14th International ERCOFTAC symposium on engineering, turbulence, modelling and measurements (ETMM)
Barcelona, Spain, 6-8 September, 2023

Robust and reliable DNS and LES on unstructured grids

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Key words: DNS, LES, symmetry-preserving, Unstructured grids, collocated

Abstract. Preserving the operators symmetries at discrete level is the key aspect to enable reliable DNS and LES simulations of turbulent flows. On the other hand, real-world applications demand robust and stable numerical methods suitable for complex geometries. In this regard, this work presents a symmetry-preserving discretization for unstructured collocated grids that, apart from being virtually free of artificial dissipation, it is shown to be unconditionally stable.

1 Introduction

The essence of turbulence are the smallest scales of motion [1]. They result from a subtle balance between convective transport and diffusive dissipation. Mathematically, these terms are governed by two differential operators differing in symmetry: the convective operator is skew-symmetric, whereas the diffusive is symmetric and negative-definite. At discrete level, operator symmetries must be retained to preserve the analogous (invariant) properties of the continuous equations [2, 3]: namely, the convective operator is represented by a skew-symmetric coefficient matrix, the diffusive operator by a symmetric, negative-definite matrix and the divergence is minus the transpose of the gradient operator. It is noteworthy to mention that in the last decade, many DNS reference results have been successfully generated using this type of discretization (see Figure 1, for example). Moreover, we also consider that symmetry-preserving discretizations form a solid basis for testing sub-grid scale models for LES. Namely, for coarse grids, the energy of the resolved scales of motion is convected in a stable manner, *i.e.* the discrete convective operator transports energy from a resolved scale of motion to other resolved scales without dissipating any energy, as it should be from a physical point-of-view.

2 Symmetry-preserving discretization on unstructured grids

For unstructured meshes, it is (still) a common argument that accuracy should take precedence over the properties of the operators. Contrary to this, our philosophy is that operator symmetries are critical to the dynamics of turbulence and must be preserved. With this in mind, a fully-conservative discretization method for general unstructured grids was proposed in Ref. [3]: it exactly preserves the symmetries of the underlying differential operators on a collocated mesh. In summary, and following the same notation as in Ref. [3], the method is based on a set of five basic operators: the cell-centered and staggered control volumes (diagonal matrices), Ω_c and Ω_s, the matrix containing the face normal vectors, N_s, the cell-to-face scalar field interpolation, $\Pi_{c\rightarrow s}$ and the cell-to-face divergence operator, M. Once these

