Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

JOHANNES KEPLER UNIVERSITY LINZ

On a proper tensor-diffusivity model for large-eddy simulations of Rayleigh-Bénard convection

F.Xavier Trias[1], Firas Dabbagh[1,2], Daniel Santos[1], Andrey Gorobets[1,3], Assensi Oliva[1]

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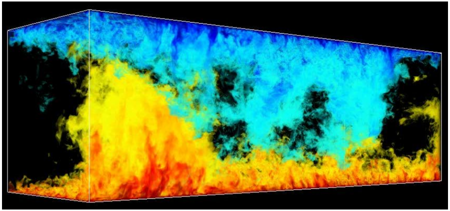
[2]Department of Particulate Flow Modelling, Johannes Kepler University, Linz, Austria
[3]Keldysh Institute of Applied Mathematics of RAS, Russia

Contents

Motivation

Research question:

- Can we find a nonlinear SGS heat flux model with **good physical** and **numerical properties**, such that we can obtain satisfactory predictions for a turbulent Rayleigh-Bénard convection?



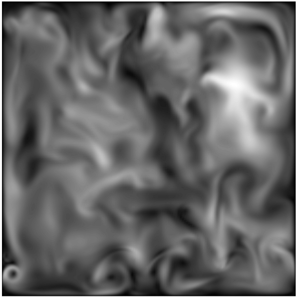
DNS of an air-filled Rayleigh-Bénard convection at $Ra = 10^{10}$

[1] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *On the evolution of flow topology in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 28:115105, 2016.

Motivation

Air-filled RB: $Pr = 0.7$

$Ra = 10^8$



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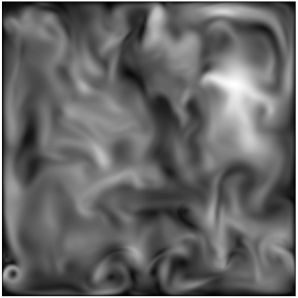

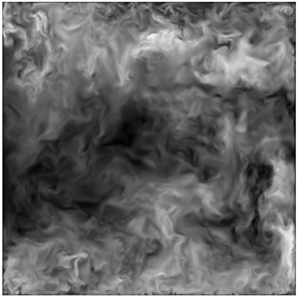

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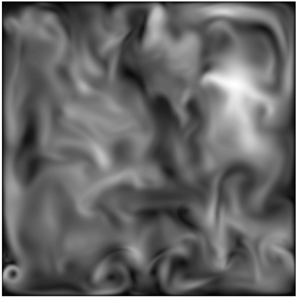

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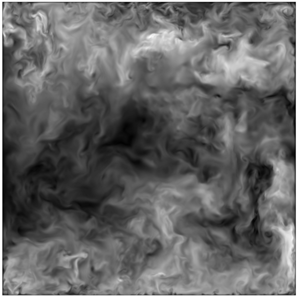
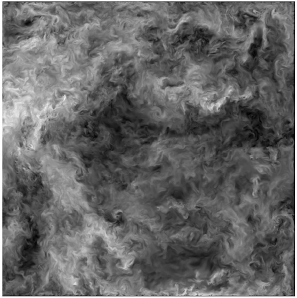


$Ra = 10^{11}$



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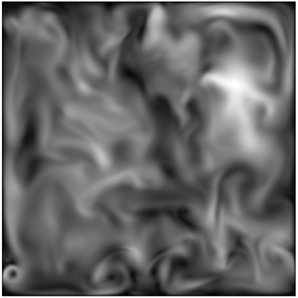
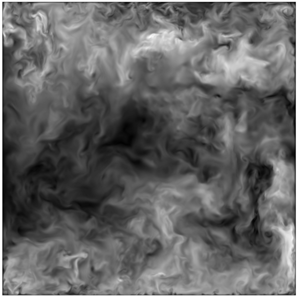


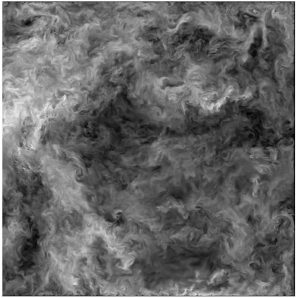
$208 \times 208 \times 400$

17.5M



$768 \times 768 \times 1024$

607M



$1662 \times 1662 \times 2048$

5600M

[2] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, (submitted).

