Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Robust and reliable DNS and LES on unstructured grids

F.Xavier Trias[1], Jannes Hopman[1], Daniel Santos[1],
Andrey Gorobets[2], Assensi Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia
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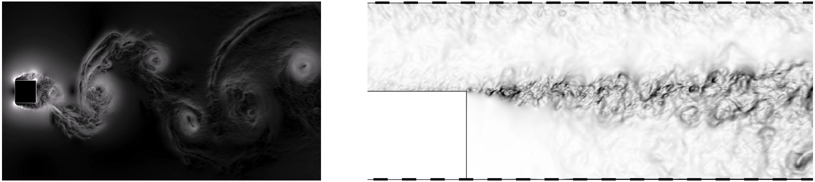

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Robust and reliable DNS and LES on unstructured grids:

playing with matrices to preserve symmetries using a small set of algebraic kernels

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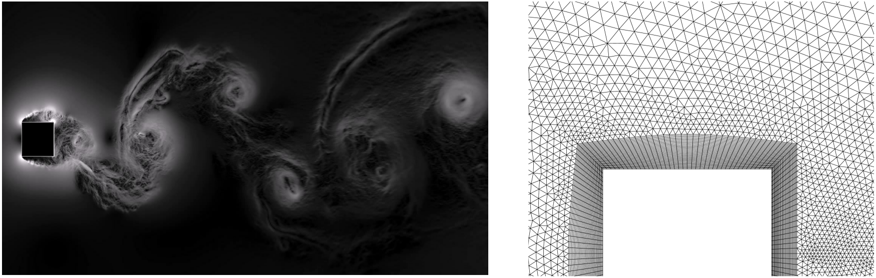
Contents

Motivation

Research question #1:

- Can we construct numerical discretizations of the Navier-Stokes equations suitable for **complex geometries**, such that the **symmetry properties** are exactly preserved?



DNS[1] of the turbulent flow around a square cylinder at $Re = 22000$

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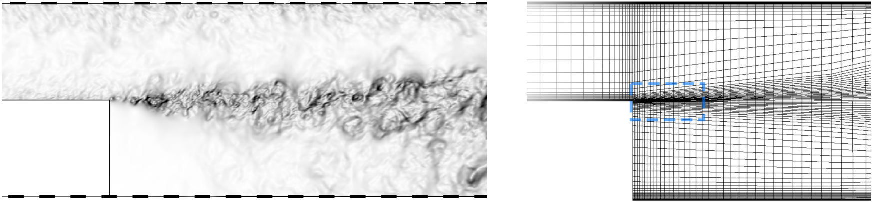

DNS[2] of backward-facing step at $Re_\tau = 395$ and expansion ratio 2

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[2] A.Pont-Vílchez, F.X.Trias, A.Gorobets, A.Oliva. *DNS of Backward-Facing Step flow at $Re_\tau = 395$ and expansion ratio 2*. **Journal of Fluid Mechanics**, 863:341-363, 2019.

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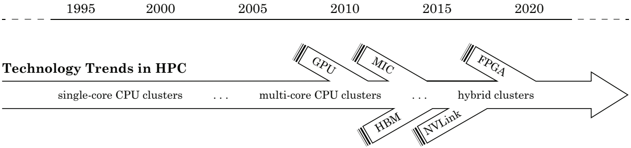
Research question #2:

- How can we develop **portable** and **efficient** CFD codes for large-scale simulations on modern supercomputers?



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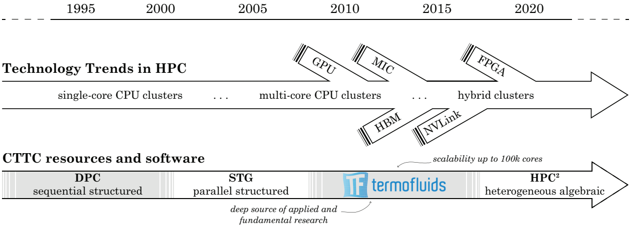

HPC^2: portable, algebra-based framework for heterogeneous computing is being developed[3,4]. Traditional stencil-based data and sweeps are replaced by algebraic structures (sparse matrices and vectors) and kernels. `SpMM`-based strategies to increase the arithmetic intensity are presented in this conference[5].

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Frequently used general purpose CFD codes:

- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM

CD-adapco SIEMENS

ANSYS FLUENT

CODE SATURNE eDF GPL

Open∇FOAM® GPL 3 Free Software

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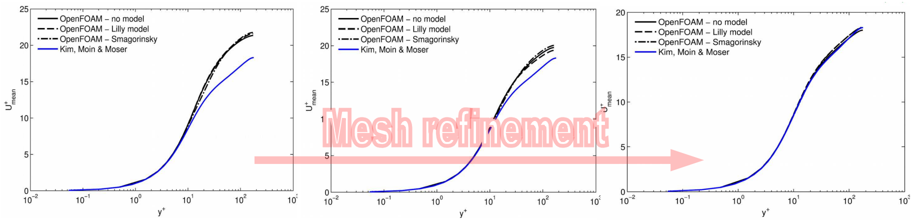
Main common characteristics of LES in such codes:

- **Unstructured finite volume** method, **collocated** grid
- Second-order spatial and temporal discretisation
- Eddy-viscosity type LES models

Motivation

Open∇FOAM® LES[6]results of a turbulent channel for at $Re_\tau = 180$



$13 \times 76 \times 20$ $\Delta x^+ = 90, \Delta y^+_{wall} = 0.5, \Delta z^+ = 30$

$19 \times 78 \times 28$ $\Delta x^+ = 60, \Delta y^+_{wall} = 0.5, \Delta z^+ = 20$

$38 \times 78 \times 57$ $\Delta x^+ = 30, \Delta y^+_{wall} = 0.5, \Delta z^+ = 10$

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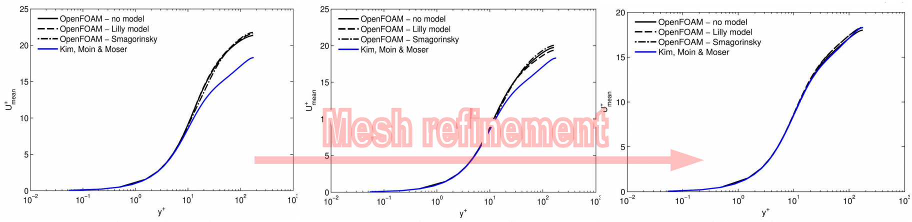

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- Are LES results are merit of the SGS model?

