

ON THE COMBINED EFFECT OF MESH ANISOTROPY AND SCHEME ACCURACY IN THE PERIODIC HILLS CASE

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Summary. *This article presents the investigation on the combined effect of mesh anisotropy, scheme accuracy, and turbulence model closure selection towards their effects on obtaining accurate numerical results of the periodic hills test case. Obtained results show that, when stretching exceeds a certain threshold, results obtained by low-order numerical algorithms can match the performance of higher-accuracy options.*

1 INTRODUCTION

Before starting a numerical simulation within Computational Fluid Dynamics (CFD), the user in front of the computer must make some critical decisions that will affect, ultimately, the quality of the obtained results.

The first selection is the mesh. The domain under study must be discretized in order to apply a numerical method; however, there is not a single option but a wide variety. Ranging from structured to non-structured grids, or octree grids where the shape is not exactly preserved, or body-fitted grids that completely capture the geometrical profiles under study, a mesh must be proposed. Among others, the most common approach within industrial frameworks is to employ some stretching, so that the mesh is no longer uniform but may still be structured, or not. This significantly reduces the computational effort as the number of control volumes required to mesh the domain under study decreases.

The second thing that the user needs to select, is the numerical scheme. The discretized operators that constitute the set of numerical equations to be resolved require both a discrete physical space, provided by the mesh, and a scheme. Again, the selection is not restricted to a single option. Schemes can be both low-order or high-order, depending on the order of convergence they provide. They can be centered or upwinded, depending on the influence, i.e.,

the weights, the surrounding nodes, or cells have in the numerical approximation. They can even be a mixture of low, high, centered, and upwinded simultaneously.

And finally, when turbulence is involved, the user needs to select a closure; in other words, how to deal with turbulence. The most straightforward option relies on Direct Numerical Simulation (DNS), in which turbulence is directly solved by refining the mesh sufficiently so that all the scales of motion, ranging from the most energetic ones up to the smallest, i.e., Kolmogorov length scale, are resolved. Large Eddy Simulations (LES) alleviate the computational cost by modeling a portion of the kinetic energy spectrum, whereas Reynolds Averaged Navier-Stokes (RANS) techniques model it completely. Or we can even have something in between LES and RANS, such as Detached Eddy Simulations (DES), with all its related subfamilies.

Consequently, all the selections in this initial effect will have an impact on the quality of the results obtained. Academia and industry do not usually have the same simulation targets. For this reason, the methods and selections used on one do not necessarily match those on the other. For example, there is a critical distinction between the meshes used in the academy and those used in industry daily: their "beauty". How meshes used in academic cases can show smooth transitions while preserving all geometric details. Or how meshes used in the industry require more aggressive stretching to reduce the overall computational cost for an otherwise excessively costly simulation. Within an academic environment, the periodic hills [1–3] case has been used as a simplified model to reproduce cases with a non-geometric flow detachment point and reattachment present in real-world phenomena. Its geometrical simplification has enabled the use of very smooth meshes that transition from the wall, where a finer mesh is required along the normal direction, to the furthest regions, which do not have as strict mesh requirements when LES or RANS models are used.

In this work, we aim to analyze the combined effect of anisotropies present in real-world-like meshes, along with the formal order of accuracy of the scheme used. To achieve our goal, we have divided this study into two main blocks. In the first, the periodic hills test case will be studied on a set of refined meshes with smooth transitions using LES models and two different numerical codes with different orders of accuracy: NOISEtte [4, 5], a high-accuracy EBR in-house code, and OpenFOAM, an open-source low-order code widely used in both academia and industry. In the second part, we will use another set of refining meshes with distorted regions and increased stretching factors to mimic what a mesh used in more industrial work would look like. This will help us to assess the differences between low- and high-accuracy schemes when applied to a set of smooth-transitioning meshes and to understand how these differences change as meshes begin to lose their uniformity.

2 CASE FORMULATION AND NUMERICAL ALGORITHMS

The Periodic hill test case at $Re_H = 10595$ based on the hill height, H , is considered. The domain's length and spanwise width are based on the hill's height, i.e., $L = 9H$ and $W = 4.5H$.

In order to fix the Reynolds number, the average velocity at the inlet, U_b , should be a fixed parameter. To do so, the pressure gradient is computed recursively at each time step following Benocci and Pinelli [6] and Krank et. al. [3], using a PID-form such that desired U_b is achieved.

Table 1: Uniform meshes parameters.

