

## Conditions for the Conservative Discretization of Surface Tension in Interface-Capturing Methods.

N. Valle<sup>\*1</sup>, R.W.C.P Verstappen<sup>2</sup> and F.X. Trias<sup>3</sup>

<sup>1</sup> Delft University of Technology, n.vallemarchante@tudelft.nl

<sup>2</sup> University of Groningen, r.w.c.p.verstappen@rug.nl

<sup>3</sup> Technical University of Catalonia, francesc.xavier.trias@upc.edu

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The Direct Numerical Simulation (DNS) of multiphase flows is essential for the understanding, assessment and design of multiphase flow systems. DNS is essential in elucidating the fundamental flow features of these complex systems and can provide invaluable insights into the development of closure models for other engineering models. Its strength lies not only in the resolution of all relevant scales of the well-known Navier-Stokes equations, but also in constructing symmetry- (or, in general, structure-) preserving discretizations that mimic the physical laws at the discrete level. This implies the conservation of both primary (mass, momentum) but also secondary (e.g.: energy) properties, which have proven invaluable in the understanding of, for example, single-phase turbulent flows [1].

However, the inclusion of surface tension in interface-capturing methods does not preserve, in general, either momentum or energy [2]. In addition, its highly complex form is prone to the introduction of numerical pathologies, most remarkably spurious currents. While we have already made some advancements by developing an energy-preserving method for surface tension [3], the conservation of linear momentum is still elusive.

In this work, we review the geometric identities that ensure momentum conservation at the continuum level. Introducing a Conservative Level Set [4] regularization of the interface together with a Continuum Surface Force [5] model for the surface tension, we prove its validity at the regularized level. Subsequently, we extend these results, along with additional conditions, to the discrete level.

### References

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