Symmetry-preserving discretizations in (unstructured) staggered meshes

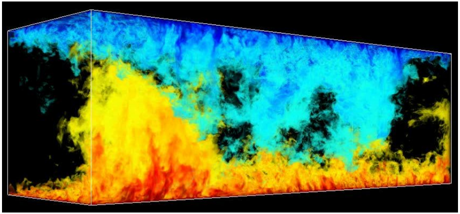
N. Valle[1,2], F.X. Trias[1] and R.W.C.P Verstappen[2]

[1]Technical University of Catalonia and [2]University of Groningen

ETMM 13, Rhodes, Greece, 15-17 September 2021

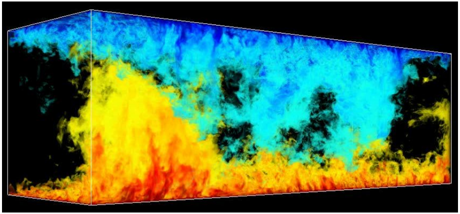
Overview

Context: DNS of Turbulence



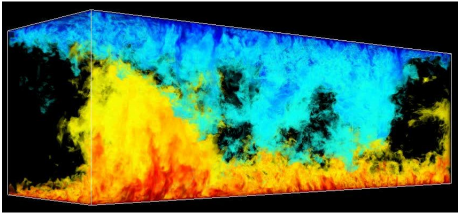
- Expensive
- Reliable
- Effective

Context: DNS of Turbulence



- Expensive
- Reliable
- Effective

Context: DNS of Turbulence



- Expensive
- Reliable
- Effective

Context: The physics

The incompressible Navier-Stokes.

$$\nabla \cdot \vec{u} = 0 \qquad \mathsf{D}\mathsf{u_f} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + C(\vec{u}, \vec{u}) = -\nabla p + \nu \nabla^2 \vec{u} \qquad \Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G} \mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Mathematical → physical properties

$$\int_\Omega ab d\Omega = (a, b) \qquad (a_h, b_h) = \mathsf{a_h}^T \Omega \mathsf{b_h}$$

$$(C(\vec{u}, \phi), \rho) = -(\phi, C(\vec{u}, \rho)) \qquad \mathsf{C}(\mathsf{u_f}) = -\mathsf{C}(\mathsf{u_f})^T$$

$$(\nabla \cdot \vec{u}, p) = (\vec{u}, \nabla p) \qquad \mathsf{D} = -\Omega \mathsf{G}^T$$

$$\left(\nu \nabla^2 \vec{u}, \vec{u}\right) \leq 0 \qquad (\mathsf{L}\phi_h, \phi_h) \leq 0$$

Context: The physics

The incompressible Navier-Stokes.

$$\nabla \cdot \vec{u} = 0 \qquad \mathsf{D}\mathsf{u_f} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + C\left(\vec{u}, \vec{u}\right) = -\nabla p + \nu \nabla^2 \vec{u} \qquad \Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G} \mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Mathematical $\rightarrow$ physical properties

$$\int_\Omega ab d\Omega = (a, b) \qquad (a_h, b_h) = \mathsf{a_h}^T \Omega \mathsf{b_h}$$

$$\left(C(\vec{u}, \phi), \rho\right) = -\left(\phi, C(\vec{u}, \rho)\right) \qquad \mathsf{C}(\mathsf{u_f}) = -\mathsf{C}(\mathsf{u_f})^T$$

$$\left(\nabla \cdot \vec{u}, p\right) = \left(\vec{u}, \nabla p\right) \qquad \mathsf{D} = -\Omega \mathsf{G}^T$$

$$\left(\nu \nabla^2 \vec{u}, \vec{u}\right) \leq 0 \qquad (\mathsf{L}\phi_h, \phi_h) \leq 0$$

Context: The physics

The incompressible Navier-Stokes.

$$\nabla \cdot \vec{u} = 0 \qquad \mathrm{D}\mathrm{u_f} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + C(\vec{u}, \vec{u}) = -\nabla p + \nu \nabla^2 \vec{u} \qquad \Omega \frac{d\mathrm{u_f}}{dt} + \mathrm{C}(\mathrm{u_f})\mathrm{u_f} = -\Omega \mathrm{G}\mathrm{p_c} + \mathrm{L}\mathrm{u_f}$$

Mathematical $\rightarrow$ physical properties

$$\int_\Omega ab\, d\Omega = (a, b) \qquad (a_h, b_h) = \mathrm{a_h}^T \Omega \mathrm{b_h}$$

$$(C(\vec{u}, \phi), \rho) = -(\phi, C(\vec{u}, \rho)) \qquad \mathrm{C}(\mathrm{u_f}) = -\mathrm{C}(\mathrm{u_f})^T$$

$$(\nabla \cdot \vec{u}, p) = (\vec{u}, \nabla p) \qquad \mathrm{D} = -\Omega \mathrm{G}^T$$

$$\left(\nu \nabla^2 \vec{u}, \vec{u}\right) \leq 0 \qquad (\mathrm{L}\phi_h, \phi_h) \leq 0$$

Context: The physics

The incompressible Navier-Stokes.

$$\nabla \cdot \vec{u} = 0 \qquad \mathsf{D}\mathsf{u_f} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + C(\vec{u}, \vec{u}) = -\nabla p + \nu \nabla^2 \vec{u} \qquad \Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G} \mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Mathematical → physical properties

$$\int_\Omega ab\, d\Omega = (a, b) \qquad (a_h, b_h) = \mathsf{a_h}^T \Omega \mathsf{b_h}$$

$$(C(\vec{u}, \phi), \rho) = -(\phi, C(\vec{u}, \rho)) \qquad \mathsf{C}(\mathsf{u_f}) = -\mathsf{C}(\mathsf{u_f})^T$$

$$(\nabla \cdot \vec{u}, p) = (\vec{u}, \nabla p) \qquad \mathsf{D} = -\Omega \mathsf{G}^T$$

$$\left(\nu \nabla^2 \vec{u}, \vec{u}\right) \leq 0 \qquad (\mathsf{L}\phi_h, \phi_h) \leq 0$$

Context: The physics

The incompressible Navier-Stokes.

$$\nabla \cdot \vec{u} = 0 \qquad \mathsf{D}\mathsf{u}_\mathsf{f} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + C(\vec{u}, \vec{u}) = -\nabla p + \nu \nabla^2 \vec{u} \qquad \Omega \frac{d\mathsf{u}_\mathsf{f}}{dt} + \mathsf{C}(\mathsf{u}_\mathsf{f})\mathsf{u}_\mathsf{f} = -\Omega \mathsf{G}\mathsf{p}_\mathsf{c} + \mathsf{L}\mathsf{u}_\mathsf{f}$$