Mesh characteristics	Coarse	Medium	Fine
$N_x N_y N_z$	140x110x100	210x165x150	280x220x200
Total cell count (CV)	1.54M	5.2M	12.32M
$\Delta x/H$	0.064	0.043	0.032
$\Delta z/H$	0.045	0.03	0.0225
$\max(\Delta y/H)$	0.06	0.04	0.03
$\max(\Delta y/H)$ wall	0.0024	0.0016	0.0012
$\max(\Delta y/\Delta x, z)$	26.26	26.54	26.08

Table 2: Stretched meshes parameters.

Mesh characteristics	Smooth	Medium	Aggressive
$N_x N_y N_z$	167x110x140	138x110x140	110x110x140
Total cell count (CV)	2.6M	2.1M	1.7M
Stretching	5%	5-10%	10%
$\Delta x/H$ at detachment	0.015	0.015	0.015
$\Delta x/H$ at reattachment	0.017	0.017	0.017
$\max(\Delta y/H)$	0.06	0.06	0.06
$\max(\Delta y/H)$ bottom wall	0.0024	0.0024	0.0024
$\max(\Delta y/\Delta x, z)$	180	344	295.7

A set of 3 refining meshes is used in both uniform and stretched studies. Regarding uniform meshes, the finest mesh is based on Breuer's [2] LES mesh. The coarsest is obtained by halving the number of CV in each direction, while keeping the mesh resolution near the walls at $\Delta y^+ \sim 1$.

Regarding non-uniform meshes, i.e., stretched in the streamwise direction, they are obtained by refining the mesh near experimental detachment and reattachment points. An average stretching of around 5% is imposed in the smoothest mesh, whereas a stronger stretching of around 10% is imposed in the most aggressive one. The minimum mesh size in the streamwise direction at detachment and reattachment zones has been fixed at half the mesh size of the finest uniform grid; this helps to ensure that maximum mesh sizes, a function of both stretching factor and minimum mesh size, are still small enough so LES can capture all the relevant scales. We have summarized the meshes' main parameters in Tables 1 and 2.

As noted in the introduction, one of the critical choices in a numerical simulation is which discretization scheme is used. In order to assess its effect on the quality of the obtained results, two codes with considerably different numerical schemes have been used.

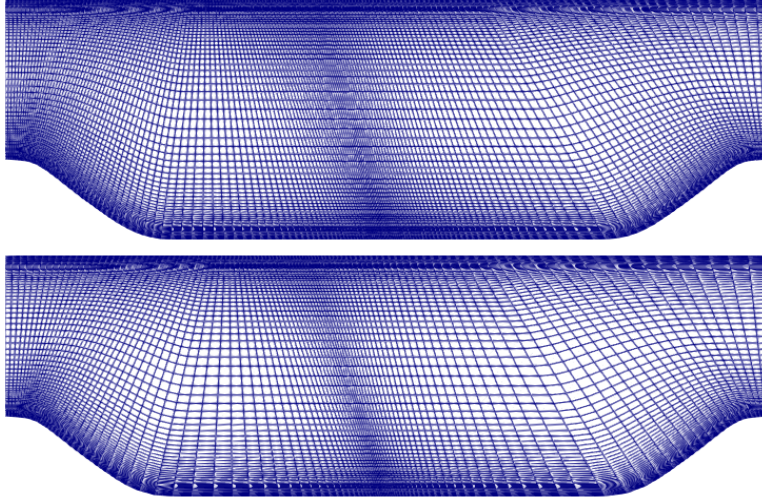


Figure 1: Used grids on the second part. Top: Smooth stretched; Bottom: Aggressive stretched.

The first code is the NOISEtte in-house research code [4, 5]. NOISEtte is based on quasi-1D vertex-centred Edge-Based Reconstruction (EBR) [7]. The convective term uses a blending of central-difference and 1% upwind schemes [8] with the special hybridizing function extracted from Guseva et al. [9]. The second code is OpenFOAM [10, 11]. OpenFOAM code is based on a collocated unstructured finite-volume approach. The convective term consists of a pure centered 2nd-order central-difference scheme.

The initial state is obtained using a RANS simulation to speed up the whole simulation process. To avoid initial-state effects from contaminating the results, simulations are run for approximately 16 flow-through time units. Once this statistically stationary state is achieved, results are averaged in both the spanwise direction and time over 80 flow-through times.

