Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Building a proper tensor-diffusivity model for large-eddy simulation of buoyancy-driven flows

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6th European Conference on Computational Mechanics (Solids, Structures and Coupled Problems) ECCM 6

7th European Conference on Computational Fluid Dynamics ECFD 7

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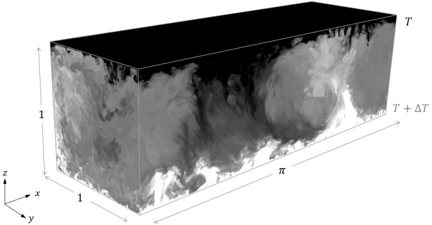
Contents

Motivation

Research question:

- Can we find a nonlinear SGS heat flux model with **good physical** and **numerical properties**, such that we can obtain satisfactory predictions for a turbulent Rayleigh-Bénard convection?



DNS of an air-filled Rayleigh-Bénard convection at $Ra = 10^{10}$

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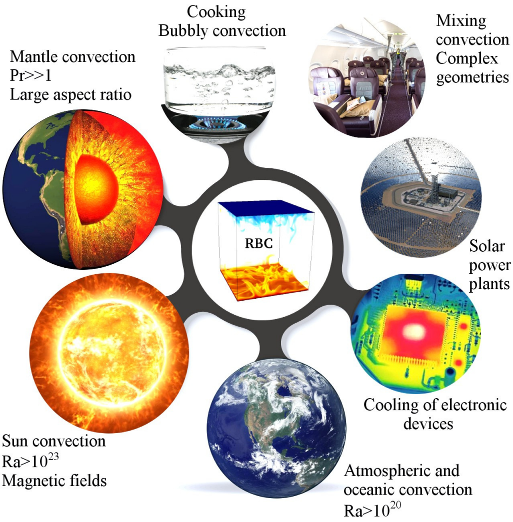
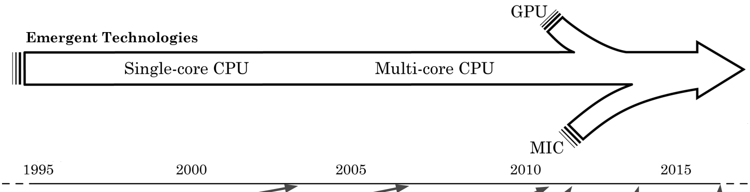
[1] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *On the evolution of flow topology in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 28:115105, 2016.

Motivation


Cooking
Bubbly convection
Mixing convection
Complex geometries
Mantle convection
Pr>>1
Large aspect ratio
RBC
Solar power plants
Cooling of electronic devices
Sun convection
Ra>10^23
Magnetic fields
Atmospheric and oceanic convection
Ra>10^20

Motivation

And of course... saving the planet!



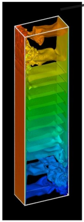
Emergent Technologies

Single-core CPU

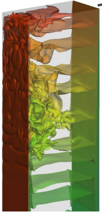
Multi-core CPU

GPU

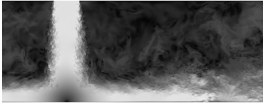
MIC



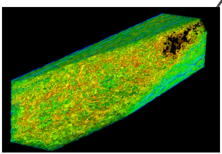
DHC
$Ra=10^9$
(3.2M)



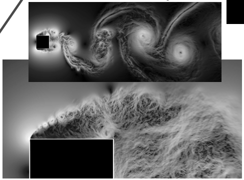
DHC
$Ra=10^{11}$
(111M)



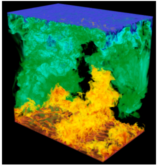
Impinging jet
Re=20000
(102M)



DUCT Re_t=1200 **(172M)**



SqCyl Re=22000 **(324M)**



RB
$Ra=10^{10}$ **(607M)**
$Ra=10^{11}$ **(5600M)**

RES
RED ESPAÑOLA DE SUPERCOMPUTACIÓN

PRACE

Eddy-viscosity models for LES

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nabla^2 \overline{u} - \nabla \overline{p} - \nabla \cdot \tau(\overline{u}) \;\; ; \qquad \nabla \cdot \overline{u} = 0$$

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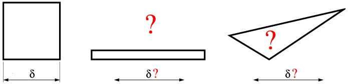

δ?

