An Efficient Two-Layer Wall Model for High Reynolds Number Large Eddy Simulation

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Outline

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4. Tests and results.
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Background and motivation

- **High Reynolds number aerodynamics** are of capital importance since they are present in key industries such as wind energy, aeronautics or automotive industry.
- Large Eddy Simulation is still **prohibitively expensive** at high Reynolds Number, specially if solid bounds are involved.
- Wall models are intended to **reduce the mesh requirements** in wall areas. An efficient Two-Layer Model is proposed.
- The proposed model featrues a **one-step low-cost methodology** intended to overcome the recurrent problems of Two Layer Models.

Wall modeling benefits

- Grid size requirements: According Choi and Moin (2011):
  - Wall Modeled LES: $N_i \sim Re_{Lx}$
  - Resolved Wall LES: $N_i \sim Re_{Lx}^{1.85}$

Flat plate Airfoil Test:

Re_c	N_i (wall modeled LES)	N_i (wall resolved LES)
10^6	3.63×10^7	5.23×10^7
10^7	8.20×10^8	7.76×10^9
10^8	9.09×10^9	5.98×10^{11}
10^9	9.26×10^{10}	4.34×10^{13}

Wall modeling benefits

- Time step benefits:

According the diffusive CFL condition, the bound value for the time step is:

- Convective: $\Delta t \leq \Delta t_{bound} = C_{conv} \left[\frac{\Delta x_i}{v_i} \right]_{min}$
- Diffusive: $\Delta t \leq \Delta t_{bound} = C_{diff} \left[\frac{\Delta x_i^2}{\nu} \right]_{min}$

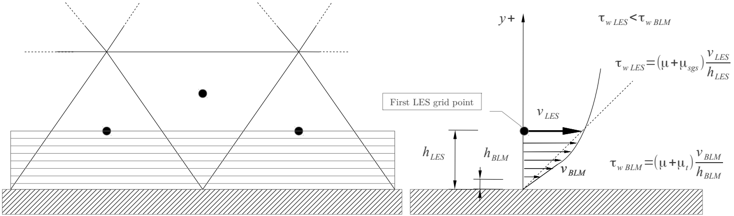
Wall modeling benefits

- Total computational cost derived from the temporal and spatial resolution requirements:

  - Wall Modeled LES: $T_{cc}^{WM} \sim Re_{Lx}^{4/3} \quad \approx Re_{Lx}^{1.33}$
  - Wall Resolved LES: $T_{cc}^{WR} \sim Re_{Lx}^{65/21} \approx Re_{Lx}^{3.09}$

General strategy

- The Two-Layer models are based on the computation of the near-wall flow field in order to obtain an **accurate wall shear stress.** This new value is used when evaluating the diffusive term of the LES equations.
- Governing equations are solved in an **embedded mesh** that is generated by extruding the superficial mesh of the solid between the wall itself and the first off-wall node of the LES mesh.



Two-Layer Wall models: Governing Equations.

The **range of applicability** of the wall model depends on the governing equations solved within the wall mesh:

-**Equilibrium** flows: attached boundary layers, no adverse pressure gradients: **Diffusive term**

$$\frac{d}{dn}\left[(\mu + \mu_{Twm})\frac{d\mathbf{U}_{||}}{dn}\right] = 0 \tag{1}$$

-General **non-equilibrium** flows: deatached boundary layers, large adverse pressure gradients: **TBLE or RANS equations**

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}\cdot\nabla)\,\mathbf{U} = \nabla\cdot[\mathbf{2}(\nu + \nu_{\mathbf{Twm}})\mathbf{S}(\mathbf{U})] - \nabla\mathbf{P} \tag{2}$$

General Mathematical and Numerical Model.

- Unsteady Reynolds Averaged Navier-Stokes

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla)\,\mathbf{U} = \nabla \cdot [\mathbf{2}(\nu + \nu_{\mathbf{Twm}})\mathbf{S}(\mathbf{U})] - \nabla \mathbf{P} \qquad (3)$$

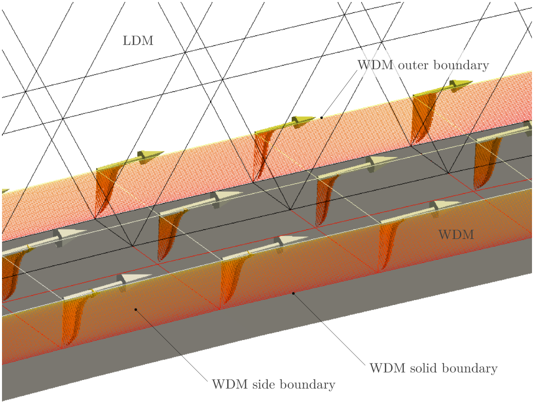
- RANS Model: Mixing-length eddy viscosity

$$\nu_{Twm} = \left(\kappa y^{+}\right)^{2} |S| \left[1 - exp\left(-y^{+}/A^{+}\right)\right]^{2}, \qquad (4)$$

Numerical Model: Boundary Conditions.