3 RESULTS AND DISCUSSION

In this section, the numerical results will be discussed and compared with reference data [3].

3.1 Uniform grids results

The time-averaged results for both streamwise velocity and kinetic energy are shown. For the sake of brevity, results in the finest mesh are only shown in OpenFOAM in Figure 2. As can be seen, both magnitudes show excellent agreement with the reference results, regardless of the used turbulence model or the filtering length scale. The main differences are located near the walls, between $x/L = 3$ and 5. This is precisely the reported reattachment point, around 4.51 ± 0.06 in [3], and 4.69 in [2]. However, the reattachment point found in this simulation is below both references and around 4.4, closer to the estimated point by Rapp [12], about 4.21 – 4.28.

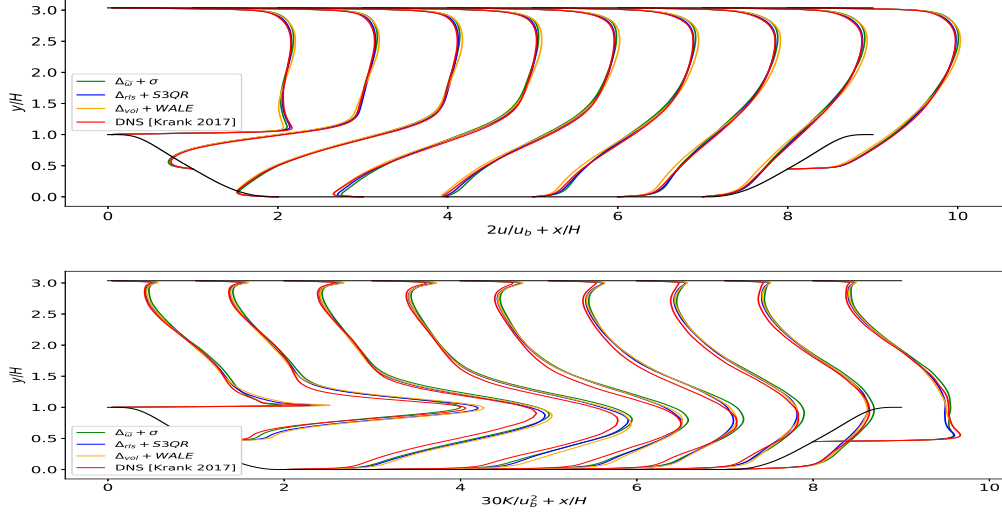


Figure 2: Normalized streamwise velocity, U , and kinetic energy, K , in the finest mesh in OpenFOAM.

Nonetheless, it should be noted that the reported Re numbers for Rapp's experiments are slightly lower than the nominal value of 10595. The kinetic energy also exhibits the highest differences in the same region. At approximately $x/L = 3$, the obtained values in the simulation are slightly higher, around 3%, than the expected ones for all the selected models.

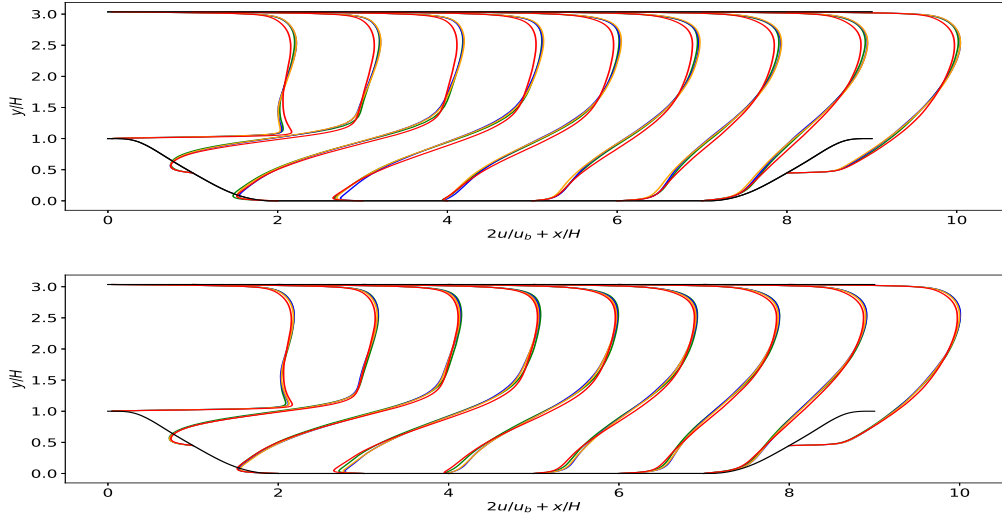


Figure 3: Normalized streamwise velocity, U , in coarse and medium mesh in OpenFOAM.