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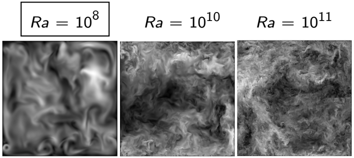
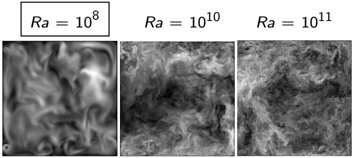


Motivation

DNS: $208 \times 208 \times 400$

$\boxed{\text{LES:} \quad 80 \times \quad 80 \times 120}$



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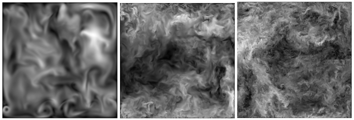
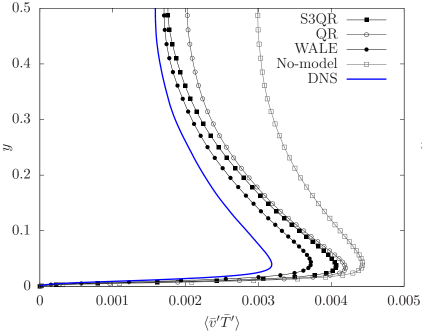
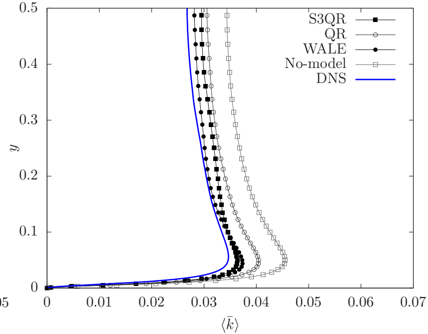


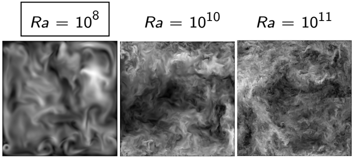


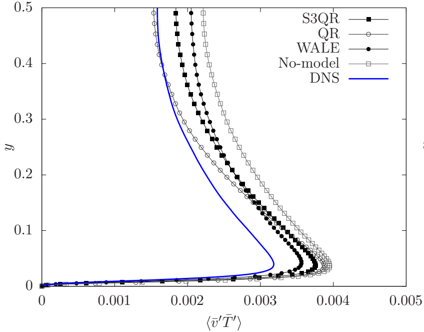


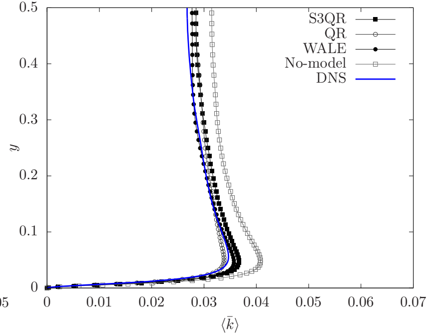
Motivation

DNS: $208 \times 208 \times 400$

$\boxed{\text{LES: } 110 \times 110 \times 168}$







How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} \qquad - \nabla \cdot \tau(\overline{u}) \;\; ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \;\; \longrightarrow \;\; \tau \;\; (\overline{u}) = -2\nu_t S(\overline{u})$$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

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$$Pr_t?$$

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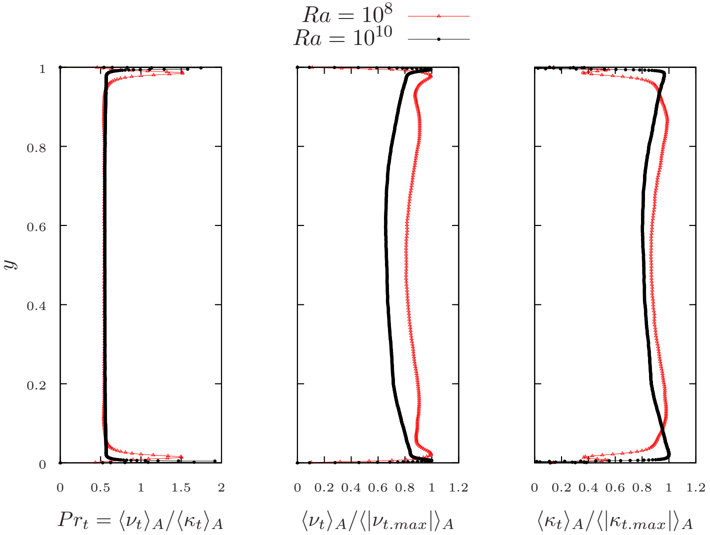

