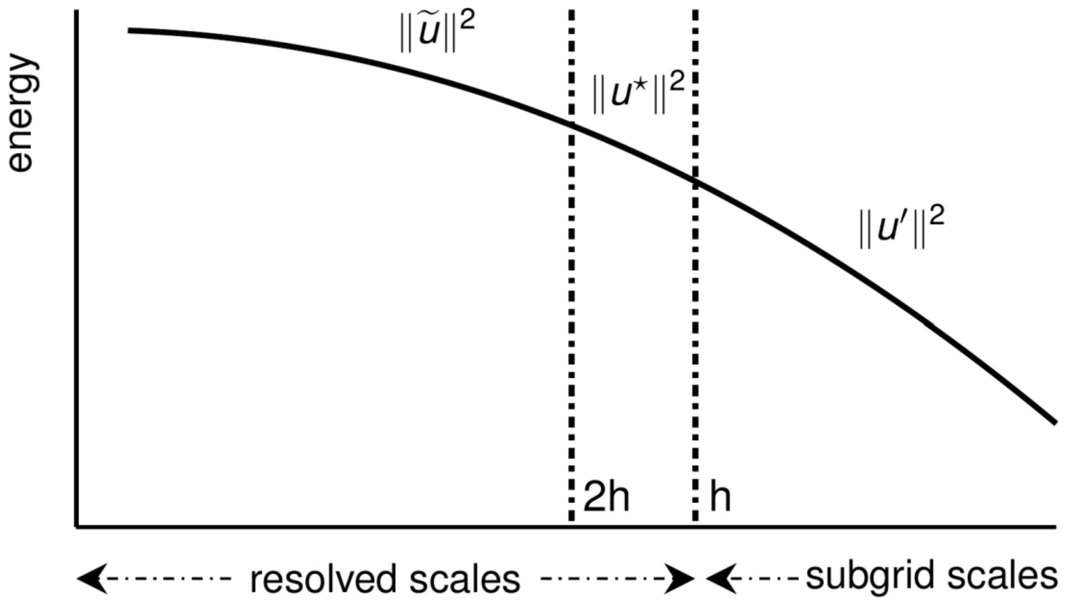
$$u = \overline{u} + u'$$

$$\overline{u}_i = \underbrace{\tfrac{1}{2}(\overline{u}_i + \overline{u}_{i+1})}_{\widetilde{u}_{i+1/2}} + \underbrace{\tfrac{1}{2}(\overline{u}_i - \overline{u}_{i+1})}_{u^\star_{i+1/2}}$$

