



Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA



UNIVERSITÀ
DEGLI STUDI
DI PADOVA

Assessing the Impact of Bulk Viscosity in Compressible Turbulence

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Transonic flows: problems and solutions

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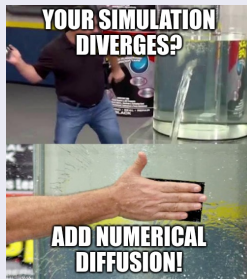
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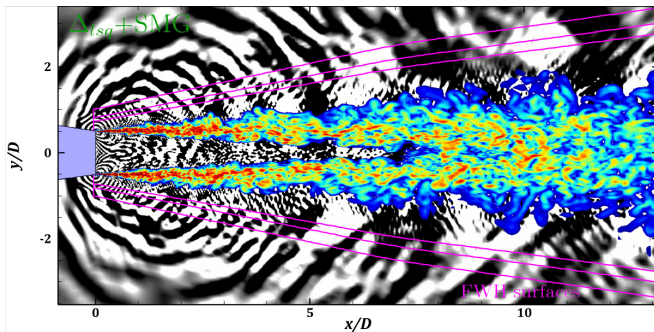
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Instantaneous flow field in a jet plume ($Re_D = 1.1e6$, $Ma = 0.9$)¹

¹A.P. Duben, J. Ruano, A.V. Gorobets, J. Rigola, and F.X. Trias. (2023). Evaluation of enhanced gray area mitigation approaches based on jet aeroacoustics. AIAA journal, 61(2), 612-625.

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Research question #3:

- Can we increase dissipation at **0 cost**?
 - Upwinding is a cheap option, so if there is an alternative it should be cheap too...

Navier-Stokes equations

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Incompressible formulation

Navier-Stokes equations

Incompressible formulation

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \tau \quad (1)$$

Navier-Stokes equations

Incompressible formulation

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Compressible formulation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u u) = -\nabla p + \nabla \cdot \tau$$

$$\frac{\partial p}{\partial t} + \nabla \cdot (p u) = -(\gamma - 1)p \nabla \cdot u + (\gamma - 1) \nabla \cdot (\kappa \nabla T) + (\gamma - 1) \Phi \quad (2)$$

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And for compressible flow??

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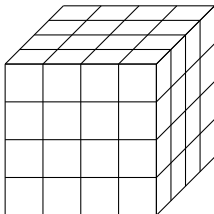
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Numerical setup

Box sized domain: $L_x = L_y = L_z = 1$.

Spectral resolution: $N_x = N_y = N_z = 128$ dealiased modes.

Algorithm: Pseudospectral with Logarithmic variables.

 Re_λ (initial): ~ 100 .

M_t (initial): 0.1-0.5-0.8.

 $Pr: 0.7$

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The other Re

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Decaying vs Forcing

- We keep the first two modes at constant energy levels.
- We avoid the temporal mismatch between viscosities.
- We can analyse the average effect of omitting Stokes hypotheses.

Logarithmic variables

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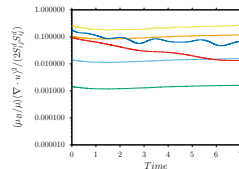
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Now, ρ and T are completely bounded!