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$$G \equiv \nabla \overline{u} \qquad\qquad q = -\frac{\delta^2}{12} G \nabla \overline{T} + \mathcal{O}(\delta^4)$$

A priori alignment trends[3]

$$\text{eddy-diffusivity} \longrightarrow \color{red}{q} \approx -\color{red}{\alpha_t}\nabla\overline{T} \qquad (\equiv q^{eddy})$$

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[3] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *A priori study of subgrid-scale features in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 29:105103, 2017.

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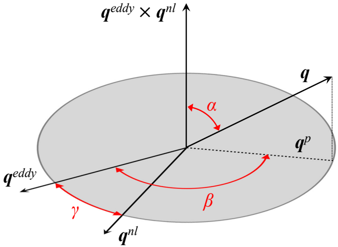

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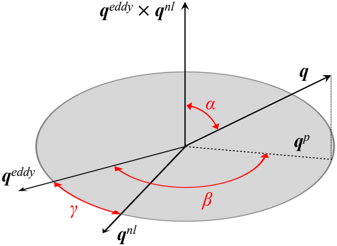

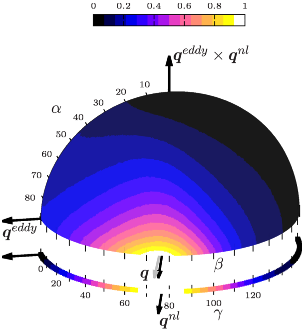

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Peng&Davidson[4] $\longrightarrow$ $q \approx -\dfrac{\delta^2}{12} S \nabla \overline{T}$ $\quad (\equiv q^{PD})$

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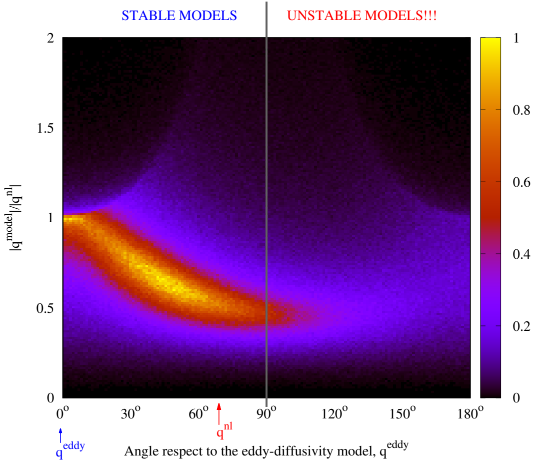
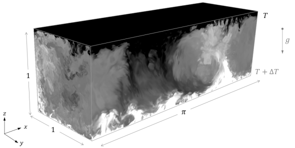
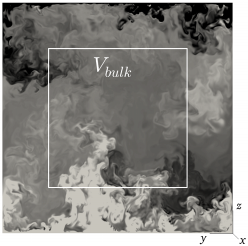
A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T} \qquad \boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$







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$$\text{mixed model} \quad \longrightarrow \quad q \approx q^{nl} + \sigma q^{eddy} \qquad\qquad (\equiv q^{mix})$$

[6] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

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$$\text{Daly\&Harlow}^6 \quad \longrightarrow \quad q \approx -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T} \quad (\equiv q^{DH})$$

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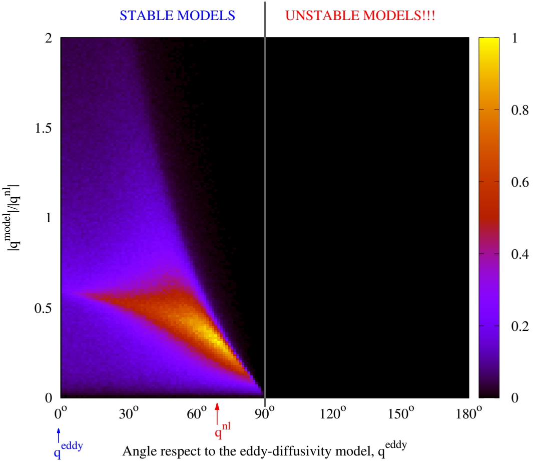
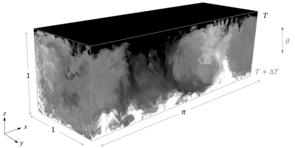
$$\mathcal{T}_{SGS} = 1/|S|$$