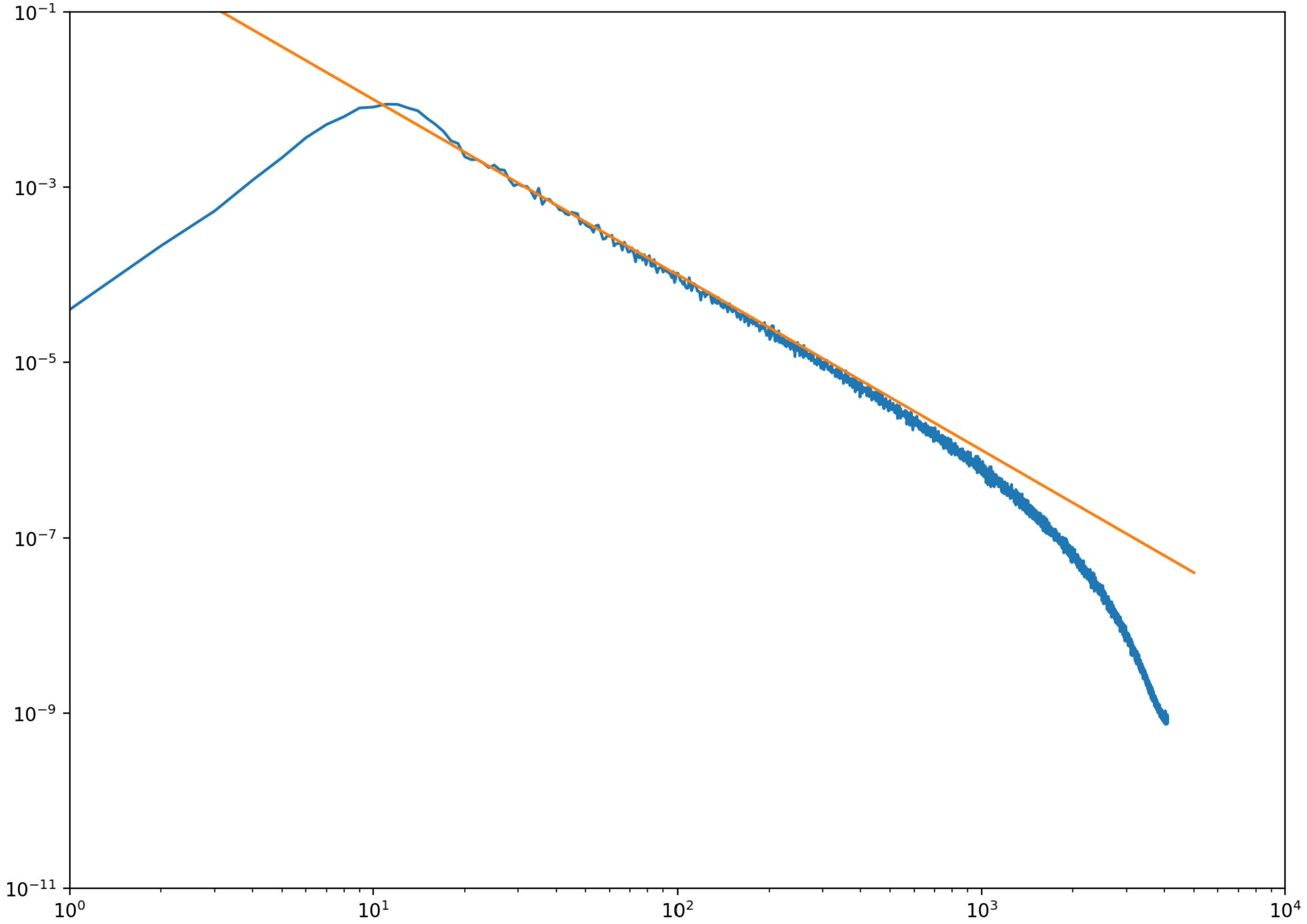
Orthogonality

$$\|\overline{u}\|^2 = \|\widetilde{u}\|^2 + \|u^\star\|^2$$

NB: this orthogonality holds on 3D unstructured meshes too