Regarding the mesh refinement effect, coarse and medium meshes underpredict the velocity on the crest of the hill, with this underprediction becoming smaller as the mesh is refined, as seen in Figures 3 and 4. The main differences between codes are, unsurprisingly, located at the bottom wall between $x/L = 3$ and 5. OpenFOAM seems to be converging towards a reattachment point closer to the experimental ones, indicating a slightly higher diffusive component than NOISEtte, which seems to be converging towards a reattachment point closer to the DNS-reported ones.

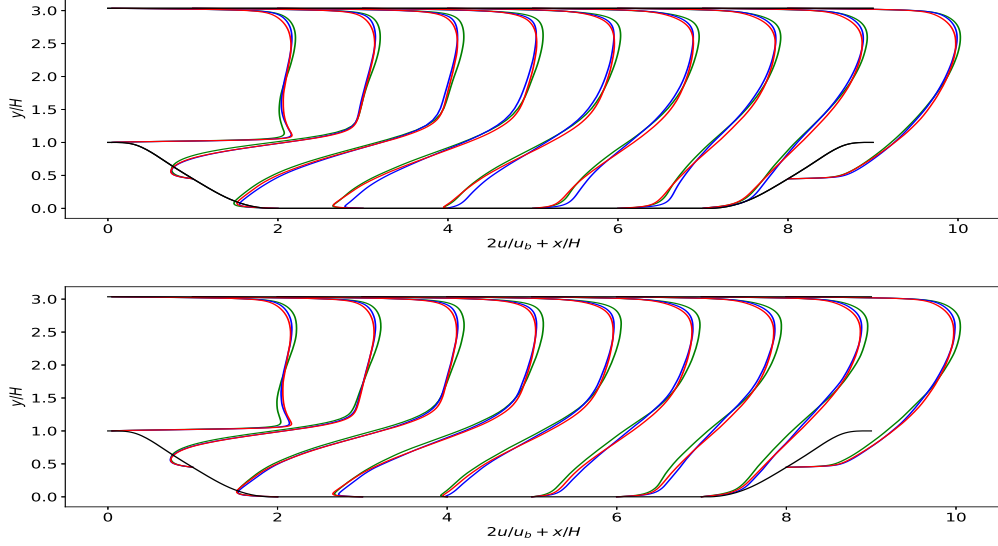


Figure 4: Normalized streamwise velocity, U , in coarse and medium mesh in NOISEtte.

The kinetic energy clearly captures the trend of the reference results, with NOISEtte showing near-perfect agreement on the medium mesh, as shown in Figure 6, whereas the OpenFOAM results, in Figure 5, still differ significantly from them.

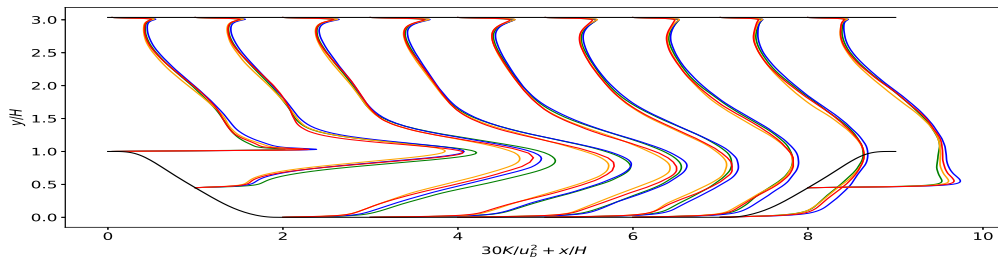


Figure 5: Normalized kinetic energy, K , in medium mesh in OpenFOAM.

As previously commented, differences observed in Figure 5 between OpenFOAM results and the Direct Numerical Simulation from [3] may be attributed to the dissipative nature of OpenFOAM observed by Komen [13], as the effective Re number is slightly different, i.e., OpenFOAM converges towards a more laminar regime than the reference one.