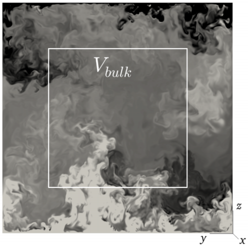
[6] B.J.Daly and F.H.Harlow. **Physics of Fluids**, 13:2634, 1970.

A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T} \qquad \boxed{q^{DH} \equiv -\mathcal{T}_{SGS} \frac{\delta^2}{12} G G^T \nabla \overline{T}}$$

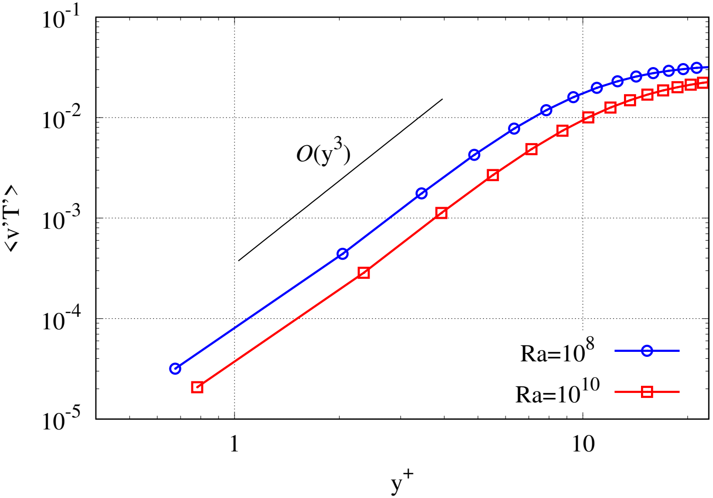






What about near-wall scaling?

What about near-wall scaling? $\Longrightarrow$ Answer: it should be $\mathcal{O}(y^3)$



Near-wall scaling for DH model?

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\,; \qquad \mathcal{T}_{SGS} = 1/|S|}$$

Near-wall scaling for DH model? $\Longrightarrow$ Answer: it is $\mathcal{O}(y^1)$ instead of $\mathcal{O}(y^3)$

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\ ; \qquad \mathcal{T}_{SGS} = 1/|S|}$$

$$u = ay + \mathcal{O}(y^2); \quad v = by^2 + \mathcal{O}(y^3); \quad w = cy + \mathcal{O}(y^2); \quad T = dy + \mathcal{O}(y^2)$$

$$G = \begin{pmatrix} y & 1 & y \\ y^2 & y & y^2 \\ y & 1 & y \end{pmatrix}; \ \nabla\overline{T} = \begin{pmatrix} y \\ 1 \\ y \end{pmatrix} \Longrightarrow \boxed{GG^T\nabla\overline{T} = \begin{pmatrix} y \\ y^2 \\ y \end{pmatrix} = \mathcal{O}(y^1)}$$

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$$\boxed{\mathcal{T}_{SGS} = 1/|S| = \mathcal{O}(y^0)}$$

Idea: build a $\mathcal{T}_{SGS}$ with the proper $\mathcal{O}(y^2)$ scaling!!!