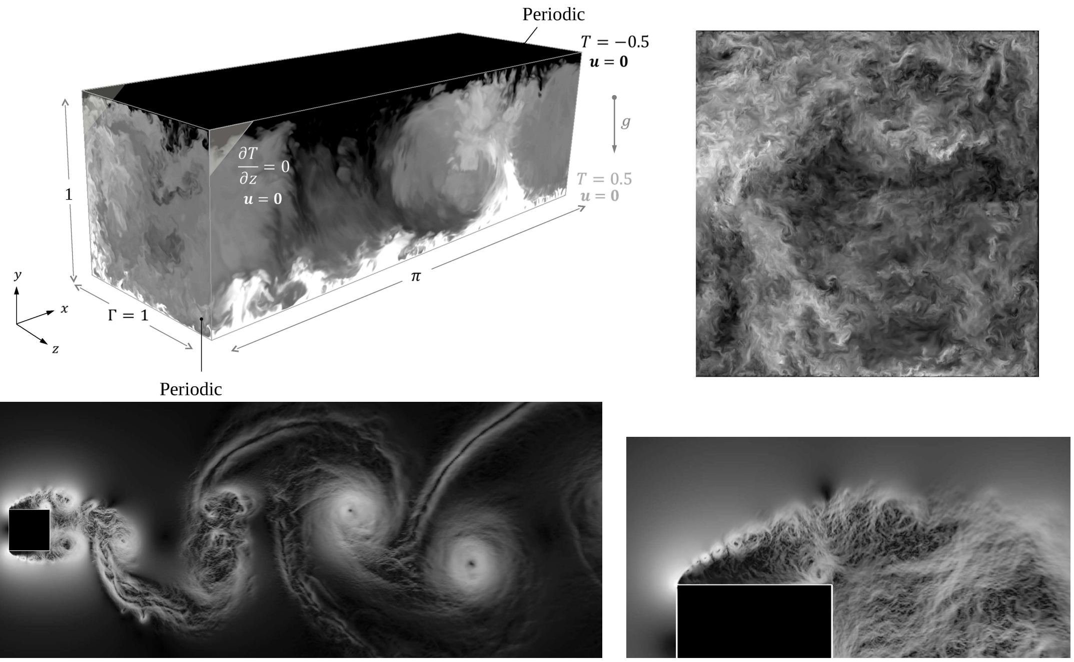

Figure 1: Examples of DNSs computed using symmetry-preserving discretizations. Top: air-filled ($Pr = 0.7$) Rayleigh-Bénard configuration studied in Ref. [1]. Instantaneous temperature field at $Ra = 10^{10}$ (left) and instantaneous velocity magnitude at $Ra = 10^{11}$ (right) for a span-wise cross section are shown. The latter was computed on 8192 CPU cores of the MareNostrum 4 supercomputer on a mesh of 5.7 billion grid points. Bottom: DNS of the turbulent flow around a square cylinder at $Re = 22000$ computed on 784 CPU cores of the MareNostrum 3 supercomputer on a mesh of 323 million grid points [4] .

operators are constructed, the rest follows straightforwardly from them. Therefore, the proposed method constitutes a robust and easy-to-implement approach to solve incompressible turbulent flows in complex configurations that can be easily implemented in already existing codes such as OpenFOAM® [5].

3 Reconciling accuracy and robustness

Pressure-correction methods on collocated grids suffer two inherent drawbacks: the cell-centered velocity field is not exactly incompressible and some artificial dissipation is inevitably introduced [3, 6]. The former error can have severe implications for DNS and LES simulations of turbulent flows since this artificial dissipation can significantly affect the dynamics of the small scales, even overwhelming the dissipation introduced by the subgrid-scale LES models. This was clearly observed for LES simulations using the standard implementation of OpenFOAM® [7]. Figure 3 schematically represents some of the existing pressure-velocity coupling algorithms. It is worth to notice that the ideal target, *i.e.* no artificial dissipation and no checkerboard, can be achieved by explicitly removing those nonphysical components of the pressure field that belong to the kernel of the so-called wide-stencil Laplacian. However, in practice, this can only be applied to Cartesian meshes where these nonphysical pressure modes are known *a priori* [8]. Alternatively, it is possible to minimize the amount of dissipation while still keeping the solution virtually free of checkerboard modes by preserving the symmetries of the discrete operators.

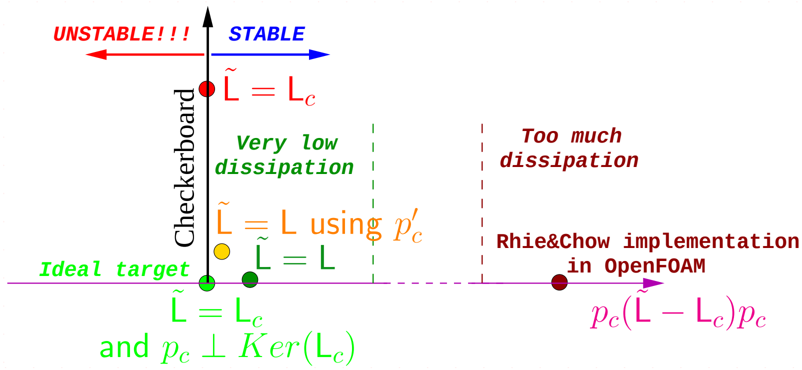

Figure 2: Schematic summary of existing pressure-velocity coupling approaches. Horizontal axis represents the amount of artificial dissipation introduced by the pressure gradient term in the momentum equation whereas vertical axis represents the appearance of checkerboard modes in the numerical solutions.

Special attention must be paid to the construction of the face-to-cell and cell-to-face interpolations, in order to guarantee that the numerical method is unconditionally stable. Apart from this, other relevant issues such as the time-integration method or the portability challenges imposed by current HPC trends will be discussed during the Symposium.

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