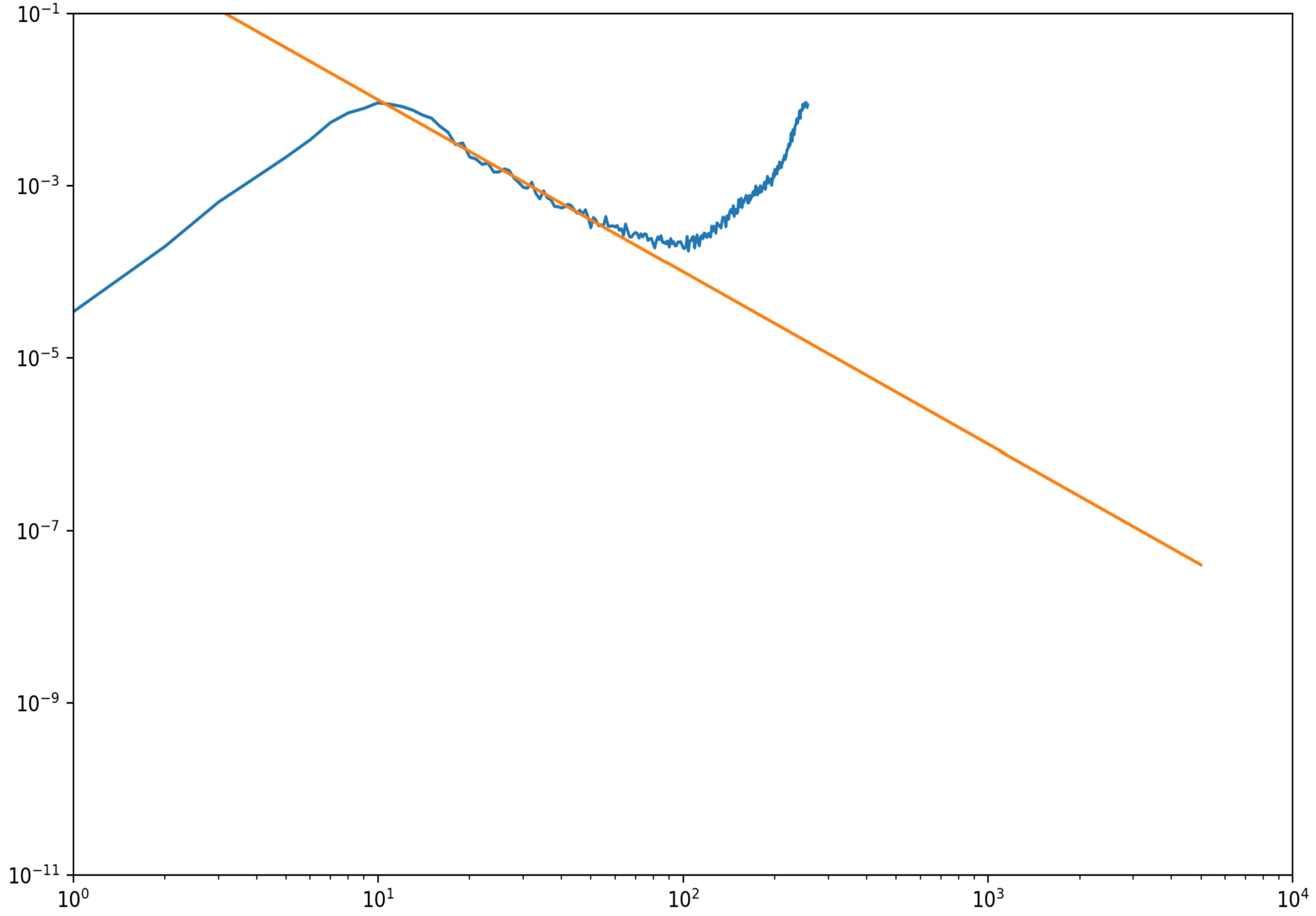


DECAYING BURGULENCE



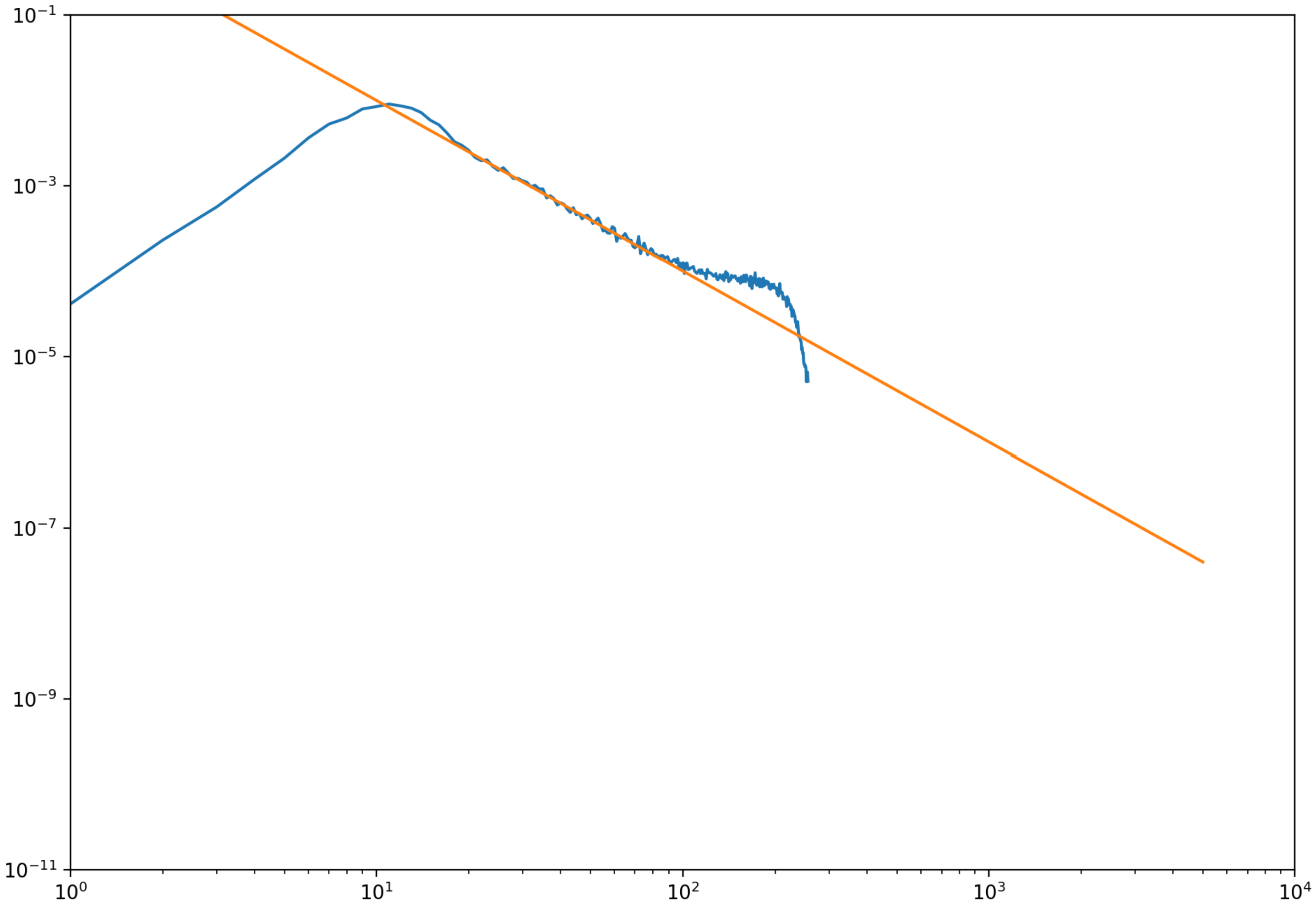
Direct Numerical Simulation

DECAYING BURGULENCE



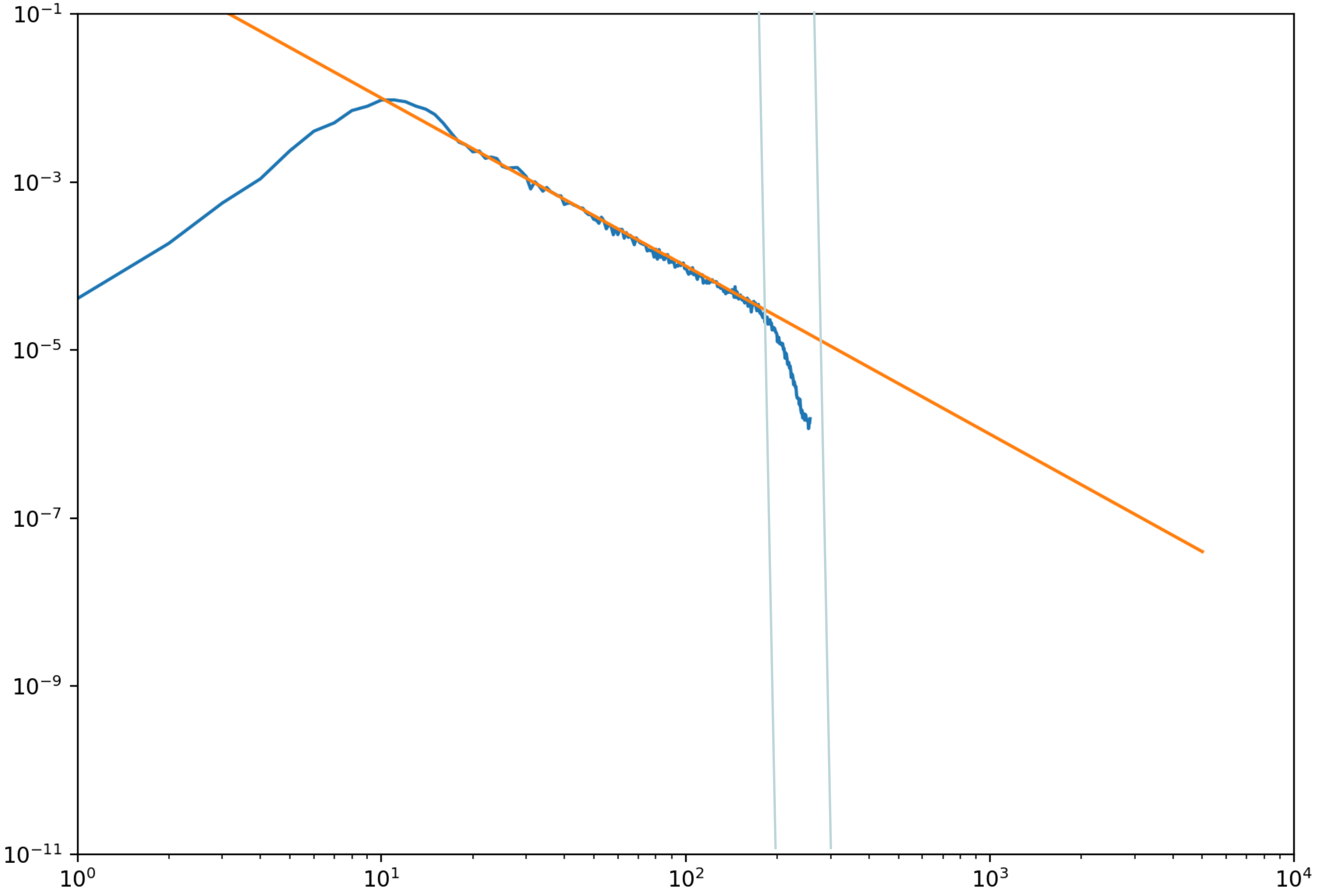
No Subgrid Model Simulation

DECAYING BURGULENCE



LES Smagronsky filterlength h

DECAYING BURGULENCE



LES filterlength 2h

TAKE-HOME

Cell-to-face interpolation introduces a second filter with a larger filter length