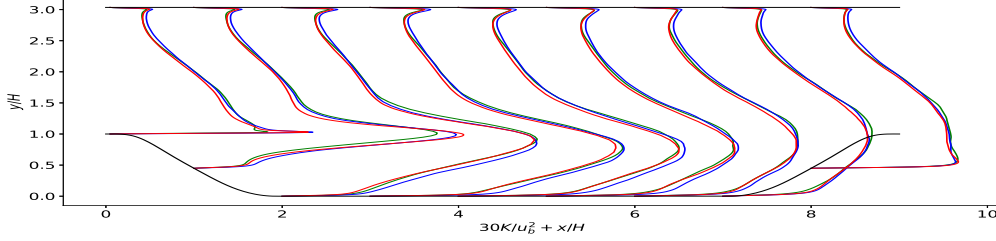


Figure 6: Normalized kinetic energy, K , in medium mesh in NOISEtte.

3.2 Stretched grids results

Regarding the results obtained with stretched grids, more differences are observable now. In OpenFOAM, in Figure 7, all the results are characterized by an overprediction of the maximum velocity in the upper part of the domain. Main differences between the aggressive and medium stretching are located in the bottom part, near the walls, where the option $\Delta_{vol} + WALE$ differs from the other options by underpredicting the velocity field.

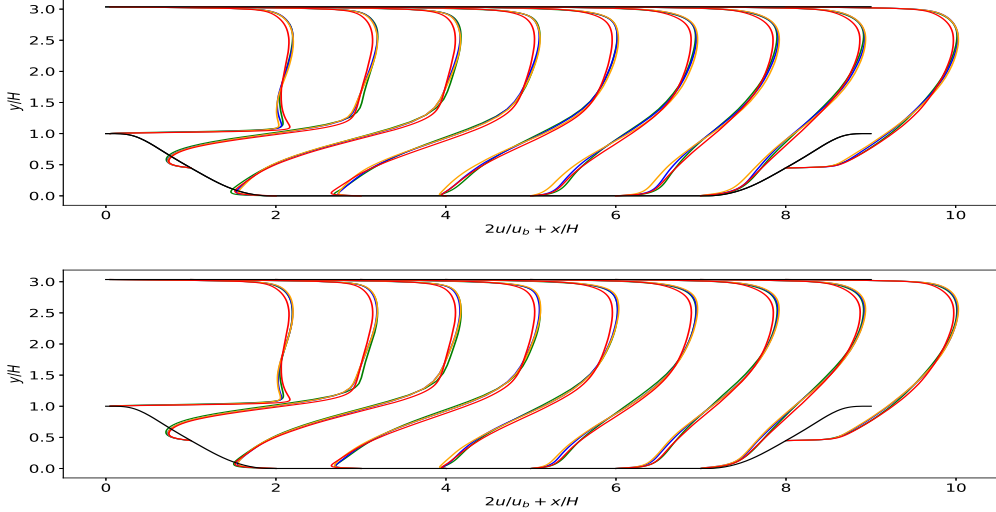


Figure 7: Normalized streamwise velocity, U , in aggressive and medium stretching in OpenFOAM.

NOISEtte, on the other hand, shows an overprediction of the velocity profiles in the lower part when the combination $\tilde{\Delta}_\omega$ [14] together with the σ turbulence model. Both codes exhibit better performance in the bottom part with $\tilde{\Delta}_{rls} + S3QR$ [15, 16]. Nonetheless, this option generates the previously detected overprediction of the velocity profiles in the upper part.

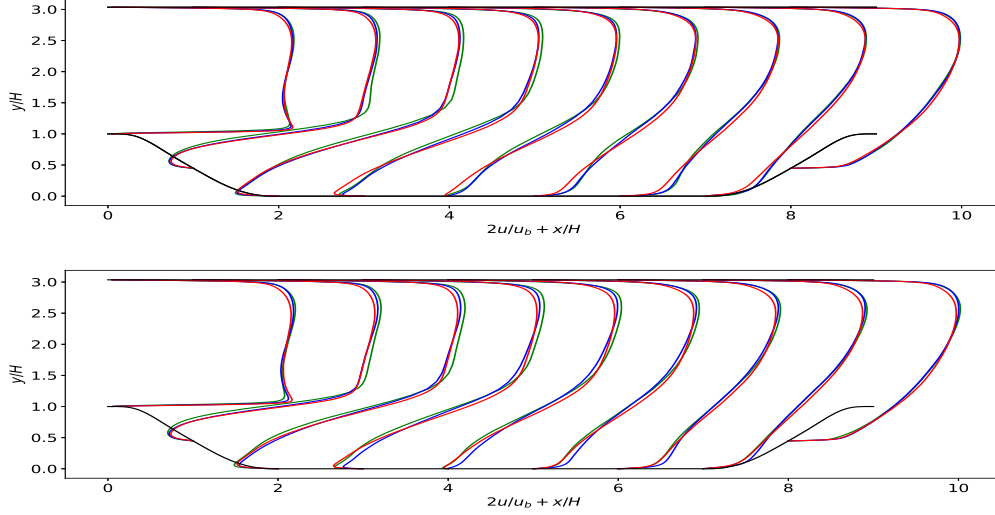


Figure 8: Normalized streamwise velocity, U , in aggressive and medium stretching in NOISEtte.