Mathematical $\rightarrow$ physical properties

$$\int_\Omega ab d\Omega = (a, b) \qquad (a_h, b_h) = \mathsf{a_h}^T \Omega \mathsf{b_h}$$

$$(C(\vec{u}, \phi), \rho) = -(\phi, C(\vec{u}, \rho)) \qquad \mathsf{C}(\mathsf{u}_\mathsf{f}) = -\mathsf{C}(\mathsf{u}_\mathsf{f})^T$$

$$(\nabla \cdot \vec{u}, p) = (\vec{u}, \nabla p) \qquad \mathsf{D} = -\Omega \mathsf{G}^T$$

$$\left(\nu \nabla^2 \vec{u}, \vec{u}\right) \leq 0 \qquad (\mathsf{L}\phi_h, \phi_h) \leq 0$$

Context: The physics

The incompressible Navier-Stokes.

$$\nabla \cdot \vec{u} = 0 \qquad \mathsf{D}\mathsf{u_f} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + C(\vec{u}, \vec{u}) = -\nabla p + \nu \nabla^2 \vec{u} \qquad \Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G} \mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Mathematical $\rightarrow$ physical properties

$$\int_\Omega ab d\Omega = (a, b) \qquad (a_h, b_h) = \mathsf{a_h}^T \Omega \mathsf{b_h}$$

$$(C(\vec{u}, \phi), \rho) = -(\phi, C(\vec{u}, \rho)) \qquad \mathsf{C}(\mathsf{u_f}) = -\mathsf{C}(\mathsf{u_f})^T$$

$$(\nabla \cdot \vec{u}, p) = (\vec{u}, \nabla p) \qquad \mathsf{D} = -\Omega \mathsf{G}^T$$

$$\left(\nu \nabla^2 \vec{u}, \vec{u}\right) \leq 0 \qquad (\mathsf{L}\phi_h, \phi_h) \leq 0$$

Context: The physics

The incompressible Navier-Stokes.

$$\nabla \cdot \vec{u} = 0 \qquad \mathsf{D}\mathsf{u_f} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + C\left(\vec{u}, \vec{u}\right) = -\nabla p + \nu \nabla^2 \vec{u} \qquad \Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G} \mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Mathematical $\rightarrow$ physical properties

$$\int_\Omega ab d\Omega = (a, b) \qquad (a_h, b_h) = \mathsf{a_h}^T \Omega \mathsf{b_h}$$

$$\left(C(\vec{u}, \phi), \rho\right) = -\left(\phi, C(\vec{u}, \rho)\right) \qquad \mathsf{C}(\mathsf{u_f}) = -\mathsf{C}(\mathsf{u_f})^T$$

$$\left(\nabla \cdot \vec{u}, p\right) = \left(\vec{u}, \nabla p\right) \qquad \mathsf{D} = -\Omega \mathsf{G}^T$$

$$\left(\nu \nabla^2 \vec{u}, \vec{u}\right) \leq 0 \qquad (\mathsf{L}\phi_h, \phi_h) \leq 0$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated
- simple
- $D \neq -\Omega G^T$

Staggered
- $D = -\Omega G^T$
- complex

Idea
Can we reuse collocated codes to construct staggered formulations?

$$\mathsf{D}\mathsf{u}_\mathsf{f} = 0_c$$

$$\Omega \frac{d\mathsf{u}_\mathsf{f}}{dt} + \mathsf{C}(\mathsf{u}_\mathsf{f})\mathsf{u}_\mathsf{f} = -\Omega \mathsf{G}\mathsf{p}_\mathsf{c} + \mathsf{L}\mathsf{u}_\mathsf{f}$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $D \neq -\Omega G^T$

Staggered

- $D = -\Omega G^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$D u_f = 0_c$$

$$\Omega \frac{d u_f}{dt} + C(u_f) u_f = -\Omega G p_c + L u_f$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $D \neq -\Omega G^T$

Staggered

- $D = -\Omega G^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$D u_f = 0_c$$

$$\Omega \frac{du_f}{dt} + C(u_f) u_f = -\Omega G p_c + L u_f$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $D \neq -\Omega G^T$

Staggered

- $D = -\Omega G^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$Du_f = 0_c$$

$$\Omega \frac{du_f}{dt} + C(u_f)u_f = -\Omega G p_c + L u_f$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $\mathsf{D} \neq -\Omega \mathsf{G}^T$

Staggered

- $\mathsf{D} = -\Omega \mathsf{G}^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$\mathsf{D}\mathsf{u_f} = 0_c$$

$$\Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G}\mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $\mathsf{D} \neq -\Omega \mathsf{G}^T$

Staggered

- $\mathsf{D} = -\Omega \mathsf{G}^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$\mathsf{D}\mathsf{u}_\mathsf{f} = 0_c$$

$$\Omega \frac{d\mathsf{u}_\mathsf{f}}{dt} + \mathsf{C}(\mathsf{u}_\mathsf{f})\mathsf{u}_\mathsf{f} = -\Omega \mathsf{G}\mathsf{p}_\mathsf{c} + \mathsf{L}\mathsf{u}_\mathsf{f}$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $\mathsf{D} \neq -\Omega \mathsf{G}^T$

Staggered

- $\mathsf{D} = -\Omega \mathsf{G}^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$\mathsf{D}\mathsf{u_f} = 0_c$$

$$\Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G}\mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $\mathsf{D} \neq -\Omega \mathsf{G}^T$

Staggered

- $\mathsf{D} = -\Omega \mathsf{G}^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$\mathsf{D}\mathsf{u_f} = 0_c$$

$$\Omega \frac{d\mathsf{u_f}}{dt} + \mathsf{C}(\mathsf{u_f})\mathsf{u_f} = -\Omega \mathsf{G}\mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $\mathsf{D} \neq -\Omega \mathsf{G}^T$

Staggered

- $\mathsf{D} = -\Omega \mathsf{G}^T$
- complex

Idea

Can we reuse collocated codes to construct staggered formulations?

$$\mathsf{D}\mathsf{u_f} = 0_c$$

$$\Omega \frac{d\mathsf{u_f}}{dt} + \boxed{\mathsf{C}(\mathsf{u_f})\mathsf{u_f}} = -\Omega \mathsf{G}\mathsf{p_c} + \mathsf{L}\mathsf{u_f}$$