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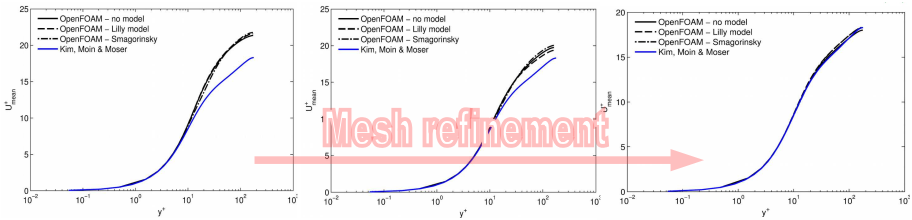

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- Are LES results are merit of the SGS model? Apparently **NOT**!!! ✗

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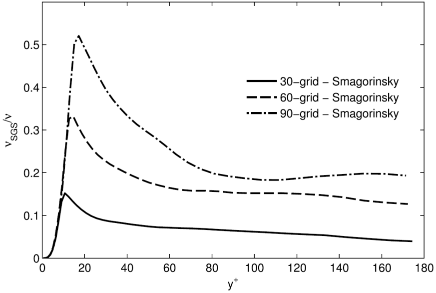
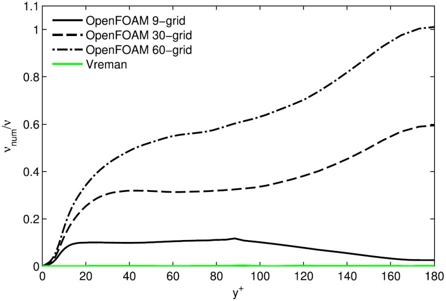
Open∇FOAM® LES[7] results of a turbulent channel for at $Re_\tau = 180$





$$\nu_{num} \neq 0$$

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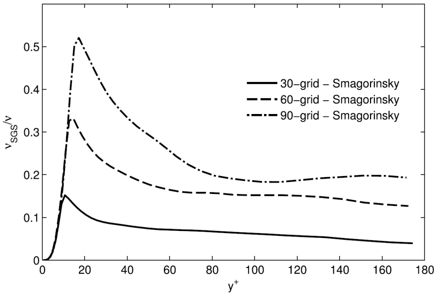

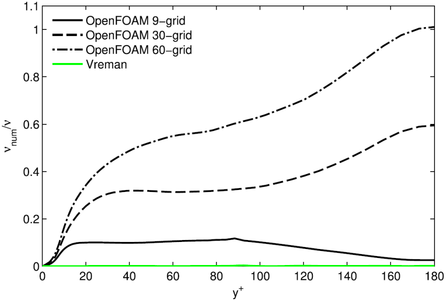

$$\nu_{SGS} < \nu_{num} \neq 0$$

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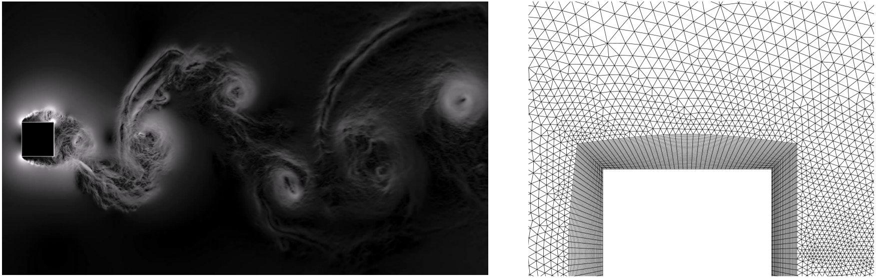

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Symmetry-preserving discretization

Continuous

$$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$$

$$\nabla \cdot \boldsymbol{u} = 0$$

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$$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$$

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Discrete

$$\Omega \frac{d \boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h) \boldsymbol{u}_h = \mathsf{D} \boldsymbol{u}_h - \mathsf{G} \boldsymbol{p}_h$$

$$\mathsf{M} \boldsymbol{u}_h = \boldsymbol{0}_h$$

Symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d \boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\, \boldsymbol{u}_h = \mathsf{D} \boldsymbol{u}_h - \mathsf{G} \boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M} \boldsymbol{u}_h = \boldsymbol{0}_h$
$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \int_\Omega \boldsymbol{a} \boldsymbol{b} d\Omega$	$\langle \boldsymbol{a}_h, \boldsymbol{b}_h \rangle_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$

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$\langle C(\boldsymbol{u}, \varphi_1), \varphi_2 \rangle = -\langle C(\boldsymbol{u}, \varphi_2), \varphi_1 \rangle$	$\mathsf{C}(\boldsymbol{u}_h) = -\mathsf{C}^T(\boldsymbol{u}_h)$

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$$\mathsf{D} = \mathsf{D}^T \qquad def -$$

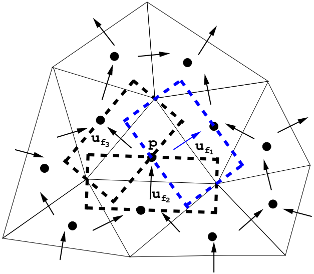
Why collocated arrangements are so popular?

- STAR-CCM+ CD-adapco SIEMENS
- ANSYS-FLUENT ANSYS FLUENT®
- Code-Saturne CODE SATURNE eDF GPL
- OpenFOAM Open∇FOAM® GPLv3 Free Software

$$\boxed{\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c}$$

In <u>staggered meshes</u>

- p-$\boldsymbol{u}_s$ coupling is naturally solved ✓
- $\mathsf{C}\left(\boldsymbol{u}_s\right)$ and D difficult to discretize ✗



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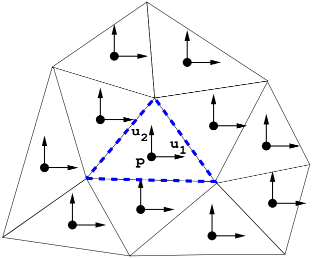
In <u>collocated meshes</u>

- p-$\boldsymbol{u}_c$ coupling is cumbersome ✗
- $\mathsf{C}\left(\boldsymbol{u}_s\right)$ and D easy to discretize ✓
- Cheaper, less memory,... ✓



Why collocated arrangements are so popular?