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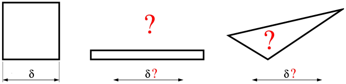

δ

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[3] F.X.Trias, A.Gorobets, M.Silvis, R.Verstappen, A.Oliva. *A new subgrid characteristic length for turbulence simulations on anisotropic grids*, **Phys.Fluids**, 26:115109, 2017.

How to model the subgrid heat flux in LES?

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$$Pr_t ?$$

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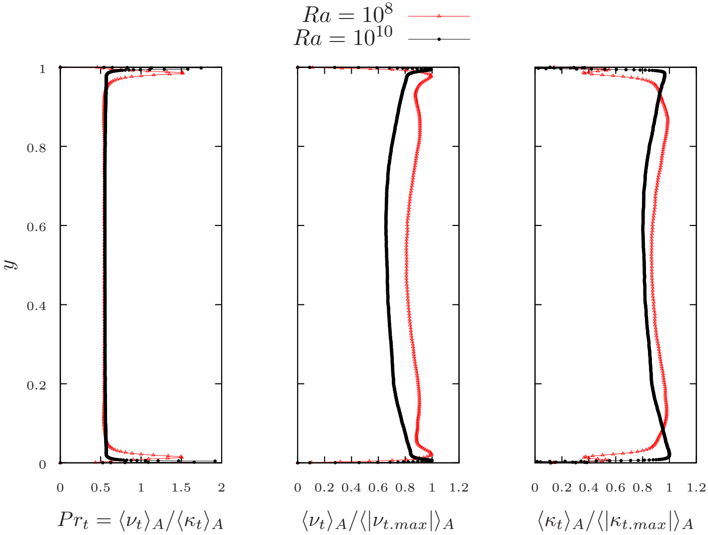

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$$G \equiv \nabla\overline{u} \qquad\qquad q = -\frac{\delta^2}{12}G\nabla\overline{T} + \mathcal{O}(\delta^4)$$

A priori alignment trends[4]

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[4] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *A priori study of subgrid-scale features in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 29:105103, 2017.

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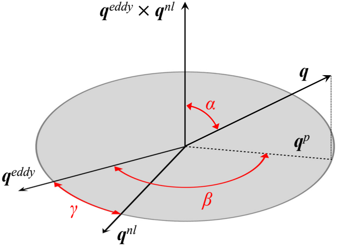

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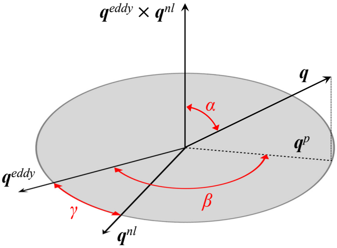

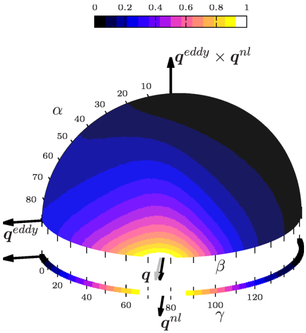

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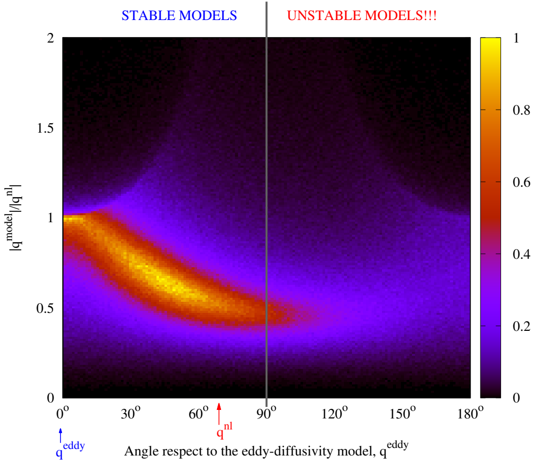
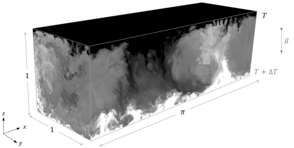
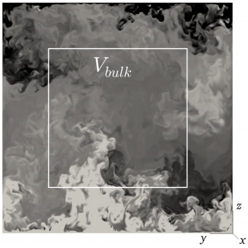
A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T} \qquad \boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$