Context: existing codes

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- $\mathsf{D} \neq -\Omega \mathsf{G}^T$

Staggered

- $\mathsf{D} = -\Omega \mathsf{G}^T$
- complex

Idea

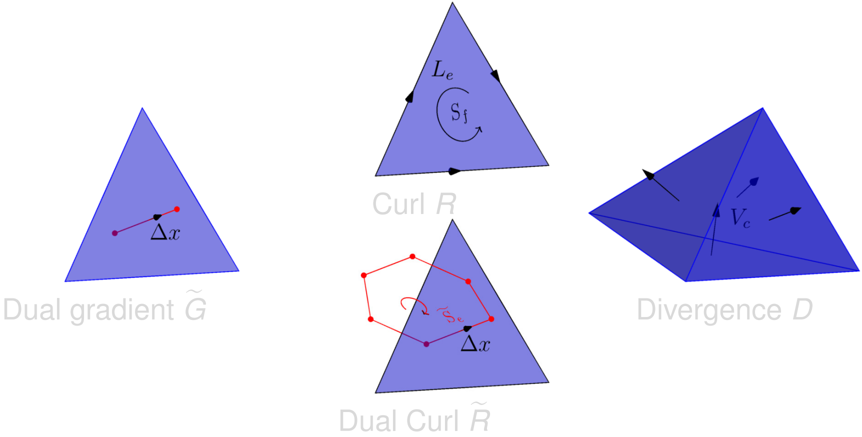
Can we reuse collocated codes to construct staggered formulations?

$$\mathsf{D}\mathsf{u_f} = 0_c$$

$$\Omega \frac{d\mathsf{u_f}}{dt} + \boxed{\mathsf{C}(\mathsf{u_f})\mathsf{u_f}} = -\Omega \mathsf{G}\mathsf{p_c} + \boxed{\mathsf{L}\mathsf{u_f}}$$

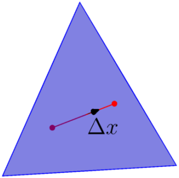
Operators

Collocated operators defined over and arbitrary unstructured mesh

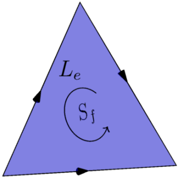


Operators

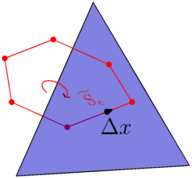
Collocated operators defined over and arbitrary unstructured mesh



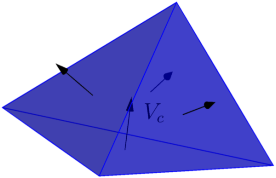
Dual gradient $\widetilde{G}$



Curl R



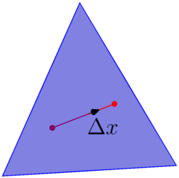
Dual Curl $\widetilde{R}$



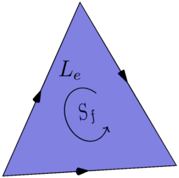
Divergence D

Operators

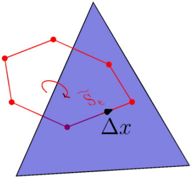
Collocated operators defined over and arbitrary unstructured mesh



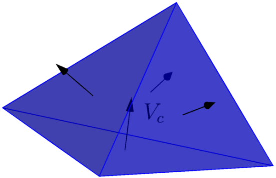
Dual gradient $\widetilde{G}$



Curl R



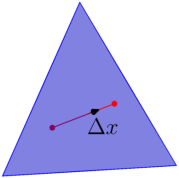
Dual Curl $\widetilde{R}$



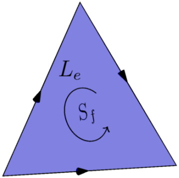
Divergence D

Operators

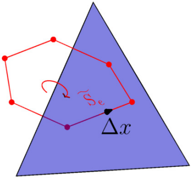
Collocated operators defined over and arbitrary unstructured mesh



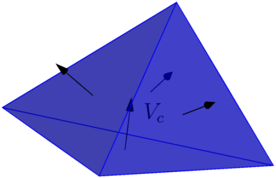
Dual gradient $\widetilde{G}$



Curl R



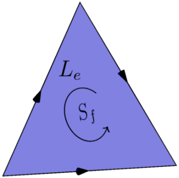
Dual Curl $\widetilde{R}$



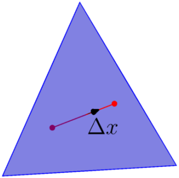
Divergence D

Operators

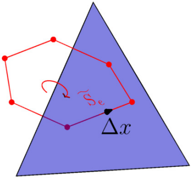
Collocated operators defined over and arbitrary unstructured mesh



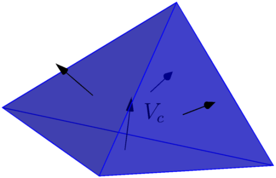
Curl R



Dual gradient $\widetilde{G}$



Dual Curl $\widetilde{R}$



Divergence D

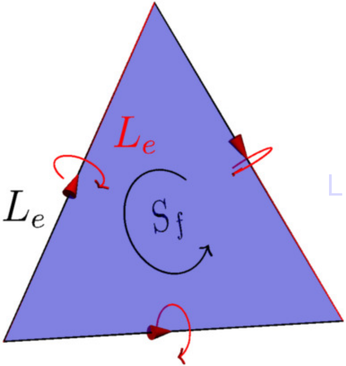
Laplacian L

Rotational formulation: $\nabla^2 \vec{u} = \nabla \times \nabla \times \vec{u} - \nabla \nabla \cdot \vec{u}$

$$\mathsf{L} = R\widetilde{R} - \widetilde{\mathsf{G}}\mathsf{D}$$

Laplacian L

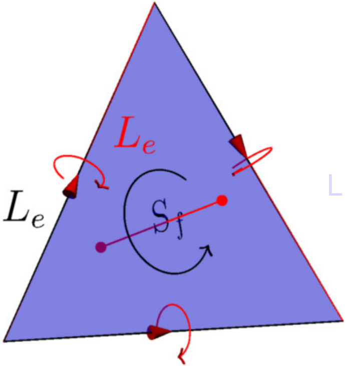
Rotational formulation: $\nabla^2 \vec{u} = \nabla \times \nabla \times \vec{u} - \nabla\nabla \cdot \vec{u}$