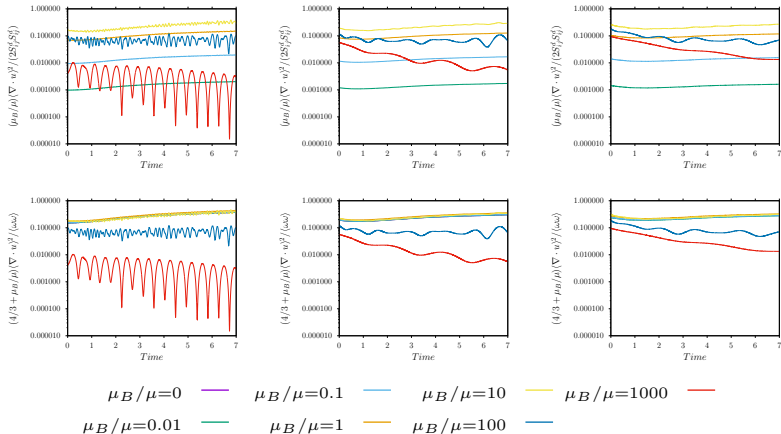
DHIT results: Dissipation ratios

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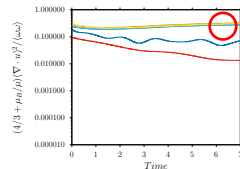
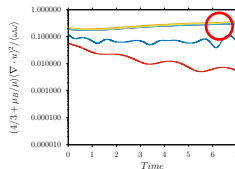
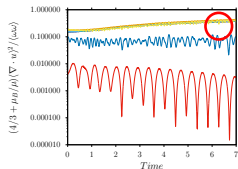
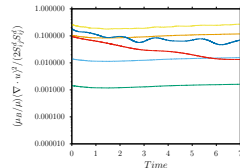
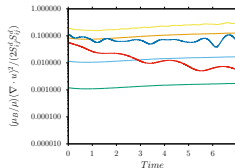
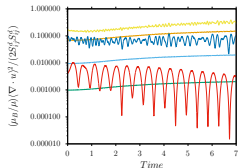


$\mu_B/\mu=0$ $\mu_B/\mu=0.1$ $\mu_B/\mu=10$ $\mu_B/\mu=1000$
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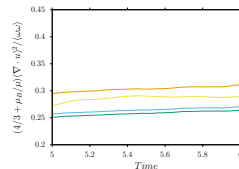
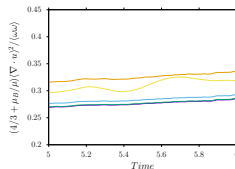
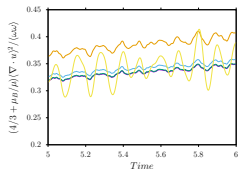
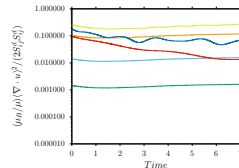
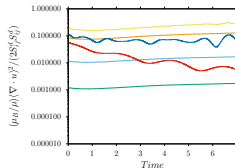
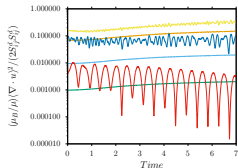
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$$\frac{(\mu_B/\mu)\langle(\nabla\cdot u)^2\rangle}{(2S_{ij}^d S_{ij}^d)}$$

M_t	μ_B/μ	0	0.01	0.1	1	10	100	1000
0.1		0	0.002	0.019	0.147	0.303	0.1	0.003
0.5		0	0.002	0.017	0.127	0.290	0.063	0.005
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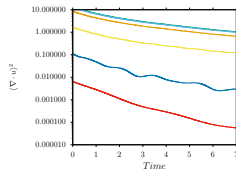
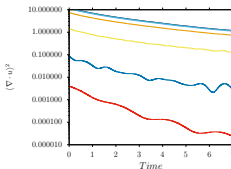
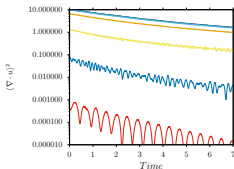
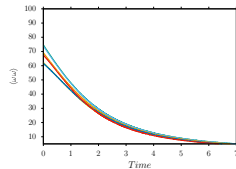
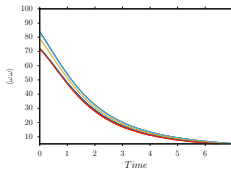
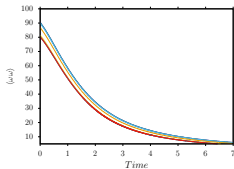
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0.8		5.10	5.095	5.047	4.727	4.303	4.337	4.200