Everything is easy except the pressure-velocity coupling...

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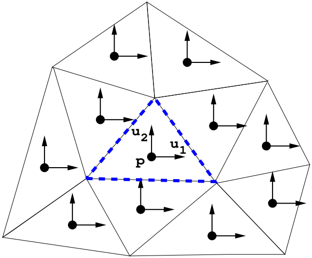

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $\mathsf{M}\Gamma_{c\rightarrow s}\boldsymbol{u}_c = \mathsf{M}\Gamma_{c\rightarrow s}\boldsymbol{u}_c - \mathsf{L}_c\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\rightarrow s}\boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c\left(\mathsf{L}-\mathsf{L}_c\right)\boldsymbol{p}_c \neq 0$ ✗

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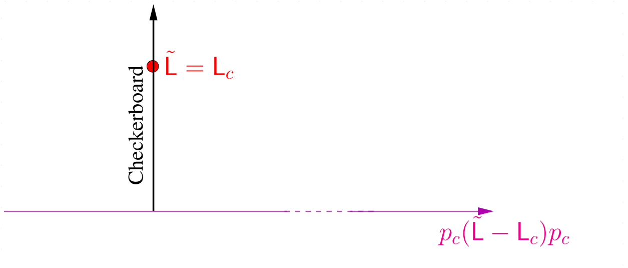

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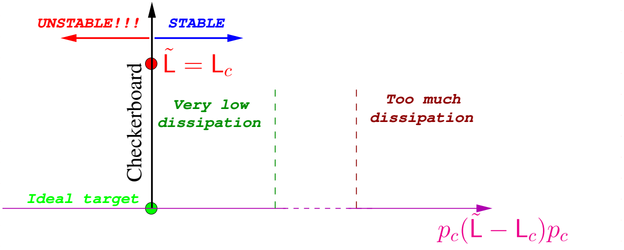

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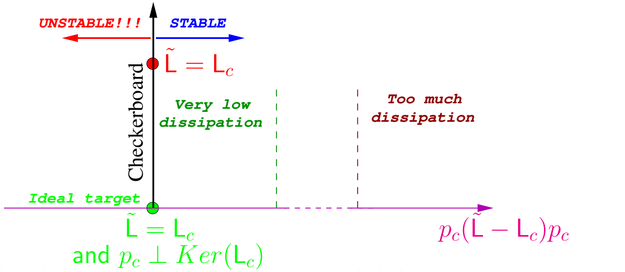

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[8] Shashank, J.Larsson, G.Iaccarino. *A co-located incompressible Navier-Stokes solver with exact mass, momentum and kinetic energy conservation in the inviscid limit*, **Journal of Computational Physics**, 229: 4425-4430,2010.

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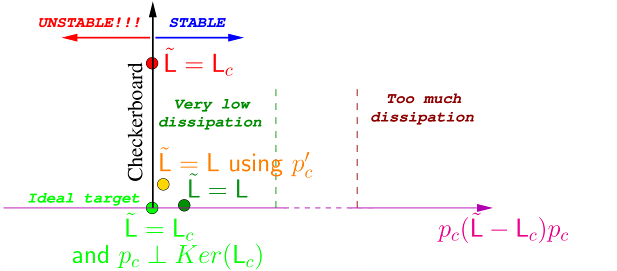

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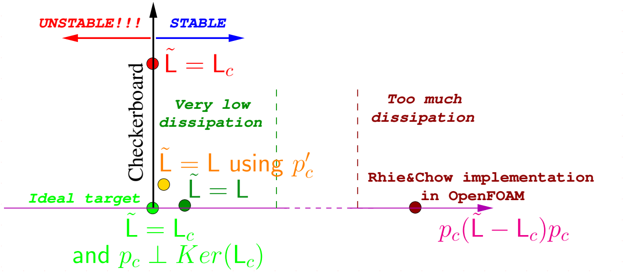
[8] F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen. *Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured grids*, **Journal of Computational Physics**, 258 (1): 246-267, 2014.

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c = \mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c - \boxed{\mathsf{L}_c\mathsf{L}^{-1}}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c\boxed{(\mathsf{L}-\mathsf{L}_c)}\boldsymbol{p}_c \neq 0$ ✗



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[8] E.Komen, J.A.Hopman, E.M.A.Frederix, F.X.Trias, R.W.C.P.Verstappen. "A symmetry-preserving second-order time-accurate PISO-based method". **Computers & Fluids**, 225:104979, 2021.

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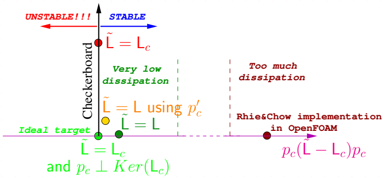


UNSTABLE!!!

STABLE

Checkerboard

$\tilde{\mathsf{L}} = \mathsf{L}_c$

Very low dissipation

Too much dissipation

$\tilde{\mathsf{L}} = \mathsf{L}$ using p_c'

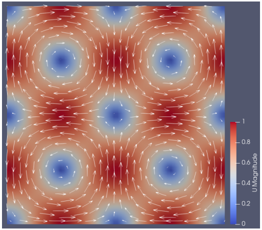
$\tilde{\mathsf{L}} = \mathsf{L}$

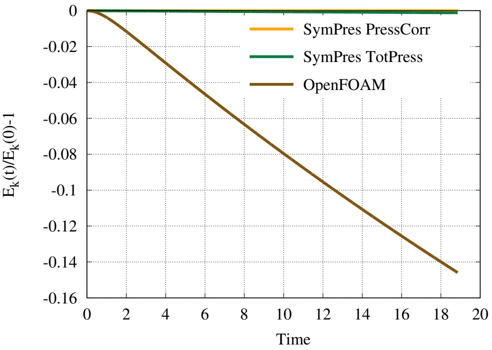
Ideal target

Rhie&Chow implementation in OpenFOAM

$\tilde{\mathsf{L}} = \mathsf{L}_c$ and $p_c \perp Ker(\mathsf{L}_c)$

$p_c(\tilde{\mathsf{L}} - \mathsf{L}_c)p_c$





Results for an inviscid Taylor-Green vortex[9]