$\mathsf{L} = R\widetilde{R} - \widetilde{\mathsf{G}}\mathsf{D}$

Laplacian L

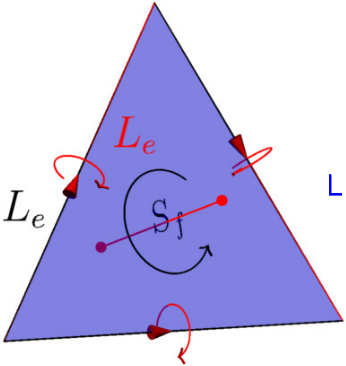
Rotational formulation: $\nabla^2 \vec{u} = \nabla \times \nabla \times \vec{u} - \nabla\nabla \cdot \vec{u}$



$\mathsf{L} = R\widetilde{R} - \widetilde{\mathsf{G}}\mathsf{D}$

Laplacian L

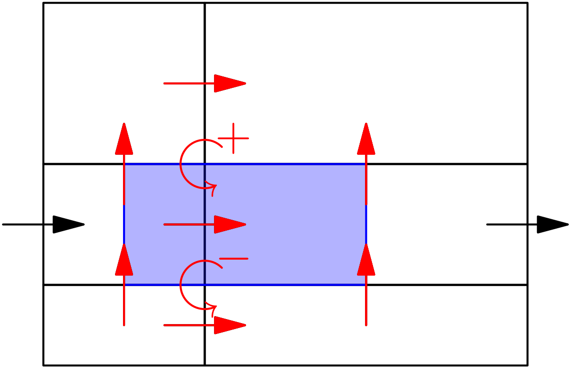
Rotational formulation: $\nabla^2 \vec{u} = \nabla \times \nabla \times \vec{u} - \nabla\nabla \cdot \vec{u}$



$\mathsf{L} = R\widetilde{R} - \widetilde{\mathsf{G}}\mathsf{D}$

Laplacian L

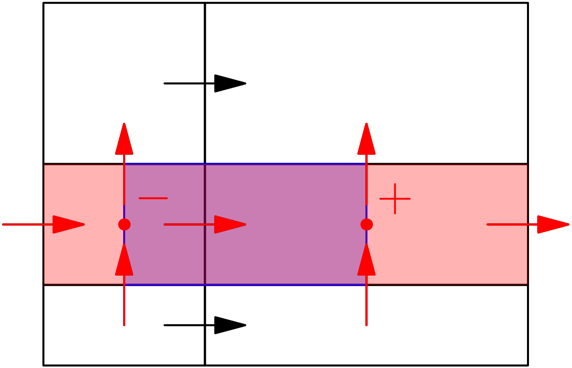
Application to Cartesian grids.



Recovers Harlow and Welch ✓

Laplacian L

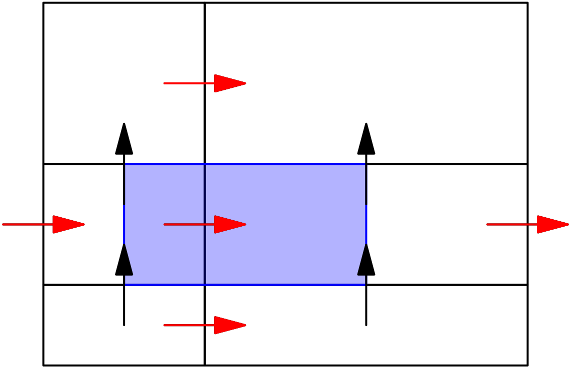
Application to Cartesian grids.



Recovers Harlow and Welch ✓

Laplacian L

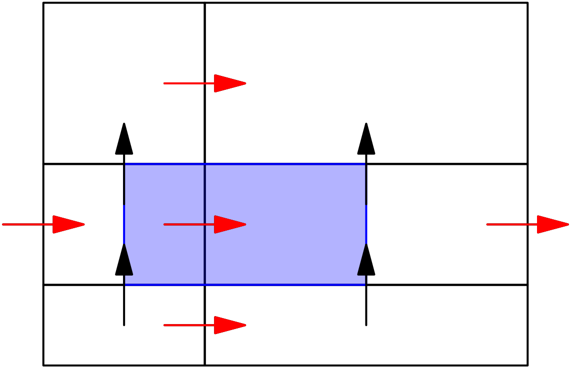
Application to Cartesian grids.



Recovers Harlow and Welch ✓

Laplacian L

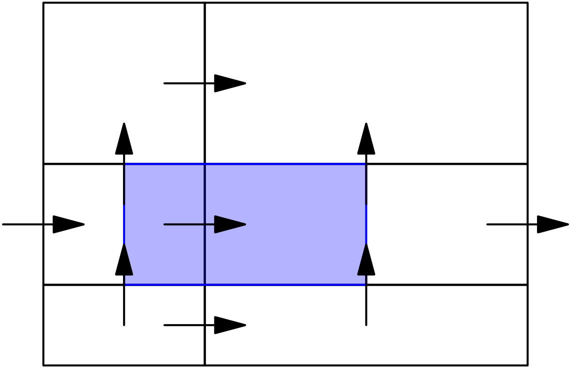
Application to Cartesian grids.



Recovers Harlow and Welch ✓

Convection $C(u_f)$

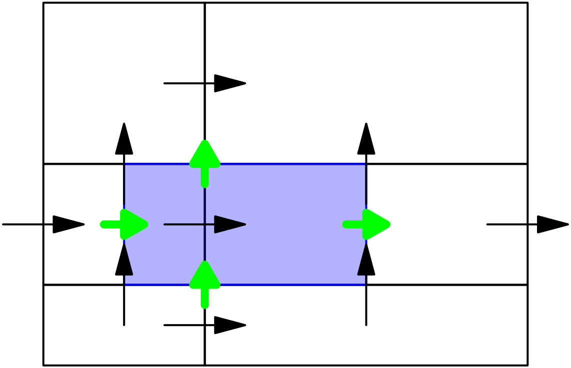
Baseline: Harlow and Welch $\nabla \cdot (\vec{u} \otimes \vec{u})$



Research question: How to define in non-Cartesian meshes?.

Convection $C(u_f)$

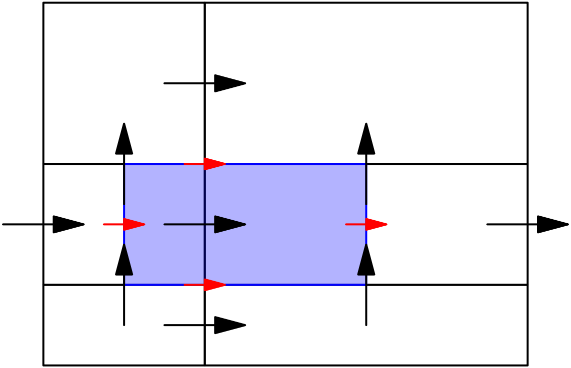
Baseline: Harlow and Welch $\nabla \cdot (\vec{u} \otimes \vec{u})$



Research question: How to define in non-Cartesian meshes?.