Building proper models for the subgrid heat flux

Let us consider models that are based on the invariants of the tensor GG^T

$$q \approx -C_M \left(P_{GG^T}^{p} Q_{GG^T}^{q} R_{GG^T}^{r} \right) \frac{\delta^2}{12} GG^T \nabla \overline{T} \qquad (\equiv q^{S2})$$

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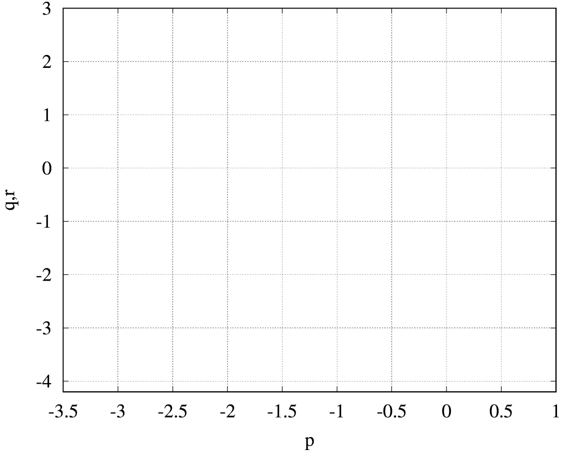
	P_{GG^T}	Q_{GG^T}	R_{GG^T}
Formula	$2(Q_\Omega - Q_S)$	$V^2 + Q_G^2$	R_G^2
Wall-behavior	$\mathcal{O}(y^0)$	$\mathcal{O}(y^2)$	$\mathcal{O}(y^6)$
Units	$[T^{-2}]$	$[T^{-4}]$	$[T^{-6}]$

$$-6r - 4q - 2p = 1 \ \ [T]; \qquad 6r + 2q = s,$$

where s is the slope for the asymptotic near-wall behavior, *i.e.* $\mathcal{O}(y^s)$.

Building proper models for the subgrid heat flux

Solutions: $q(p,s) = -(1+s)/2 - p$ and $r(p,s) = (2s+1)/6 + p/3$



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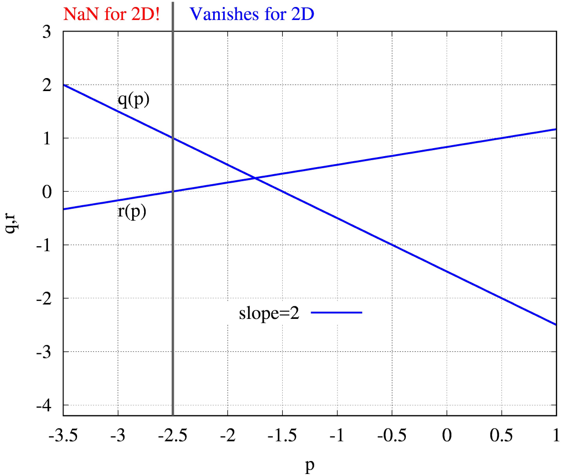

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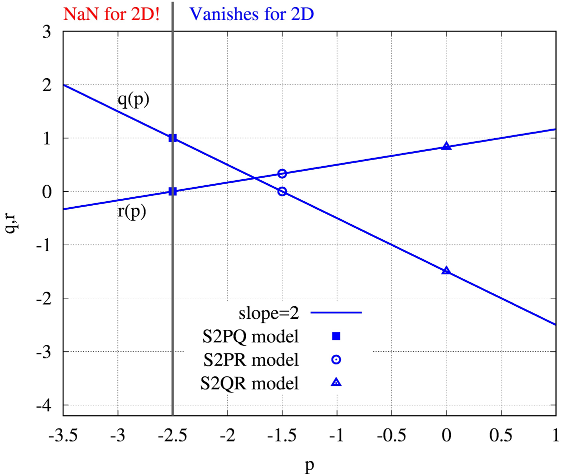

A priori alignment trends of S2PR

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

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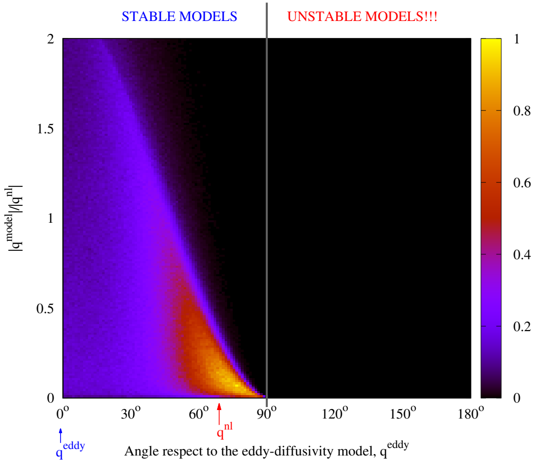