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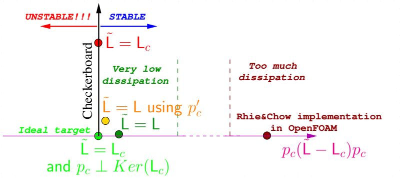

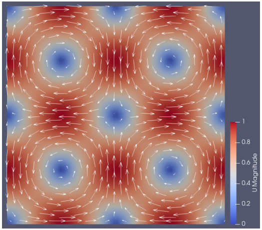

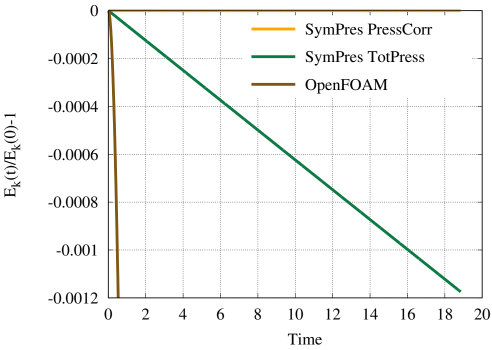

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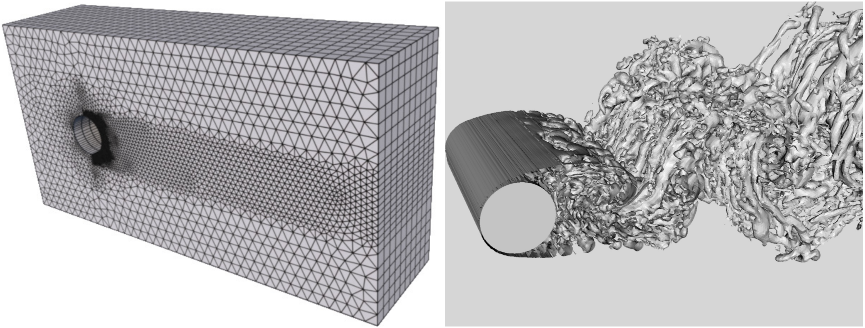
Pressure-velocity coupling on collocated grids

Examples of simulations

Despite these inherent limitations, symmetry-preserving collocated formulation has been successfully used for DNS/LES simulations[10]:



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[10] R.Borrell, O.Lehmkuhl, F.X.Trias, A.Oliva. *Parallel Direct Poisson solver for discretizations with one Fourier diagonalizable direction*. **Journal of Computational Physics**, 230:4723-4741, 2011.

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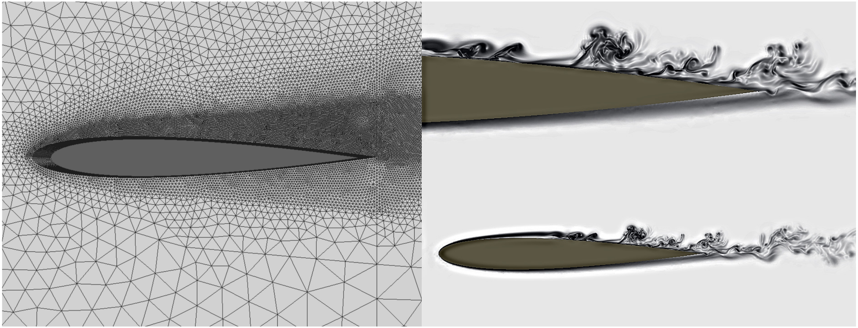

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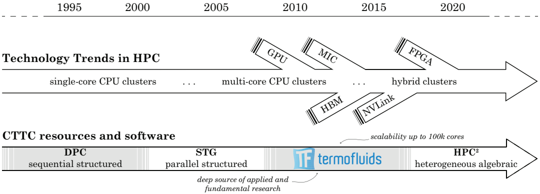
Algebra-based approach naturally leads to portability

Research question #2:

- How can we develop **portable** and **efficient** CFD codes for large-scale simulations on modern supercomputers?



1995 2000 2005 2010 2015 2020

Technology Trends in HPC

GPU
MIC
FPGA

single-core CPU clusters … multi-core CPU clusters … hybrid clusters

HBM
NVLink

CTTC resources and software

scalability up to 100k cores

DPC
sequential structured

STG
parallel structured

termofluids

HPC²
heterogeneous algebraic

deep source of applied and fundamental research

HPC^2: portable, algebra-based framework for heterogeneous computing is being developed. Traditional stencil-based data and sweeps are replaced by algebraic structures (sparse matrices and vectors) and kernels. `SpMM`-based strategies to increase the arithmetic intensity are/were presented in this conference[11].

---

[11] À.Alsalti-Baldellou, G.Colomer, J.A.Hopman, X.Álvarez-Farré, A.Gorobets, F.X.Trias, A.Oliva. *Reliable overnight industrial LES: challenges and limitations. Application to CSP technologies*. **On Friday at 15:40**

Algebra-based approach naturally leads to portability, to simple and analyzable formulations

Continuous

$$\frac{\partial \boldsymbol{u}}{\partial t} + C\left(\boldsymbol{u}, \boldsymbol{u}\right) = \nu \nabla^2 \boldsymbol{u} - \nabla p$$
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \int_\Omega \boldsymbol{a}\boldsymbol{b} d\Omega$$

$$\langle C\left(\boldsymbol{u}, \varphi_1\right), \varphi_2 \rangle = - \langle C\left(\boldsymbol{u}, \varphi_2\right), \varphi_1 \rangle$$
$$\langle \nabla \cdot \boldsymbol{a}, \varphi \rangle = - \langle \boldsymbol{a}, \nabla \varphi \rangle$$
$$\left\langle \nabla^2 \boldsymbol{a}, \boldsymbol{b} \right\rangle = \quad \left\langle \boldsymbol{a}, \nabla^2 \boldsymbol{b} \right\rangle$$