Convection $C(u_f)$

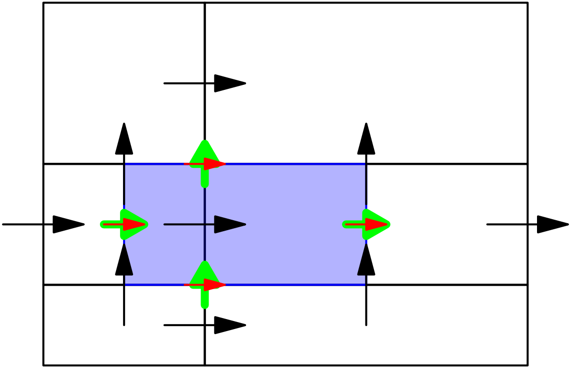
Baseline: Harlow and Welch $\nabla \cdot (\vec{u} \otimes \vec{u})$



Research question: How to define in non-Cartesian meshes?.

Convection $C(u_f)$

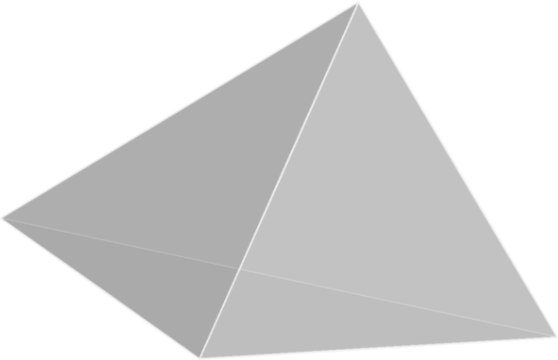
Baseline: Harlow and Welch $\nabla \cdot (\vec{u} \otimes \vec{u})$



Research question: How to define in non-Cartesian meshes?.

Convection $C(u_f)$

Baseline: Harlow and Welch $\nabla \cdot (\vec{u} \otimes \vec{u})$



Research question: How to define in non-Cartesian meshes?.

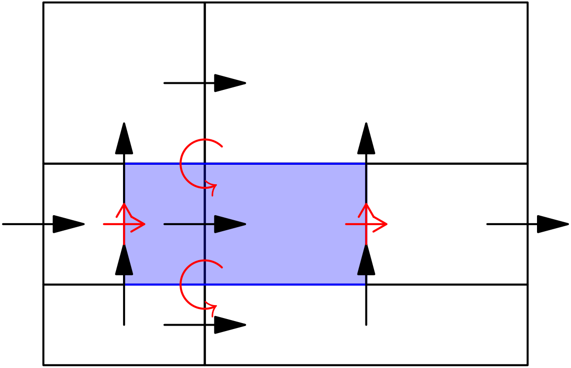
Previous attempts

Rotational formulation $\nabla \cdot (\vec{u} \otimes \vec{u}) = \vec{u} \times \nabla \times \vec{u} + \frac{1}{2}\nabla (\vec{u} \cdot \vec{u})$

Recovers Harlow and Welch $\times$
Chain rule does not hold at the discrete level.

Previous attempts

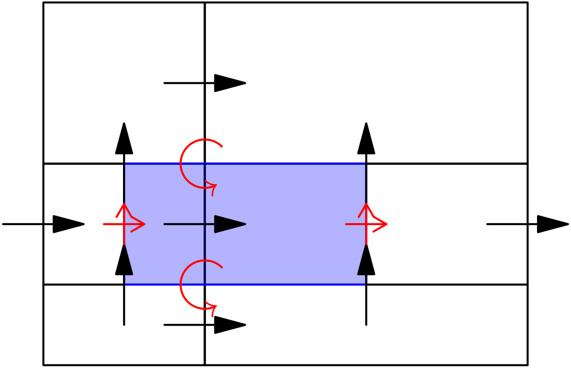
Rotational formulation $\nabla \cdot (\vec{u} \otimes \vec{u}) = \vec{u} \times \nabla \times \vec{u} + \frac{1}{2}\nabla(\vec{u} \cdot \vec{u})$



Recovers Harlow and Welch ×
Chain rule does not hold at the discrete level.

Previous attempts

Rotational formulation $\nabla \cdot (\vec{u} \otimes \vec{u}) = \vec{u} \times \nabla \times \vec{u} + \frac{1}{2}\nabla (\vec{u} \cdot \vec{u})$

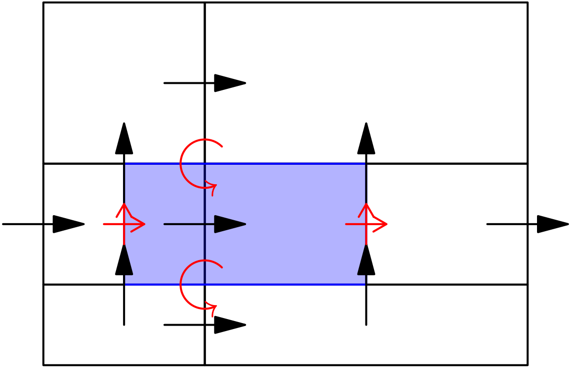


Recovers Harlow and Welch $\times$

Chain rule does not hold at the discrete level.

Previous attempts

Rotational formulation $\nabla \cdot (\vec{u} \otimes \vec{u}) = \vec{u} \times \nabla \times \vec{u} + \frac{1}{2}\nabla(\vec{u} \cdot \vec{u})$



Recovers Harlow and Welch $\times$

Chain rule does not hold at the discrete level.

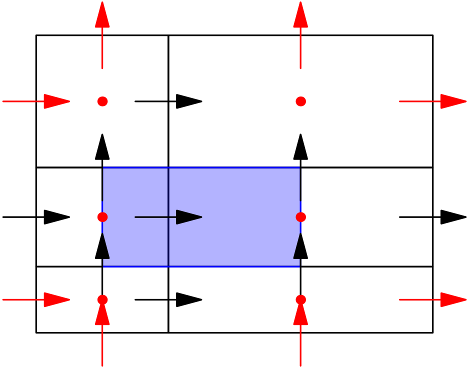
Previous attempts

Interpolated formulation $C_s^0(u) = \Gamma_{c\rightarrow s} C_c(u) \Gamma_{s\rightarrow c}$

Recovers Harlow and Welch $\times$
Larger stencil.