Finally, the kinetic energy profiles obtained by both codes differ greatly. On the one hand, OpenFOAM provides very similar results in both aggressive and medium stretching factors, as shown in Figure 9. Both $\tilde{\Delta}_\omega$ and Δ_{vol} options provide very similar solutions, being independent of the stretching factor used. Both options are characterized by underpredicting velocity fluctuations in the lower part of the domain. The profiles obtained using these options underpredict the peak in kinetic energy by about 5-10%. $\tilde{\Delta}_{rls} + S3QR$ results are very similar in both meshes and, unlike previous options, overpredict the kinetic energy peak by about the same magnitude. On the other hand, NOISEtte results, in Figure 10, differ greatly from the medium stretched mesh to the aggressive stretched one. The kinetic energy in the medium-stretched grid has no significant differences between the obtained results and the reference values. Nonetheless, the results obtained on the most aggressive stretched grid show patterns similar to those obtained by OpenFOAM. $\tilde{\Delta}_{rls} + S3QR$ overpredicts the kinetic energy profiles by a greater margin than OpenFOAM, surpassing, in the worst point, the 10% difference. $\tilde{\Delta}_\omega$ together with the σ turbulence model is less affected by the increased stretching, showing just a slight overprediction of the peak kinetic energy.

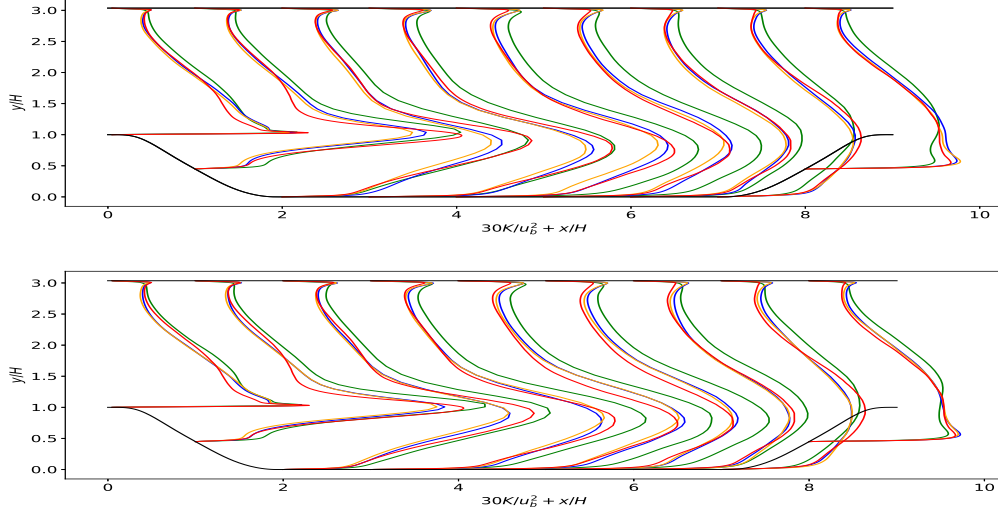


Figure 9: Normalized kinetic energy, K , in aggressive and medium stretching in OpenFOAM.

4 CONCLUSIONS

The numerical results of the periodic hill test case using two different schemes on a set of refining and stretched meshes have been presented. Overall, the selected approaches have shown good agreement with Krank reference results.

For uniform meshing strategies along the stream and spanwise directions, both codes have provided acceptable results. OpenFOAM velocity profiles are characterized by converging towards a slightly more laminar regime than the one in reference DNS results. In this sense, the reattachment point found in the first part of the work using OpenFOAM is closer to the experimental results from Rapp, with a slightly lower reported Reynolds number than the nominal 10595 than the reattachment point reported by Breuer or Krank.