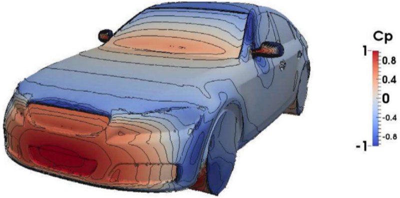
Discrete

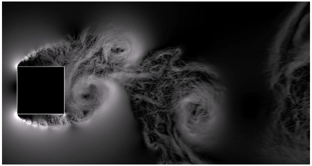
$$\Omega \frac{d \boldsymbol{u}_h}{dt} + \mathsf{C}\left(\boldsymbol{u}_h\right) \boldsymbol{u}_h = \mathsf{D} \boldsymbol{u}_h - \mathsf{G} \boldsymbol{p}_h$$
$$\mathsf{M} \boldsymbol{u}_h = \boldsymbol{0}_h$$

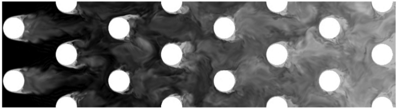
$$\langle \boldsymbol{a}_h, \boldsymbol{b}_h \rangle_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$$

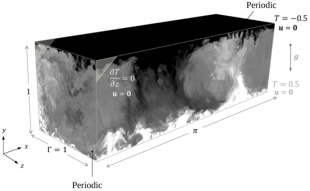
$$\mathsf{C}\left(\boldsymbol{u}_h\right) = -\mathsf{C}^T\left(\boldsymbol{u}_h\right)$$
$$\Omega \mathsf{G} = -\mathsf{M}^T$$
$$\mathsf{D} = \mathsf{D}^T \qquad def -$$

Algebra-based approach naturally leads to portability, to simple and analyzable formulations and opens the door to new strategies[12,13] to improve its perfomance...









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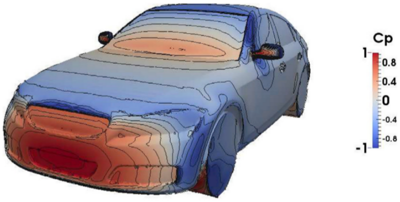
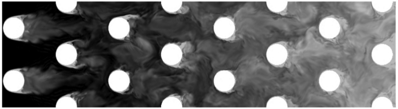
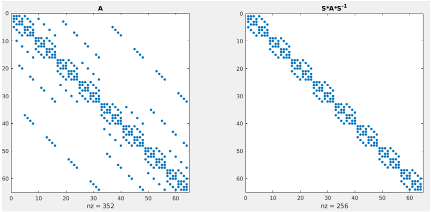




$$\hat{\mathsf{L}} = \mathsf{S}\mathsf{L}\mathsf{S}^{-1} = \mathsf{I} \otimes \mathsf{L}_{inn} + diag(\boldsymbol{d})$$



SpMM can be used $\Longrightarrow$ **higher AI**

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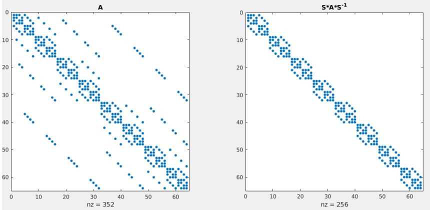
Benefits for Poisson solver are 3-fold:

$$\hat{\mathsf{L}} = \mathsf{S}\mathsf{L}\mathsf{S}^{-1} = \mathsf{I} \otimes \mathsf{L}_{inn} + diag(\boldsymbol{d})$$

- Higher arithmetic intensity (AI)
- Reduction of memory footprint
- Reduction in the number of iterations



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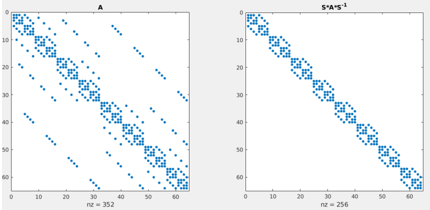

→ Overall speed-up up to **x2-x3** ✓

→ Memory reduction of ≈**2** ✓

`SpMM` can be used ⟹ **higher AI**

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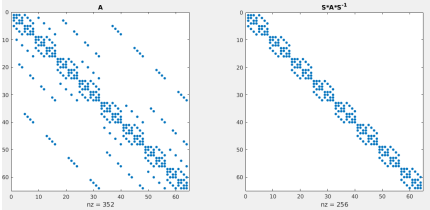
Other `SpMM`-based strategies to **increase AI** and **reduce memory** footprint:

- Multiple transport equations
- Parametric studies
- Parallel-in-time simulations
- Go to higher-order?

$$\hat{\mathsf{L}} = \mathsf{S}\mathsf{L}\mathsf{S}^{-1} = \mathsf{I} \otimes \mathsf{L}_{inn} + diag(\boldsymbol{d})$$



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Rethinking standard CFD operations

In summary...

<u>Leitmotiv</u>: relying on a **minimal set of (algebraic) kernels** is crucial for code **portability** and **maintenance**!!!

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- Computational challenge: `SpMV` has a low AI
    - Solution: make use of `SpMM` whenever possible (multiple transport equations, spatial symmetries, parallel-in-time simulations, parametric studies,...) to increase AI and, therefore, perfomance.
    - Positive side-effects: reduction of memory footprint (crucial for GPUs), improvement of the convergence for the Poisson solver...