Previous attempts

Interpolated formulation $C_s^0(u) = \Gamma_{c\rightarrow s} C_c(u) \Gamma_{s\rightarrow c}$

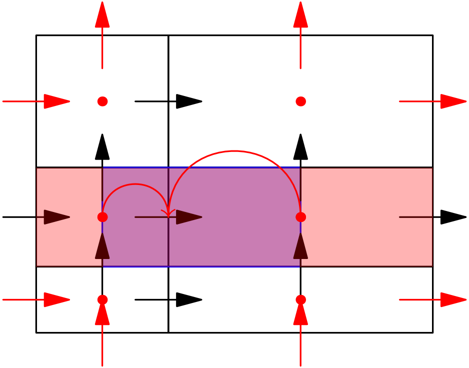


Recovers Harlow and Welch $\times$

Larger stencil.

Previous attempts

Interpolated formulation $C_s^0(u) = \Gamma_{c\to s} C_c(u) \Gamma_{s\to c}$

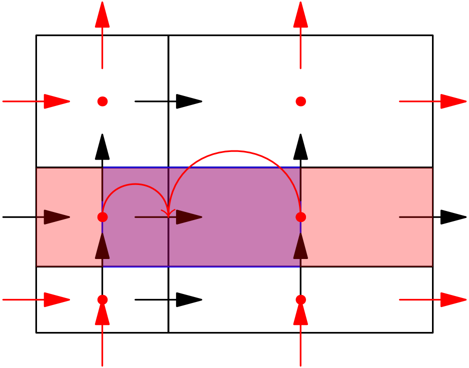


Recovers Harlow and Welch $\times$

Larger stencil.

Previous attempts

Interpolated formulation $C_s^0(u) = \Gamma_{c\to s} C_c(u) \Gamma_{s\to c}$

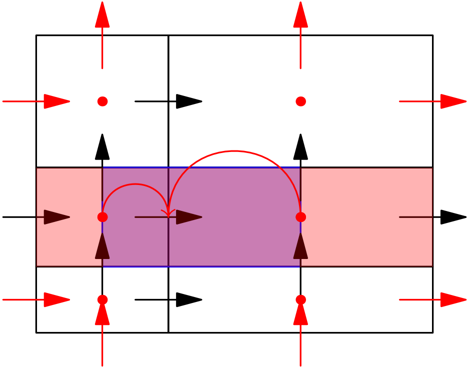


Recovers Harlow and Welch $\times$

Larger stencil.

Previous attempts

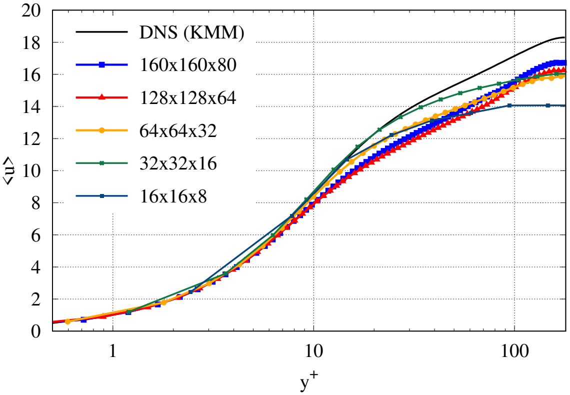
Interpolated formulation $C_s^0(u) = \Gamma_{c \to s} C_c(u) \Gamma_{s \to c}$



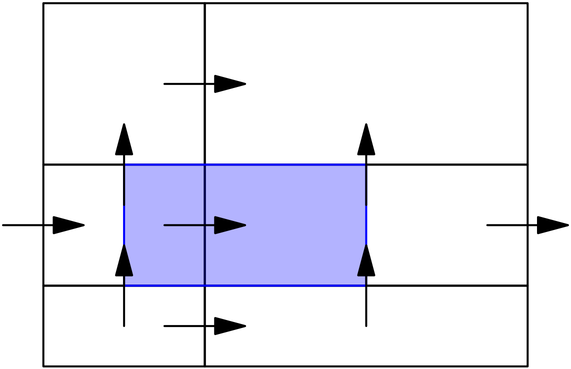
Recovers Harlow and Welch $\times$
Larger stencil.

CF180 - Interpolated

Channel flow at $Re_\tau = 180$. Cartesian mesh.