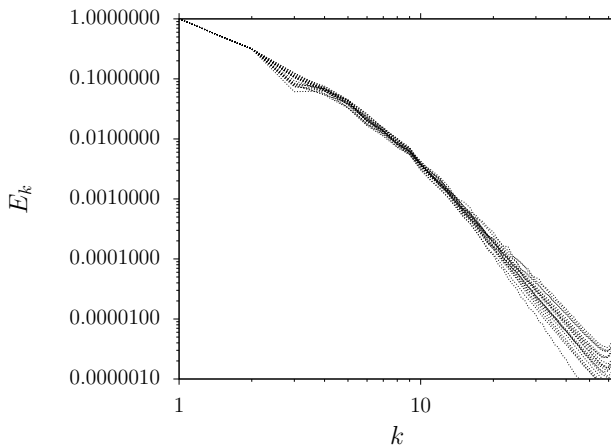
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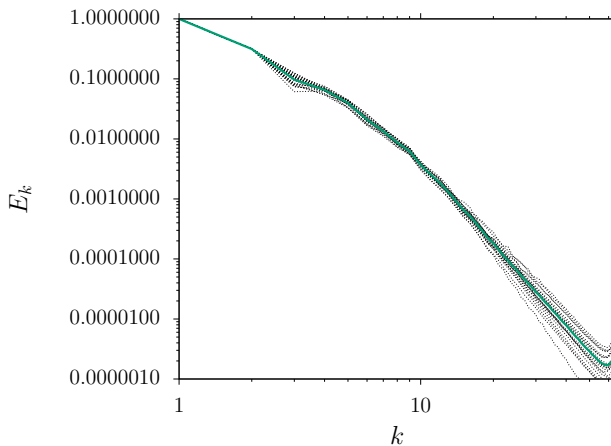
$\langle (\nabla \cdot u)^2 \rangle$								
M_t	μ_B/μ	0	0.01	0.1	1	10	100	1000
0.1		1.633	1.622	1.533	0.965	0.140	0.004	1e-5
0.5		1.185	1.180	1.120	0.737	0.130	0.003	2e-5
0.8		1.05	1.05	0.995	0.662	0.120	0.003	5e-5

FHIT results: Energy Spectra

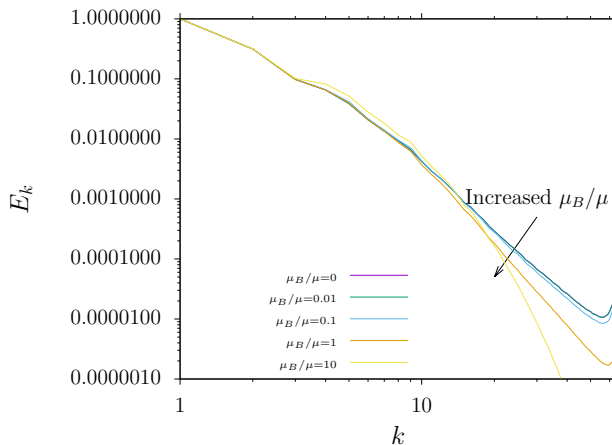
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FHIT results: Average values

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$(4/3 + \mu_B/\mu)\langle(\nabla \cdot u)^2\rangle/\langle\omega\omega\rangle$	0.649	0.716	0.823	0.972	0.902

Concluding remarks

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DHIT and FHIT results

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- **Two different maximums** appear:
 - A maximum bulk to shear dissipation rate around $\mu_B/\mu \sim 1-10$.
 - A maximum dilatational to solenoidal dissipation rate around $\mu_B/\mu \sim 1$.
- A slight reduction of the final enstrophy of the system when μ_B/μ increases.
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General conclusions

- Dissipation is increased, at maximum, **around 15-20%**.
 - But at 0 cost; $\nabla \cdot u$ is already being computed.

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General conclusions

- Dissipation is increased, at maximum, **around 15-20%**.
 - But at 0 cost; $\nabla \cdot u$ is already being computed.
- Dilatation is **extremely sensitive** to the bulk viscosity ratio; fluid becomes incompressible.
 - It may help to stabilize the simulation.

Further work

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High μ_B/μ and M_t results

We want to compute FHIT at the highest μ_B/μ ratios to assess extracted conclusions. Additionally, we want to compute results at $M_t = 0.5$ and 0.8

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Maximum dissipation regime

We want to determine the mechanisms that trigger the maximum dissipation regime.

Thanks for your attention