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- Implementation challenge: there still exists a list of standard CFD methods that do not seem to fit well on an algebraic framework (*e.g.* flux limiters[14], boundary conditions, CFL condition,...).
    - Dilemma: "add more and more specific kernels" vs "rethink them"

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CFL-like condition

Step #1: forget about classical formulae from textbooks...

$$\xcancel{\Delta t \leqslant C_{conv} \left(\frac{\Delta x}{U} \right)_{\min}} \quad \text{and} \quad \xcancel{\Delta t \leqslant C_{diff} \left(\frac{\Delta x^2}{\nu} \right)_{\min}}$$

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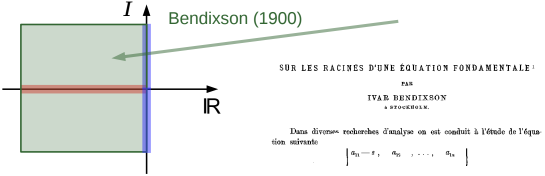
...and replace it by an eigenbounding problem of $\mathsf{C}(\boldsymbol{u}_s)$ and D matrices

$$\boxed{\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}(\boldsymbol{u}_s)\,\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c}$$

$$\left(-\mathsf{C}(\boldsymbol{u}_s) + \mathsf{D}\right)\boldsymbol{v} = \lambda \boldsymbol{v}$$

$$\mathsf{C}(\boldsymbol{u}_s)\,\boldsymbol{v} = \lambda \boldsymbol{v}$$

$$\mathsf{D}\boldsymbol{v} = \lambda \boldsymbol{v}$$



SUR LES RACINES D'UNE ÉQUATION FONDAMENTALE[1]

PAR

IVAR BENDIXSON

À STOCKHOLM.

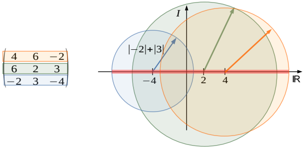
Dans diverses recherches d'analyse on est conduit à l'étude de l'équation suivante

$$\begin{vmatrix} a_{11} - s, & a_{12} & , \ldots, & a_{1n} \end{vmatrix}$$

Rethinking standard CFD operations

CFL-like condition

Step #2: compute eigenbounds of $\mathsf{C}\left(\boldsymbol{u_s}\right)$ and D in an inexpensive way[15]



$$\rho(\mathsf{D}) \leqslant \rho^{Gersh}(\mathsf{D})$$

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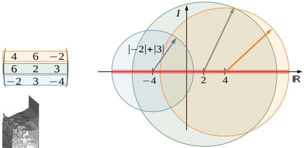

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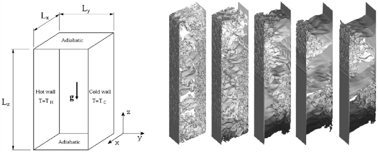


Table 1. Tests for the air-filled (Pr = 0.71) differentially heated cavity at Rayleigh number Ra = 3×10^{10} and height aspect ratio 4; averaged results correspond to the statistically steady state

	N_x	N_y	N_z	$\varphi/(\pi/2)$	$\overline{\delta t}_{\mathrm{CFL+AB2}}$	$\overline{\delta t}_{\mathrm{EigenCD+\kappa1L2}}$	$\overline{\delta t}_{\mathrm{EigenCD+\kappa1L2}}/\overline{\delta t}_{\mathrm{CFL+AB2}}$
MeshA	128	338	778	0.072	1.04×10^{-4}	3.02×10^{-4}	2.90
MeshB	64	168	338	0.158	4.31×10^{-4}	1.21×10^{-3}	2.80
MeshC	32	84	168	0.252	1.80×10^{-3}	4.69×10^{-3}	2.59
MeshD	32	56	112	0.408	4.21×10^{-3}	8.75×10^{-3}	2.08
MeshE	16	42	84	0.504	6.88×10^{-3}	1.35×10^{-3}	1.96

---

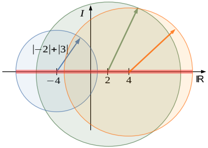
[15] F.X.Trias and O.Lehmkuhl. *A self-adaptive strategy for the time-integration of Navier-Stokes equations*. **Numerical Heat Transfer, part B**, 60(2):116-134, 2011.

Rethinking standard CFD operations

CFL-like condition

Step #2: compute eigenbounds of $\mathsf{C}\,(\boldsymbol{u_s})$ and D in an inexpensive way[15]

$$\begin{pmatrix} 4 & 6 & -2 \\ 6 & 2 & 3 \\ -2 & 3 & -4 \end{pmatrix}$$



$$\rho(\mathsf{D}) \leqslant \rho^{Gersh}(\mathsf{D})$$

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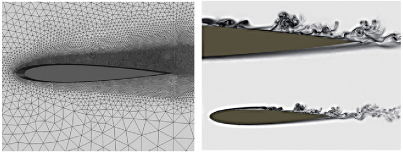

Table 2. Tests for the flow around a NACA 0012 airfoil at Reynolds number 5×10^4 and an angle of attack of 5°; averaged results correspond to the statistically steady state

	N_x	Mesh2D	$\varphi/(\pi/2)$	$\overline{\delta t}_{\mathrm{CFL+AB2}}$	$\overline{\delta t}_{\mathrm{EigenCD+\kappa 1L2}}$	$\overline{\delta t}_{\mathrm{EigenCD+\kappa 1L2}}/\overline{\delta t}_{\mathrm{CFL+AB2}}$
UMeshA	64	$\approx 2.65 \times 10^5$	0.593	4.69×10^{-5}	1.30×10^{-4}	2.77
UMeshB	32	$\approx 4.69 \times 10^4$	0.956	1.61×10^{-4}	6.86×10^{-4}	4.27

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[15] F.X.Trias and O.Lehmkuhl. *A self-adaptive strategy for the time-integration of Navier-Stokes equations*. **Numerical Heat Transfer, part B**, 60(2):116-134, 2011.