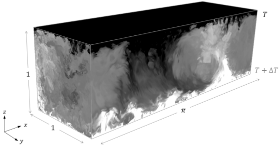

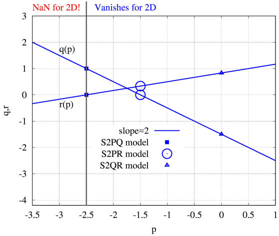

A posteriori results?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} + \overline{f} - \nabla \cdot \tau(\overline{u}) \;\; ; \qquad \nabla \cdot \overline{u} = 0$$

eddy-viscosity $\longrightarrow \;\; \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

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⚠ But first we need to answer the following **research question**:

- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

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- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

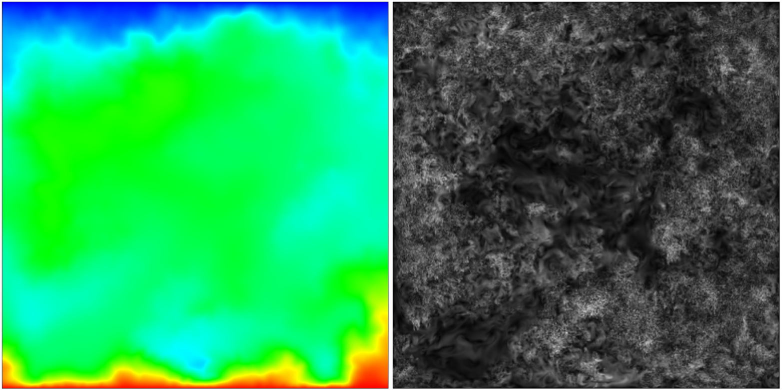
Idea: let's do an LES for momentum and a DNS for temperature!

DNS at very low Pr number

Why? scale separation scales with $Pr^{0.5}$

DNS at very low *Pr* number

Why? scale separation scales with $Pr^{0.5}$ (≈ 0.07 is our case)



DNS of a RB at $Ra = 7.14 \times 10^6$ and $Pr = 0.005$ (liquid sodium)

$488 \times 488 \times 1280 \approx$ **305M**

DNS at very low Pr number

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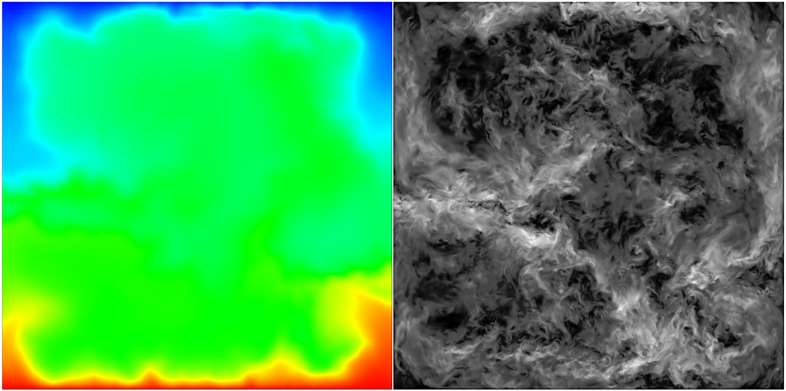

DNS of a RB at $Ra = 7.14 \times 10^7$ and $Pr = 0.005$ (liquid sodium)

$966 \times 966 \times 2048 \approx$ **1911M**

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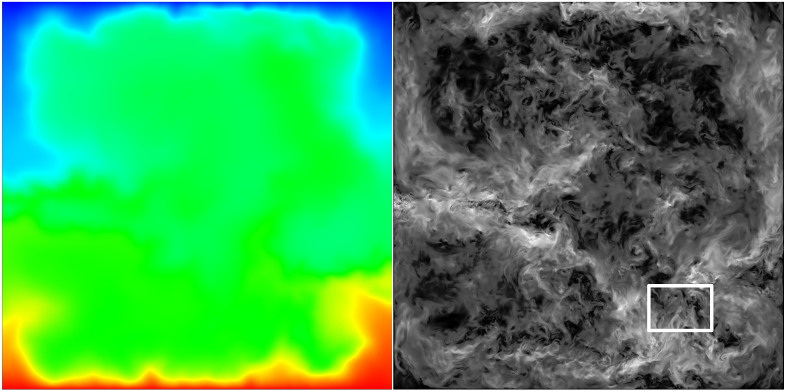