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$$\text{mixed model} \quad \longrightarrow \quad q \approx q^{nl} + \sigma q^{eddy} \qquad\qquad (\equiv q^{mix})$$

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[7] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

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$$\partial_t \overline{T} + (\overline{u}\cdot\nabla)\overline{T} = \alpha\nabla^2\overline{T} - \nabla\cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

$$\text{mixed model} \quad \longrightarrow \quad q \approx q^{nl} + \sigma q^{eddy} \qquad (\equiv q^{mix})$$

$$\text{Daly\&Harlow}^7 \quad \longrightarrow \quad q \approx -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T} \qquad (\equiv q^{DH})$$

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[7] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \; ; \qquad \nabla \cdot \overline{u} = 0$$

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A priori alignment trends

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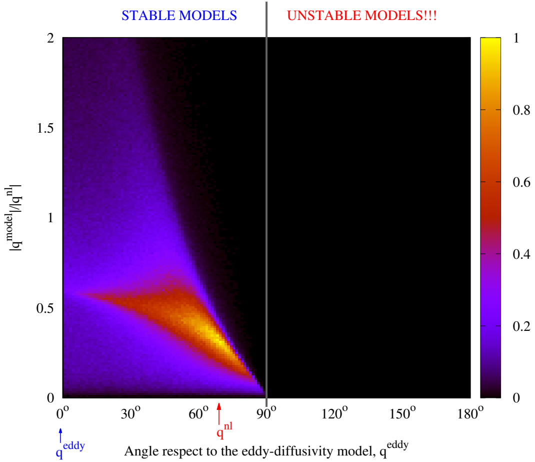
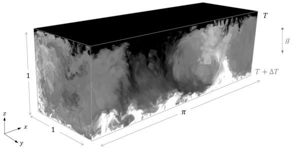
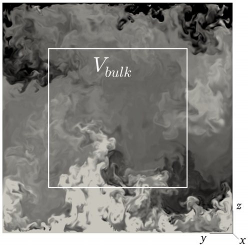
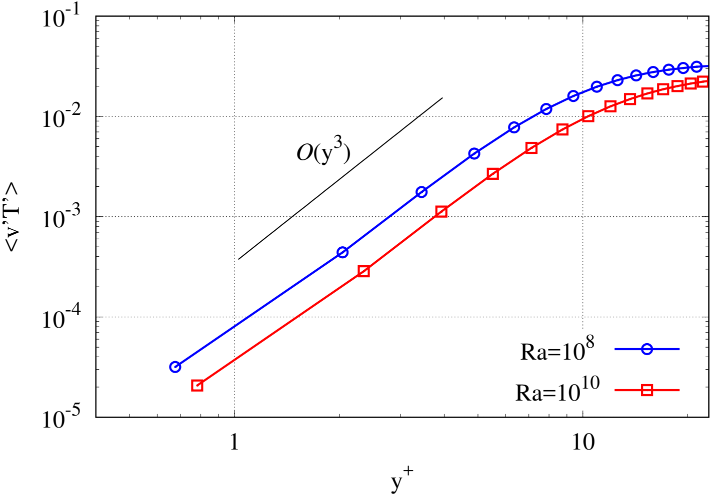






Near-wall scaling



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$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\;; \qquad \mathcal{T}_{SGS} = 1/|S|}$$

$$u = ay + \mathcal{O}(y^2);\;\; v = by^2 + \mathcal{O}(y^3);\;\; w = cy + \mathcal{O}(y^2);\;\; T = dy + \mathcal{O}(y^2)$$

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$$G = \begin{pmatrix} y & 1 & y \\ y^2 & y & y^2 \\ y & 1 & y \end{pmatrix}; \;\; \nabla T = \begin{pmatrix} y \\ 1 \\ y \end{pmatrix}$$

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$$\boxed{\mathcal{T}_{SGS} = 1/|S| = \mathcal{O}(y^0)}$$

Idea: build a $\mathcal{T}_{SGS}$ with the proper $\mathcal{O}(y^2)$ scaling!!!