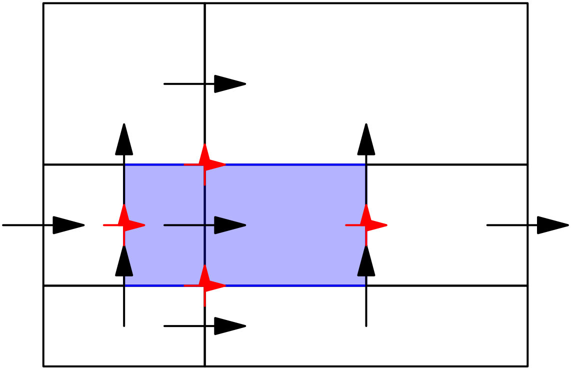


Our attempt



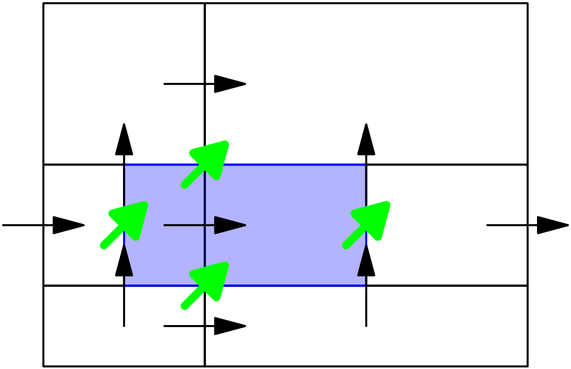
Recovers Harlow and Welch ✓

Our attempt



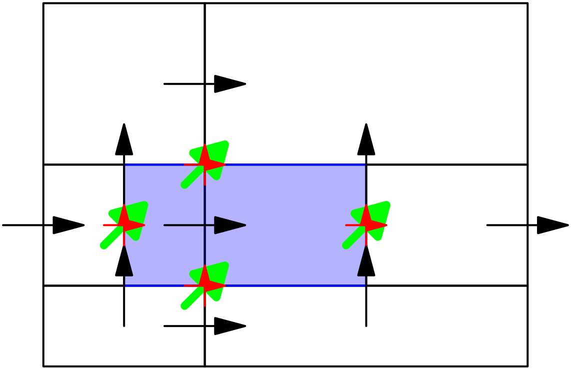
Recovers Harlow and Welch ✓

Our attempt



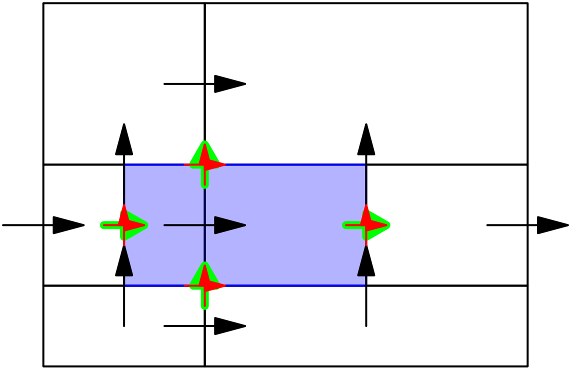
Recovers Harlow and Welch ✓

Our attempt



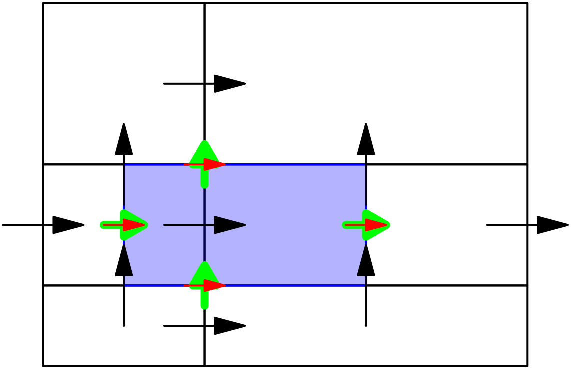
Recovers Harlow and Welch ✓

Our attempt



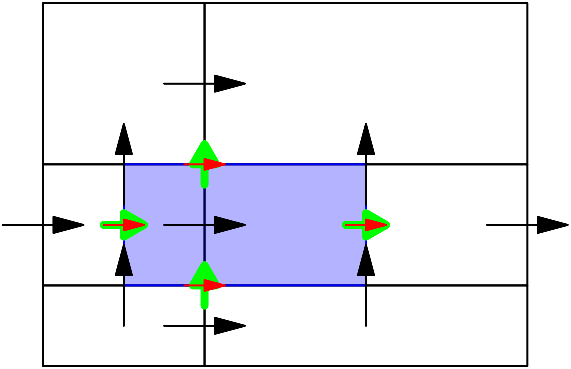
Recovers Harlow and Welch ✓

Our attempt



Recovers Harlow and Welch ✓

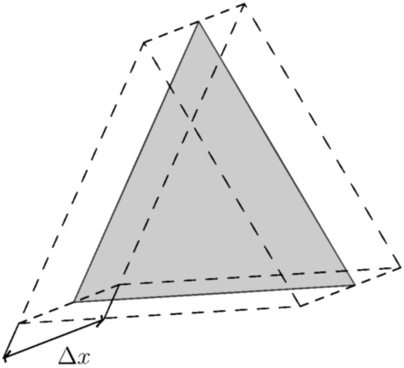
Our attempt



Recovers Harlow and Welch ✓

Our attempt

Construct the explicit staggered control volume.



$$\vec{F}^i_c = \vec{u}\, \boxed{SP^i_{f\to c}}\, u_f$$

$$(\Delta x)^{-1}\, \hat{n}_f \cdot \sum_{c\in f} \pm \vec{F}^i_c$$

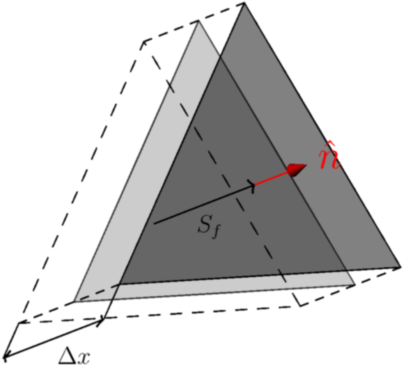
$$F^i_e = \vec{u}_e \boxed{SP^i_{f\to e}}\, u_f$$

$$(\Delta x S_f)^{-1} \sum_{e\in f} \pm \vec{F}_{ei} \cdot \left(\hat{n}_f \times \hat{t}_e\right) L_e \Delta x$$

Project over the face normal

Our attempt

Construct the explicit staggered control volume.



$$\vec{F}_c^i = \vec{u}\, \boxed{SP^i_{f\to c}}\, u_f$$

$$(\Delta x)^{-1}\, \hat{n}_f \cdot \sum_{c\in f} \pm \vec{F}_c^i$$

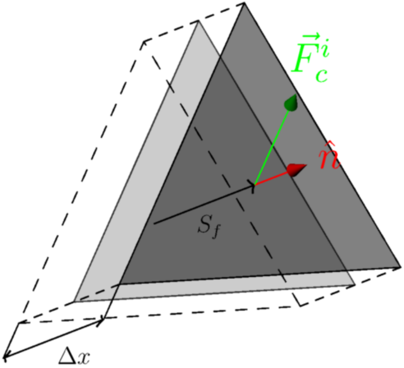
$$F_e^i = \vec{u}_e\, \boxed{SP^i_{f\to e}}\, u_f$$

$$(\Delta x S_f)^{-1} \sum_{e\in f} \pm \vec{F}_{ei} \cdot \left(\hat{n}_f \times \hat{t}_e\right) L_e \Delta x$$

Project over the face normal

Our attempt

Construct the explicit staggered control volume.



$$\vec{F}_c^i = \vec{u} \boxed{SP^i_{f\rightarrow c}} u_f$$

$$(\Delta x)^{-1} \hat{n}_f \cdot \sum_{c \in f} \pm \vec{F}_c^{\,i}$$

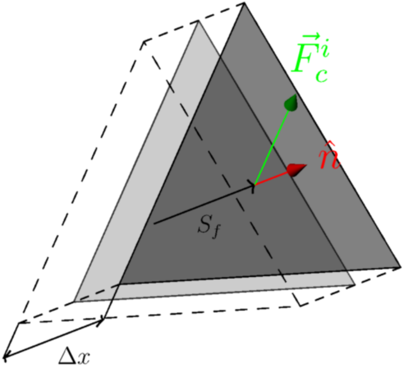
$$F_e^i = \vec{u}_e \boxed{SP^i_{f\rightarrow e}} u_f$$

$$(\Delta x S_f)^{-1} \sum_{e \in f} \pm \vec{F}_{ei} \cdot \left(\hat{n}_f \times \hat{t}_e\right) L_e \Delta x$$