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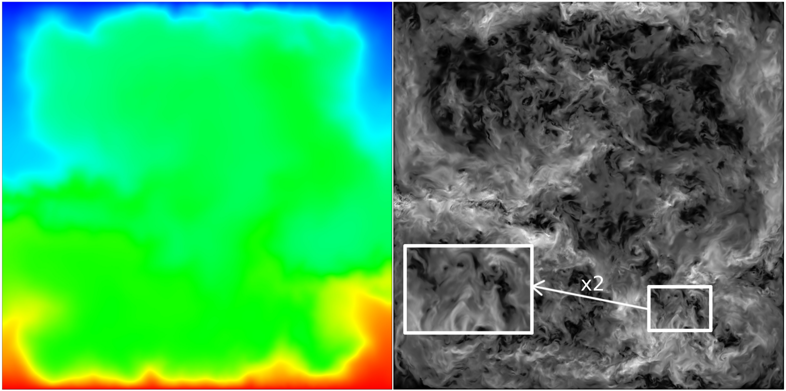

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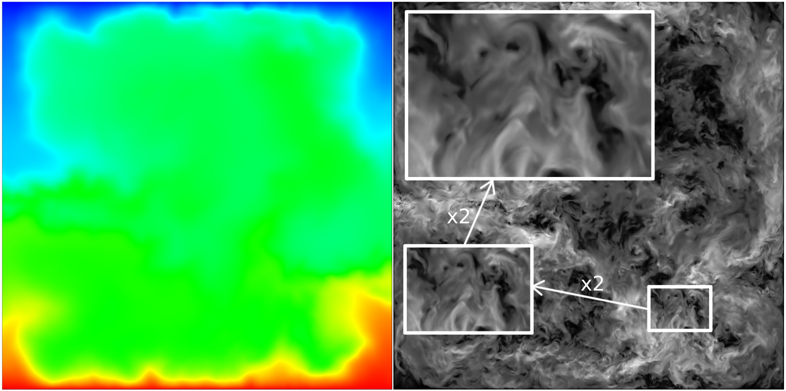

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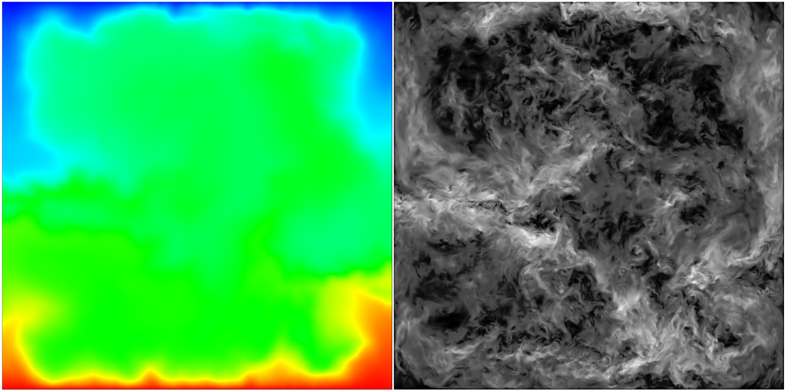

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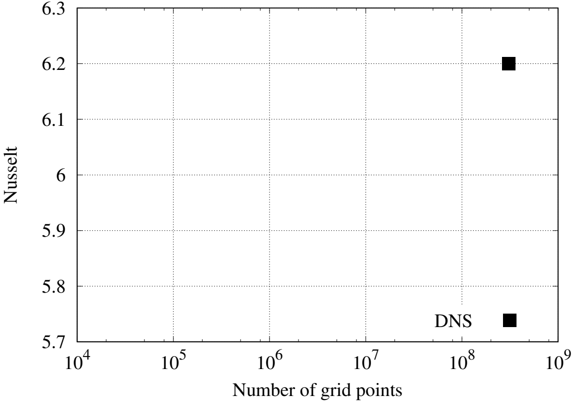
LES[7] results at very low Pr number