Building proper models for the subgrid heat flux

Let us consider models that are based on the invariants of the tensor GG^T

$$q \approx -C_M \left(P^p_{GG^T} Q^q_{GG^T} R^r_{GG^T} \right) \frac{\delta^2}{12} GG^T \nabla T \qquad (\equiv q^{S2})$$

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	P_{GG^T}	Q_{GG^T}	R_{GG^T}
Formula	$2(Q_\Omega - Q_S)$	$V^2 + Q_G^2$	R_G^2

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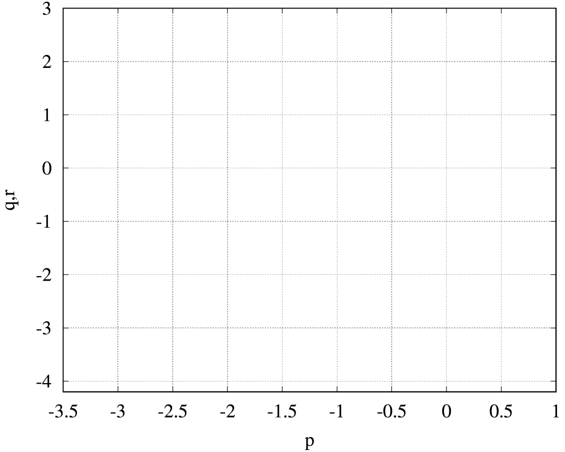
$$-6r - 4q - 2p = 1 \;\; [T]; \qquad 6r + 2q = s,$$

where s is the slope for the asymptotic near-wall behavior, *i.e.* $\mathcal{O}(y^s)$.

Building proper models for the subgrid heat flux

Solutions: $q(p, s) = -(1 + s)/2 - p$ and $r(p, s) = (2s + 1)/6 + p/3$



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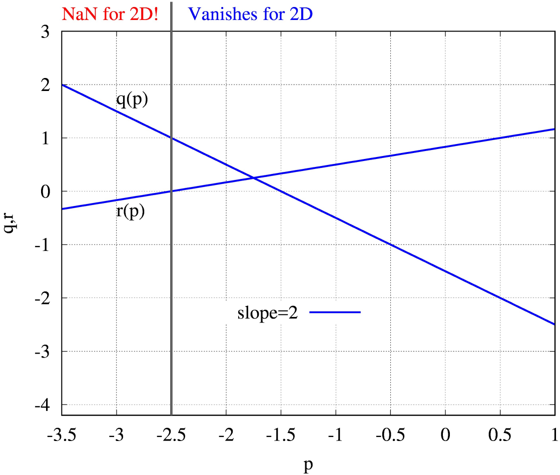

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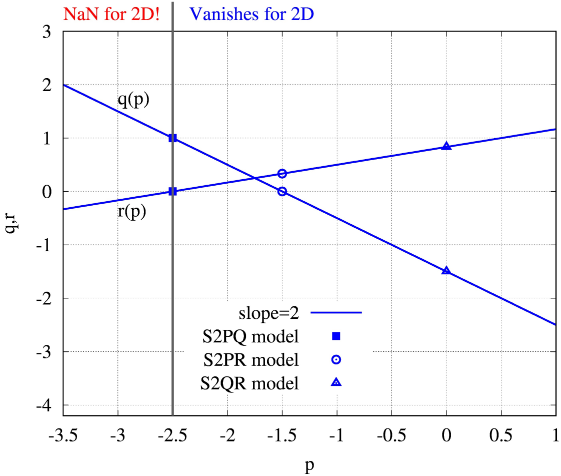