Rethinking standard CFD operations

CFL-like condition

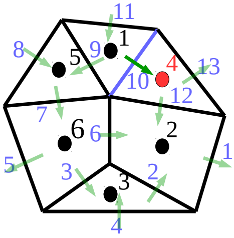
Step #3:

reformulate the problem in a way that we avoid constructing $\mathsf{C}\left(\boldsymbol{u}_s\right)$ and D

$$\boxed{\mathsf{C}\left(\boldsymbol{u}_s\right) \equiv -1/2\, T_{cs}^{T} A_s U_s |T_{cs}| \qquad \mathsf{D} \equiv -T_{cs}^{T} A_s \Lambda_s \Delta_s^{-1} T_{cs}}$$

$$T_{sc} \equiv \begin{array}{c} \begin{array}{ccccccccccccc} & & & & & & & & & 10 & & 12 & 13 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

face-to-cell oriented incidence matrix



Rethinking standard CFD operations

CFL-like condition

Step #3 ... #4 (some maths that would take too long to explain): reformulate the problem in a way that we avoid constructing $\mathsf{C}\left(\boldsymbol{u}_s\right)$ and D

$$\boxed{\mathsf{C}\left(\boldsymbol{u}_s\right) \equiv -1/2\, T_{cs}^{T} A_s U_s |T_{cs}| \qquad \mathsf{D} \equiv -T_{cs}^{T} A_s \Lambda_s \Delta_s^{-1} T_{cs}}$$

$$\rho(\mathsf{D}) \leqslant \rho^{Gersh}(\mathsf{D}) \ldots$$

$$\rho(\mathsf{C}) \leqslant \rho^{Gersh}(\mathsf{C}) \ldots$$

where T_{cs} is the face-to-cell oriented incidence matrix; $\tilde{\Delta}_s \equiv A_s \Lambda_s \Delta_s^{-1}$ (diffusivity-like fluxes) and $\tilde{F}_s \equiv A_s U_s$ (mass fluxes) are diagonal matrices.

Rethinking standard CFD operations

CFL-like condition

Step #3 ... #4 (some maths that would take too long to explain): reformulate the problem in a way that we avoid constructing $\mathsf{C}(\boldsymbol{u}_s)$ and D

$$\boxed{\mathsf{C}(\boldsymbol{u}_s) \equiv -1/2 T_{cs}^T A_s U_s |T_{cs}| \qquad \mathsf{D} \equiv -T_{cs}^T A_s \Lambda_s \Delta_s^{-1} T_{cs}}$$

$$\rho(\mathsf{D}) \leqslant \rho^{Gersh} \dots (\mathsf{D}) \leqslant \rho(T_{cs} T_{cs}^T \tilde{\Delta}_s) \leqslant \max(\underbrace{\overbrace{|T_{cs} T_{cs}^T|}^{\substack{Constant \\ matrix}} diag(\tilde{\Delta}_s)}_{\mathtt{SpMV}})$$

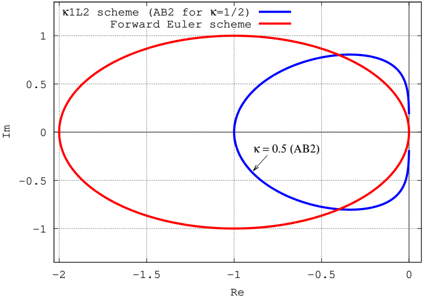
$$\rho(\mathsf{C}) \leqslant \rho^{Gersh} \dots (\mathsf{D}) \leqslant 1/4 \rho(|T_{cs} T_{cs}^T| |\tilde{F}_s|) \leqslant 1/4 \max(\underbrace{\overbrace{|T_{cs} T_{cs}^T|}^{\substack{Constant \\ matrix}} diag(|\tilde{F}_s|)}_{\mathtt{SpMV}})$$

where T_{cs} is the face-to-cell oriented incidence matrix; $\tilde{\Delta}_s \equiv A_s \Lambda_s \Delta_s^{-1}$ (diffusivity-like fluxes) and $\tilde{F}_s \equiv A_s U_s$ (mass fluxes) are diagonal matrices.

Rethinking standard CFD operations

CFL-like condition

Step #5: self-adaptation of the stability region

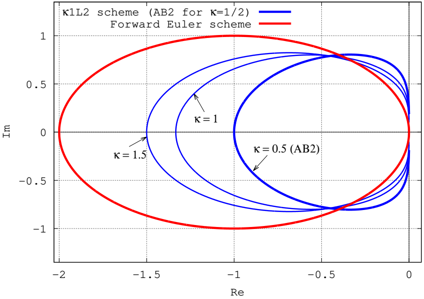


$$\frac{\left(\kappa+\frac{1}{2}\right)\phi^{n+1}-2\kappa\phi^{n}+\left(\kappa-\frac{1}{2}\right)\phi^{n-1}}{\Delta t}=f\left((1+\kappa)\phi^{n}-\kappa\phi^{n-1}\right)$$

Rethinking standard CFD operations

CFL-like condition

Step #5: self-adaptation of the stability region



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Rethinking standard CFD operations

CFL-like condition

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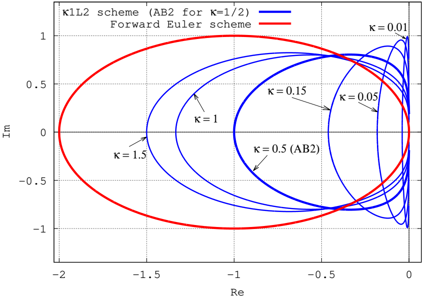

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Rethinking standard CFD operations

CFL-like condition

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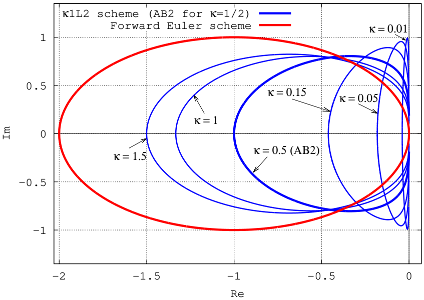

