

CONSISTENT ENERGY-EXCHANGE DISCRETIZATIONS FOR MULTIPHYSICS FLOWS

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ABSTRACT

Large-scale DNS and LES are increasingly applied to multiphysics flows in which kinetic energy continuously exchanges with other energy forms, such as potential, internal, surface, or electromagnetic energy. Classical energy-preserving and symmetry-preserving discretizations have primarily focused on the conservation of quadratic invariants [1, 2], most notably kinetic energy, in order to control numerical dissipation and improve robustness [3, 4]. While this property is essential, it is not sufficient for multiphysics simulations, where the correct representation of energy exchanges between different physical fields is often more relevant than the preservation of a single invariant.

In this work, we revisit symmetry-preserving discretizations of the incompressible Navier–Stokes equations from the perspective of discrete energy exchanges. We show that preserving fundamental operator symmetries at the discrete level not only guarantees the conservation of quadratic invariants, but also provides a consistent and physically meaningful framework for coupling kinetic energy with other energy reservoirs. Importantly, a numerical method may conserve total energy while still producing spurious inter-energy transfers, leading to qualitatively incorrect multiphysics dynamics.

This viewpoint is particularly relevant for buoyancy-driven flows [5, 6], multiphase systems with surface tension [7], and magnetohydrodynamic problems [8], where artificial energy exchanges can severely contaminate the solution. Emphasis is placed on formulations suitable for complex physics and complex geometries, requiring unstructured collocated meshes and robust pressure–velocity coupling strategies.

The proposed approach highlights consistent energy-exchange preservation as a guiding principle for the design of artificial-dissipation-free, stable, and physically consistent discretizations for multiphysics flows on unstructured grids.

References

- [1] G. Coppola, F. Capuano, and L. de Luca. Discrete energy-conservation properties in the numerical simulation of the Navier–Stokes equations. *Applied Mechanics Reviews*, 71(1):010803, 2019.
- [2] A. E. P. Veldman. A general condition for kinetic-energy preserving discretization of flow transport equations. *Journal of Computational Physics*, 398:108894, 2019.
- [3] F. X. Trias, O. Lehmkuhl, A. Oliva, C.D. Pérez-Segarra, and R.W.C.P. Verstappen. Symmetry-preserving discretization of Navier–Stokes equations on collocated unstructured meshes. *Journal of Computational Physics*, 258:246–267, 2014.
- [4] D. Santos, J. A. Hopman, C. D. Pérez-Segarra, and F. X. Trias. On a symmetry-preserving unconditionally stable projection method on collocated unstructured grids for incompressible flows. *Journal of Computational Physics*, 523:113631, 2025.
- [5] F. X. Trias, M. Soria, A. Oliva, and C. D. Pérez-Segarra. Direct numerical simulations of two- and three-dimensional turbulent natural convection flows in a differentially heated cavity of aspect ratio 4. *Journal of Fluid Mechanics*, 586:259–293, 2007.
- [6] B. Sanderse and F. X. Trias. Energy-consistent discretization of viscous dissipation with application to natural convection flow. *Computers & Fluids*, 286:106473, 2025.
- [7] N. Valle, F. X. Trias, and J. Castro. An energy-preserving level set method for multiphase flows. *Journal of Computational Physics*, 400(1):108991, 2020.
- [8] J. A. Hopman, J. Rigola, and F. X. Trias. A checkerboard-free, symmetry-preserving, conservative method for magnetohydrodynamics. *Journal of Computational Physics*, 545:114475, 2026.