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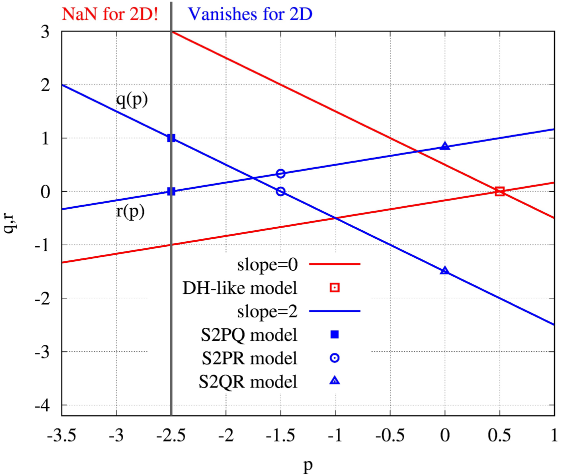
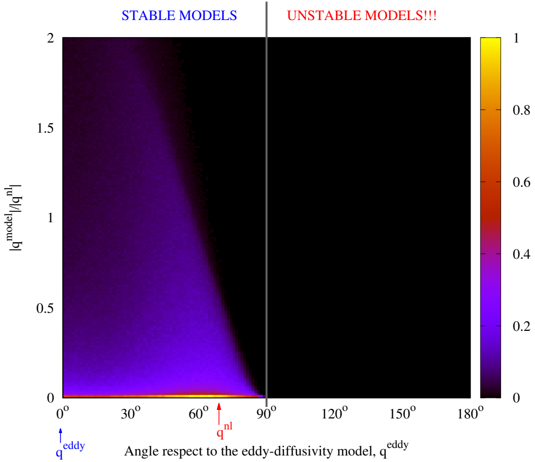
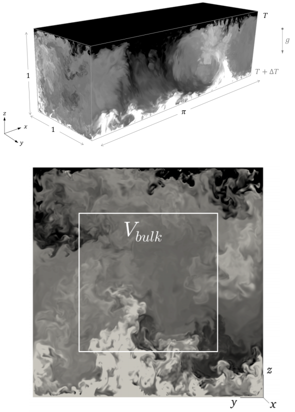


A priori alignment trends of S2QR

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

$$\boxed{q^{s2QR} \equiv -C_M Q_{GG^T}^{3/2} R_{GG^T}^{5/6} \frac{\delta^2}{12} G G^T \nabla \overline{T}}$$





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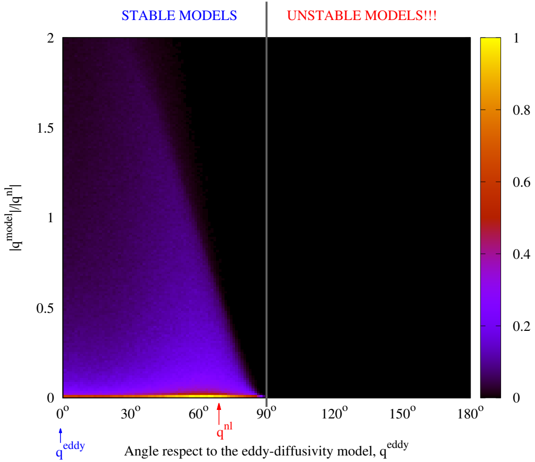

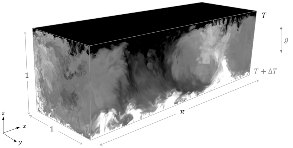

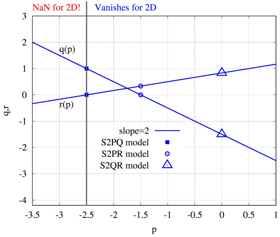

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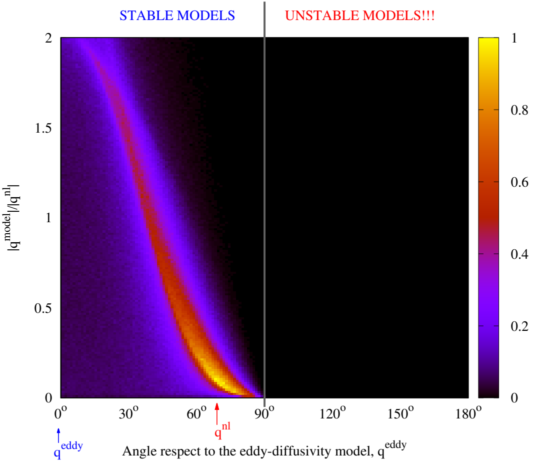