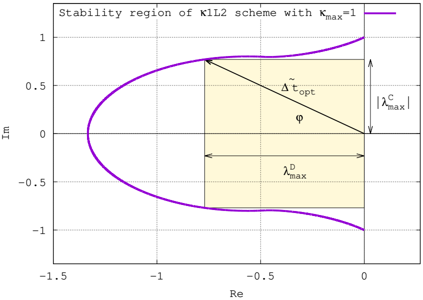

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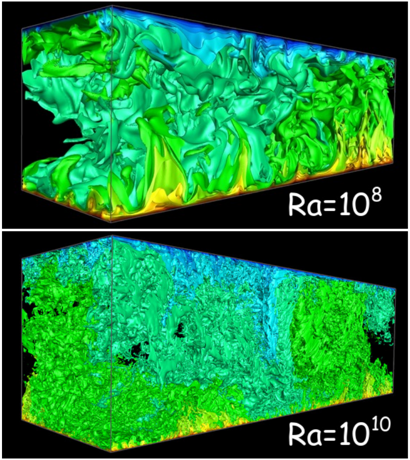
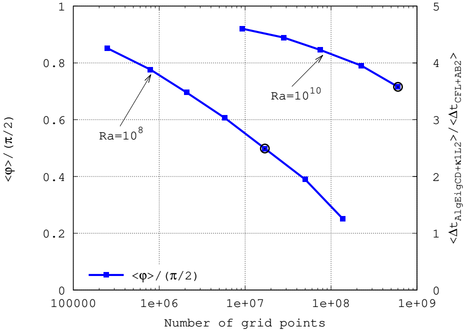
Rethinking standard CFD operations

CFL-like condition: results

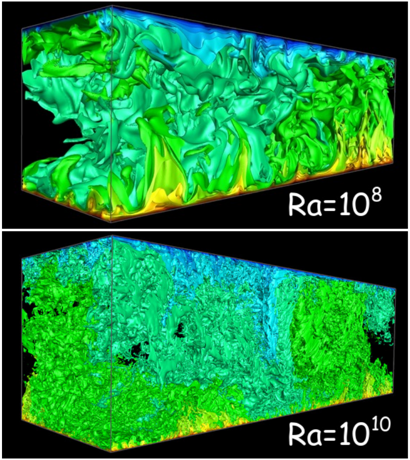


Numerical test #1:
air-filled Rayleigh–Bénard convection



Rethinking standard CFD operations

CFL-like condition: results



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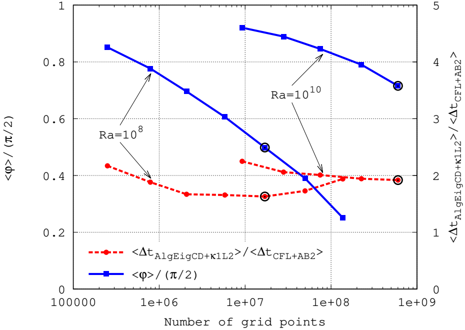
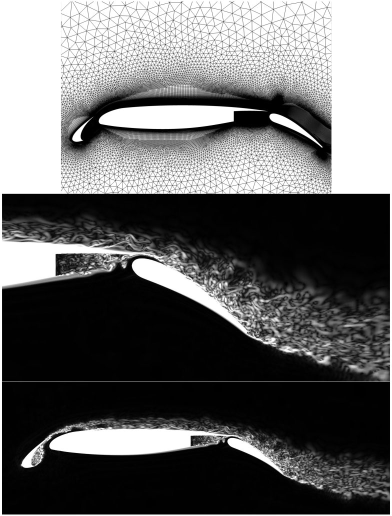
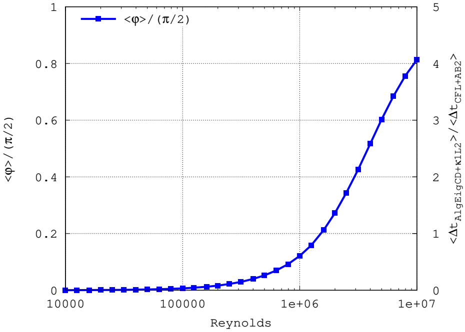


Rethinking standard CFD operations

CFL-like condition: results



Numerical test #2: 30P30N airfoil



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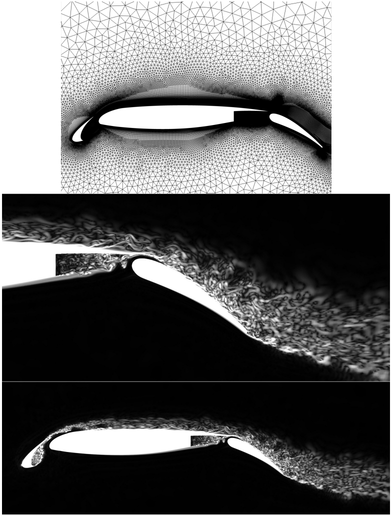

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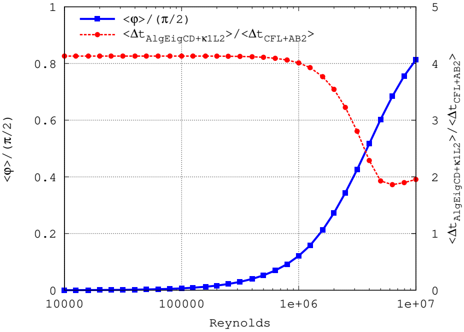
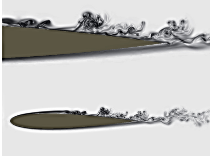


Concluding remarks

- **Preserving symmetries** either using staggered or collocated formulations is the key point for **reliable LES/DNS** simulations.



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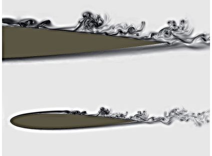
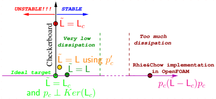
[16] N.Valle, X.Álvarez, A.Gorobets, J.Castro, A.Oliva, F.X.Trias. *On the implementation of flux limiters in algebraic frameworks*. **Computer Physics Communications**, 271:108230, 2022.

Concluding remarks

- **Preserving symmetries** either using staggered or collocated formulations is the key point for **reliable LES/DNS** simulations.
- Algebra-based approach naturally leads to **portability**, to simple and **analyzable** formulations and opens the door to **new strategies to improve its perfomance**.





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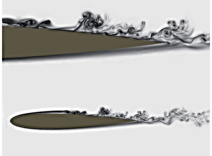

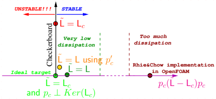

On-going research:

- **Rethinking** standard CFD operations (*e.g.* flux limiters[16], boundary conditions, **CFL**,...) to adapt them into an algebraic framework (<u>Motivation</u>: maintaining a minimal number of basic kernels is crucial for portability!!!)

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Thank you for your attendance