Both OpenFOAM and NOISEtte have shown an overprediction of less than 3% for the velocity profiles in the upper part of the domain on both coarse and medium meshes, attributed to insufficient mesh resolution. Nonetheless, mesh refinement from the coarsest to the medium mesh has reduced this overprediction.

Kinetic energy profiles have been better predicted by NOISEtte than OpenFOAM. In NOISEtte's case, both turbulent closure options have shown perfect agreement with Krank's results, with no significant differences between them. In the OpenFOAM case, Δ_{vol} has slightly underpredicted kinetic energy peaks, whereas the other two selected approaches, $\tilde{\Delta}_{rls} + S3QR$ and $\tilde{\Delta}_{\omega}$, have overpredicted the maximum values. Nonetheless, it should be noted that the differences are, in the worst case, less than 7%. Therefore, in the uniform meshing approach, although both numerical schemes achieved extremely high accuracy, NOISEtte presents a more robust option in comparison with OpenFOAM.

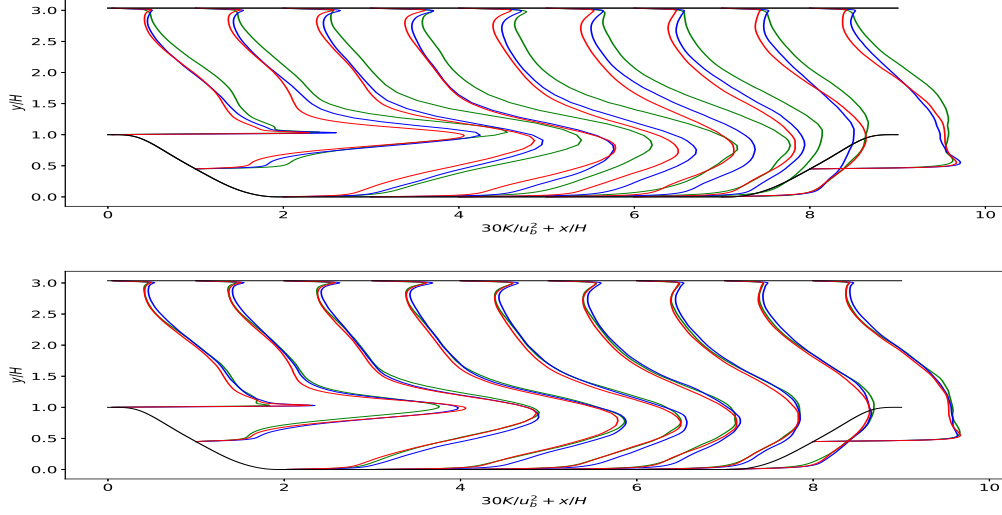


Figure 10: Normalized kinetic energy, K , in aggressive and medium stretching in NOISEtte.

Regarding the results when stretched grids are used, these showed greater differences among the codes used. The velocity profiles obtained in both codes, even now that grids are no longer uniform in the streamwise direction, still match the literature results reasonably well. OpenFOAM shows a slight overprediction of the obtained velocity magnitudes in the upper part of the domain, regardless of the turbulence closure option or the mesh used. This is explained by the lack of resolution, together with the implicit diffusion added by the velocity-pressure coupling implemented in the code [13]. In the most aggressive stretching case, using Δ_{vol} distorts the velocity profiles near the bottom wall near the reattachment point. NOISEtte's velocity profiles are characterized by two main aspects. The first one is that when stretching is used, the $\tilde{\Delta}_\omega$ option starts to deviate from the reference results regardless of the used stretching factor. On the other hand, $\tilde{\Delta}_{rls}$ exhibits a more robust behavior, and only in the most aggressive case presents deviation from selected reference results.

Kinetic energy is where both codes have exhibited greater differences. OpenFOAM is less affected by increasing the stretching factor, as the trends are the same in both the most aggressive and medium-stretched grids. $\tilde{\Delta}_{rls}$ overpredicts the maximum peak of kinetic energy by about 7% in the worst case, whereas the other selected turbulent closure options underpredict it by a similar magnitude. On the other hand, NOISEtte is more sensitive on the used stretching factors. In the case of medium-stretched grids, the results obtained in NOISEtte almost perfectly match the reference results, with only a small overprediction when $\tilde{\Delta}_\omega$ is used. On the other hand, in the most aggressive stretched grid, the results deviate. This deviation is more noticeable when $\tilde{\Delta}_{rls}$ is used, with differences slightly above 10%.

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