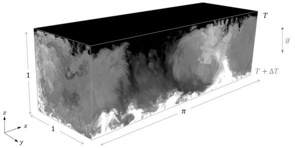

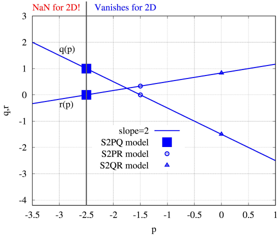

A priori alignment trends of S2PR

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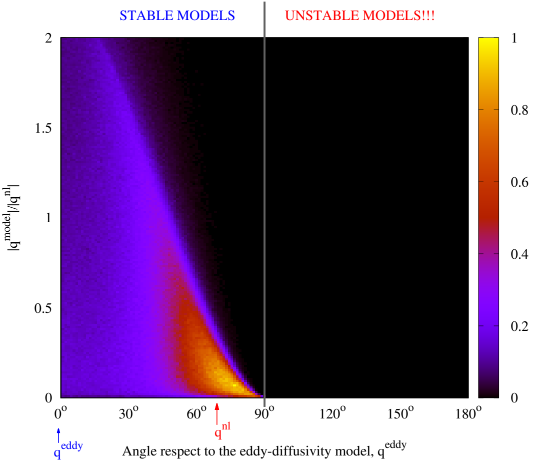

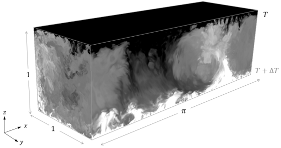

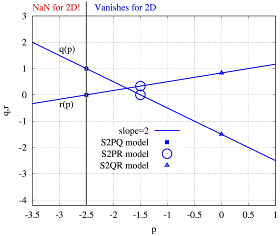

A priori alignment trends of S2PR in the near-wall region

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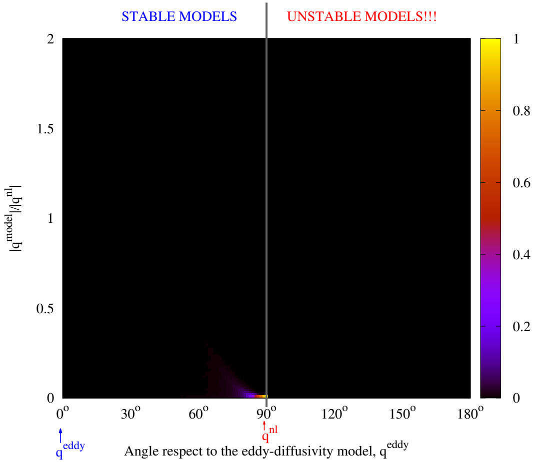

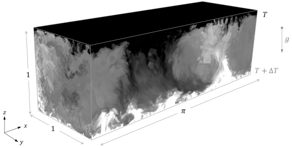

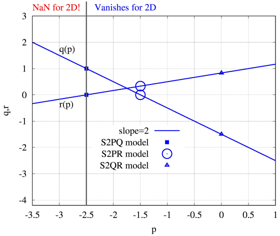

A priori alignment trends of DH in the near-wall region

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

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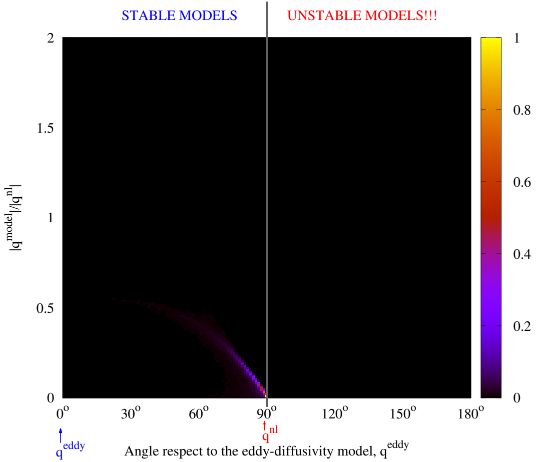

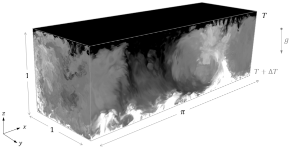

A posteriori results

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Concluding remarks

- A new tensor-diffusivity model has been proposed

$$\boxed{q^{s2PR} \equiv -C_M P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

- Locally defined, unconditionally stable and vanishes for 2D flows. ✓
- Good *a priori* alignment trends. ✓
- Proper near-wall scaling. ✓

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Future:

- *A posteriori* tests for Rayleigh-Bénard convection.

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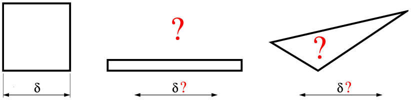
Future:

- *A posteriori* tests for Rayleigh-Bénard convection.
- How δ should be defined for highly anisotropic grids?



Thank you for your attention