Wall Domain Mesh (WDM):

- Top boundary: Dirichlet for P and $\vec{U}$ from the LES.
- Solid boundary: Neumann for P and no-slip for $\vec{U}$.
- Side boundary: The same than the LES domain.



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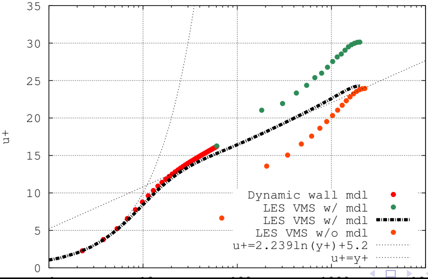
Wall shear stress models: Log-Layer Mismatch.

- **Log-layer mismatch**: a general error of wall shear stress models.
- **Source of error**: near-wall numerical and subgrid errors (Kawai et al. 2012)
- **Proposed solution**: extending the wall model mesh beyond the first off-wall row of nodes (Kawai et al. 2012)



Resolved Reynolds stresses inflow.

- **Resolved Reynolds stresses inflow**: a particular error of RANS-based models featuring advective term (Cabot 1999).
- **Source of error**: overprediction of the total Reynolds stresses within the RANS layer due to LES resolved inflow data.
- **Proposed solution**:
  - Dynamic calculation of κ coefficient in the RANS model (Cabot and Moin 1999, Kawai et al. 2012)
  - Subtraction of the Resolved stresses contribution to the RANS turbulent viscosity (Park and Moin 2014)

$$\nu_{T_{wm}} = \nu_{Tml} - \left(- \frac{R(\mathbf{U})\mathbf{S}(\mathbf{U})}{2S(\mathbf{U})\mathbf{S}(\mathbf{U})} \right), \tag{5}$$

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Time-averaging filter at WM/LES interface.

This methodology tackles the log-layer mismatch and the Resolved Reynolds stress inflow problems at once with a **single and low-computational-cost step.**

- It suppresses the turbulent fluctuations incoming from the LES domain.
- $\overline{\phi}(t)$ is the local time-average of a given variable ϕ with an exponential decaying memory.
- The memory decaying speed depends on the size of T.
- The value of T has to be of the same order of magnitude than the large flow structure characteristic time-scale.

$$\overline{\phi}(t) = \int_0^t \phi(\xi) \frac{exp[(\xi - t)/T]}{T} d\xi \qquad (6)$$

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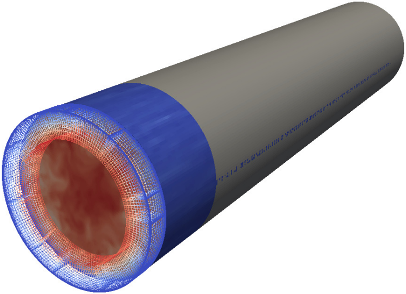
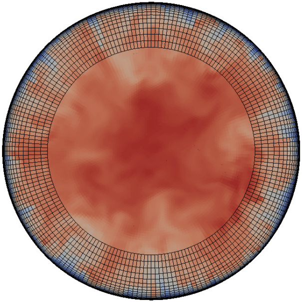
Time-averaging filter performance evaluation.

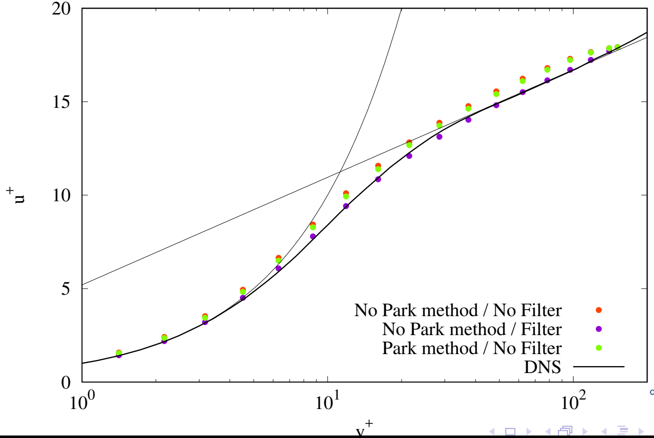
- The wall model was connected to wall-resolved $Re_\tau \approx 500$ pipe flow at a heiht of $y^+ \approx 150$.
- The purpose was to reproduce the flow physics within the wall layer.





Time-averaging filter performance evaluation.

- Near wall velocity profile. Comparison with DNS data from Chin et al. (2010)



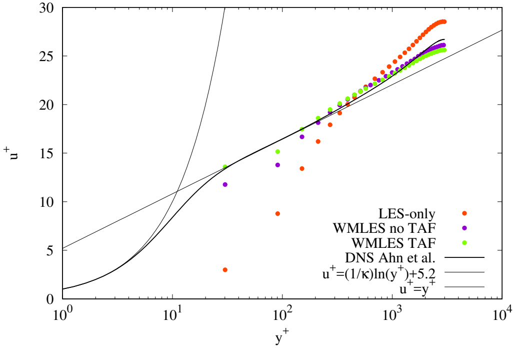
Time-averaging filter performance evaluation.