Project over the face normal

Our attempt

Construct the explicit staggered control volume.



$$\vec{F}_c^i = \vec{u} \boxed{SP^i_{f\rightarrow c}} u_f$$

$$(\Delta x)^{-1} \hat{n}_f \cdot \sum_{c \in f} \pm \vec{F}_c{}^i$$

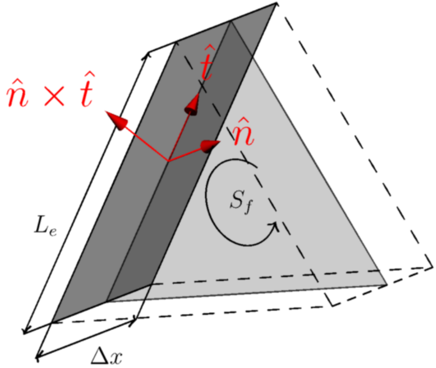
$$F_e^i = \vec{u}_e \boxed{SP^i_{f\rightarrow e}} u_f$$

$$(\Delta x S_f)^{-1} \sum_{e \in f} \pm \vec{F}_{ei} \cdot \left(\hat{n}_f \times \hat{t}_e\right) L_e \Delta x$$

Project over the face normal

Our attempt

Construct the explicit staggered control volume.



$$\vec{F}_c^i = \vec{u} \boxed{SP^i_{f\to c}} u_f$$

$$(\Delta x)^{-1} \hat{n}_f \cdot \sum_{c\in f} \pm \vec{F}_c^{\,i}$$

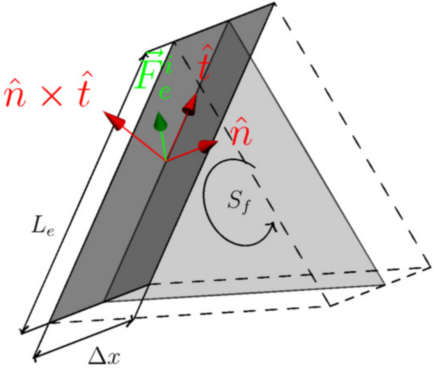
$$F_e^i = \vec{u}_e \boxed{SP^i_{f\to e}} u_f$$

$$(\Delta x S_f)^{-1} \sum_{e\in f} \pm \vec{F}_{ei} \cdot \left(\hat{n}_f \times \hat{t}_e\right) L_e \Delta x$$

Project over the face normal

Our attempt

Construct the explicit staggered control volume.



$$\vec{F}_c^i = \vec{u} \boxed{SP_{f \to c}^i} u_f$$

$$(\Delta x)^{-1} \hat{n}_f \cdot \sum_{c \in f} \pm \vec{F}_c{}^i$$

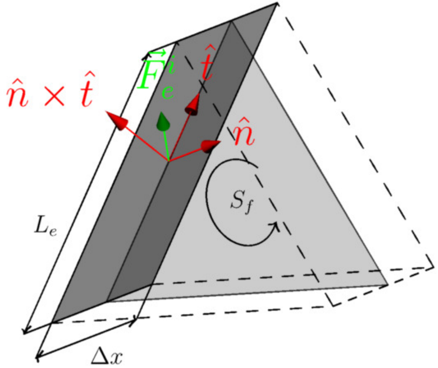
$$F_e^i = \vec{u}_e \boxed{SP_{f \to e}^i} u_f$$

$$(\Delta x S_f)^{-1} \sum_{e \in f} \pm \vec{F}_{ei} \cdot \left(\hat{n}_f \times \hat{t}_e \right) L_e \Delta x$$

Project over the face normal

Our attempt

Construct the explicit staggered control volume.



$$\vec{F}^i_c = \vec{u}\, \boxed{SP^i_{f\to c}}\, u_f$$

$$(\Delta x)^{-1}\, \hat{n}_f \cdot \sum_{c\in f} \pm \vec{F}_c^{\;i}$$

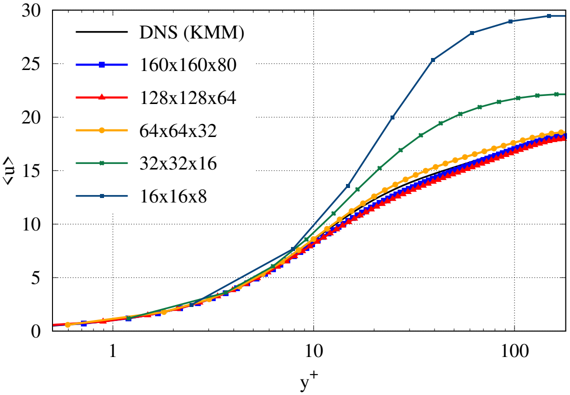
$$F^i_e = \vec{u}_e\, \boxed{SP^i_{f\to e}}\, u_f$$

$$(\Delta x S_f)^{-1} \sum_{e\in f} \pm \vec{F}_{e\,i} \cdot \left(\hat{n}_f \times \hat{t}_e\right) L_e \Delta x$$

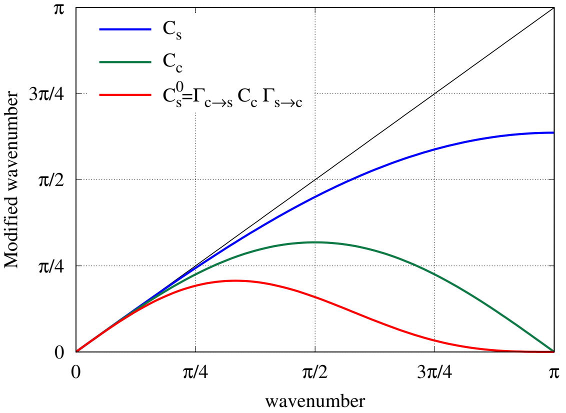
Project over the face normal

CF180 - Harlow and Welch

Channel flow at $Re_\tau = 180$. Cartesian mesh.



Dispersion relation



Closure

Conclusions

- Construction of L in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement L and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.

Closure

Conclusions

- Construction of L in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement L and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.

Closure

Conclusions

- Construction of L in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement L and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.

Closure

Conclusions

- Construction of L in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement L and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.

Closure

Conclusions

- Construction of L in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement L and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.

Closure

Conclusions

- Construction of L in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement L and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.

Closure

Conclusions

- Construction of L in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement L and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.

Thank you for your attention.