[7] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

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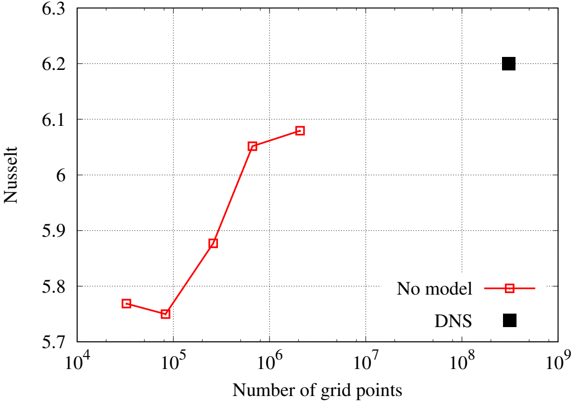

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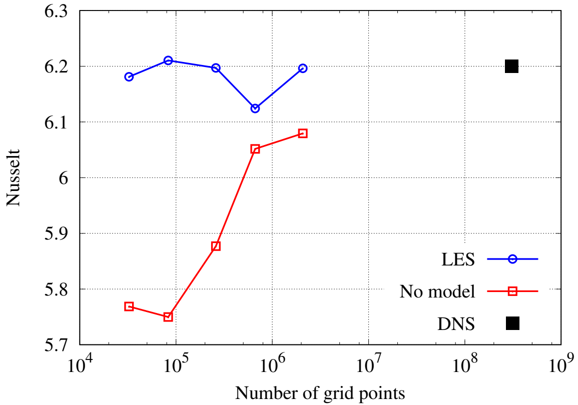

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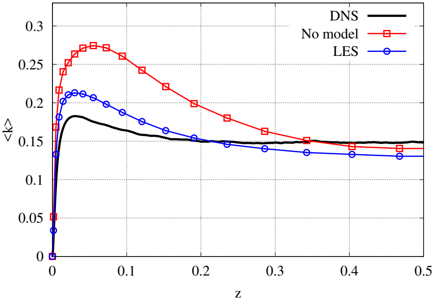
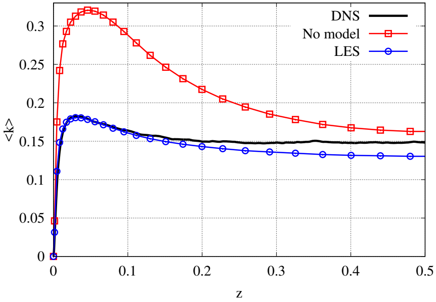


$64 \times 32 \times 32$



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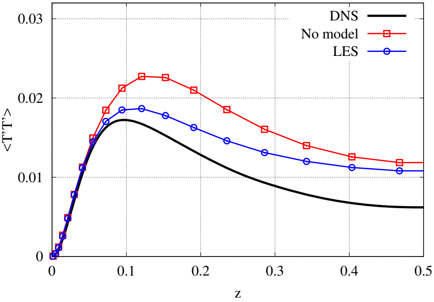

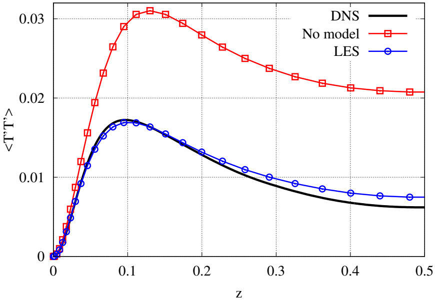

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Concluding remarks

- A new tensor-diffusivity model has been proposed

$$\boxed{q^{s2PR} \equiv -C_M P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

- Locally defined, unconditionally stable and vanishes for 2D flows ✓
- Good *a priori* alignment trends and proper near-wall scaling ✓
- Eddy-viscosity models work for RB ✓

[8] F.X.Trias, A.Gorobets, M.Silvis, R.Verstappen, A.Oliva. *A new subgrid characteristic length for turbulence simulations on anisotropic grids*, **Phys.Fluids**, 26:115109, 2017.

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- *A posteriori* tests using q^{s2PR} for Rayleigh-Bénard convection.

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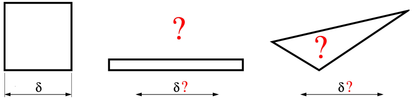
- *A posteriori* tests using q^{s2PR} for Rayleigh-Bénard convection.
- How δ should be defined for highly anisotropic grids[8]?



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Thank you for your attention