- Wall shear stress evaluation throught the computed Re_τ value

Time-Averaging filter	Park Method	Computed Re_τ	rel. error [%] ref. $Re_\tau \approx 500$
NO	NO	529.25	5.85
NO	YES	522.07	4.41
YES	NO	503.64	0.72
YES	YES	503.71	0.74

WMLES of a pipe flow at $Re_\tau \approx 3000$.

- LES model: Original Smagorinsky (1963)
- First off-wall node placed at $y^+ \approx 30$
- No presence of Log-layer mismatch (Yang and Moin 2017)



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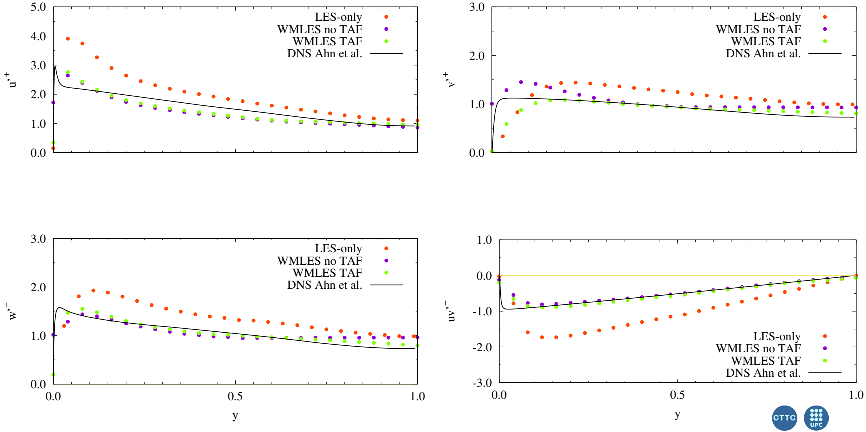
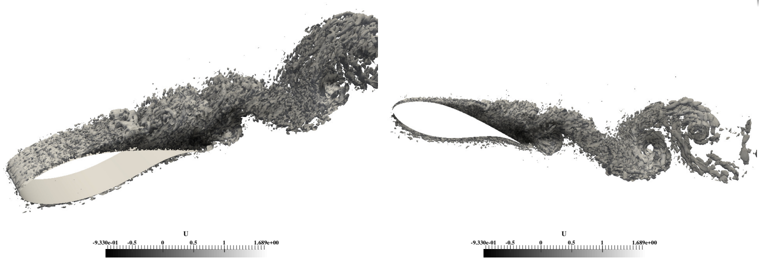
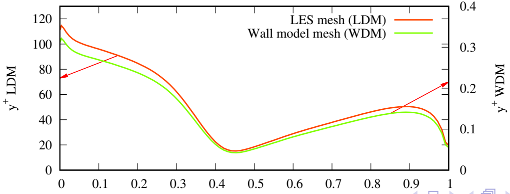
WMLES of a pipe flow at $Re_\tau \approx 3000$.



WMLES of a DU 91-W2-250 airfoil at $AoA = 15.2^\circ$ and $Re = 3 \times 10^6$.





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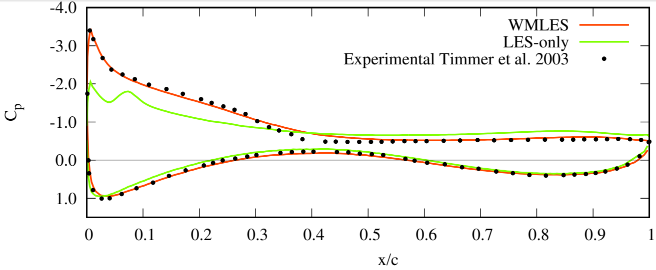
WMLES of a DU 93-W2-250 airfoil at $AoA = 15.2^{\circ}$ and $Re = 3 \times 10^{6}$.



	C_l	rel. error [%]	C_d	rel. error [%]
Experimental	1.128	-	0.1144	-
LES-only	0.93	17.55	0.2075	80.94
WMLES	1.122	0.53	0.1305	14.07

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Conclusions

- An **efficient Two-Layer wall model** has been proposed.
- A time-averaging filter is used to tackle two recurrent problemes of TLM with a **single and low-computational-cost technique**.
- The time-averaging filter is more efficient in blocking the resolved Reynolds stresses inflow than the existing methodologies.
- The methodlogy has been tested in **equilibrium and non-equilibrium** conditions obtaining good results.
- Further studies on dynamic procedures to determine the filtering period T will be carried out.

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Thank you for your attention!