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On the large-eddy simulation of a fully developed wind-turbine array boundary layer

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Abstract. The incompressible Navier-Stokes equations stand as the best mathematical model for turbulent flows. However, as direct numerical simulations at high Reynolds numbers are not yet feasible, dynamically less complex mathematical formulations have been developed. We will focus on the well-known eddy-viscosity models. Most of these models are based on the combination of invariants of a symmetric tensor that depends on the gradient of the resolved velocity field, $\mathsf{G} = \nabla\overline{u}$. Brand-new models have been constructed considering the first three invariants of the symmetric tensor $\mathsf{G}\mathsf{G}^T$ with excellent results on decaying isotropic turbulence and turbulent channel flow [1]. In this work, we have tested and compared the performance of the S3PQR and other LES models on the free boundary layer case, with a pseudo-spectral, fractional step fully explicit second-order time-integration method [2]. Then, we will deal with a simplified model [3] of a wind turbine in order to simulate a fully developed boundary layer wind farm.

1 Introduction

Large eddy simulation equations result from applying a spatial filter to the incompressible Navier-Stokes equations, that then read as follows:

$$\begin{aligned} \partial_t\overline{u} + C(\overline{u},\overline{u}) &= D(\overline{u}) - \nabla p - \nabla\cdot\tau(\overline{u}); \\ \nabla\cdot\overline{u} &= 0 \end{aligned} \tag{1}$$

where $\overline{u}$ is the filtered velocity and $\tau(\overline{u})$ is the subgrid stress (SGS) tensor that approximates the effect of the under-resolved scales. This equation needs a closure model in order to be numerically solved. The LES closure is of the type $\tau(\overline{u}) \approx -2\nu_e S(\overline{u})$ where $S(\overline{u}) = 1/2(\nabla\overline{u} + \nabla\overline{u}^T)$ is the rate-of-strain tensor. We must define an eddy viscosity: $\nu_e = (C_m\Delta)^2 D_m(\overline{u})$ where C_m is the model constant, Δ is the subgrid characteristic length, and $D_m(\overline{u})$ is the differential operator with units of frequency associated with the model [4].

The S3PQR models [1] involve invariants of the symmetric tensor $\mathsf{G}\mathsf{G}^T$. The different types of S3PQR models are obtained by restricting them to solutions with only two of those invariants. The three different obtained models are [1] $\nu_e^{S3PQ}, \nu_e^{S3PR}, \nu_e^{S3QR}$ or for simplicity, PQ, PR, QR.

There are two ways to determine the model constant [1]:

1. Imposing numerical stability and less or equal dissipation than Vreman's model. Then,

$C_{s3pq} = C_{s3pr} = C_{s3qr} = \sqrt{3}C_{Vr} \approx 0.458$

2. Granting that the averaged dissipation of the models is equal to that of the Smagorinsky model. Then, $C_{s3pq} = 0.572$, $C_{s3pr} = 0.709$, $C_{s3qr} = 0.762$

Therefore we are left with 6 possible combinations to test (3 model types x 2 constants) that we will call PQ1, PQ2, and so on. The general algorithm for a boundary layer is based on the method proposed by Spalart [5], which includes normal coordinate similarity transformations, growing terms $GT(\overline{u},\overline{U})$ and scaling factors.

There are some differences with our implementation, though. First, our algorithm is based on the strong formulation of the Navier-Stokes equations with a Poisson - pressure correction term. Second, we use the standard algebraic scaling [6], $y_\infty = L\frac{1+y}{1-y}$, for the the semi-infinite domain over the normal direction. We will test the zero mean pressure gradient case.

2 Turbine model and first results

We will follow the model stated by Calaf et al [3] which is based on the concept of a disk actuator for every wind turbine. The force of the turbine (per unit mass), in the streamwise direction, at a given grid point i, j, k, is given by

$$F(i,j,k) = -\frac{1}{2} C_T' \langle \overline{u}^T \rangle_d^2 \frac{\gamma_{j,k}}{\Delta x}$$

where C_T' is a thrust coefficient, $\langle \overline{u}^T \rangle_d^2$ is the disk averaged local velocity, $\gamma_{j,k}$ is the fraction area overlap of the disk and Δx is the distance between turbines.

We carried out our simulation without the turbine model yet. The grid of size of the domain is $Nx = 32$, $Nz = 32$, and $Ny = 64$ points, where x, y, and z, are the streamwise, normal, and spanwise directions. It is a pseudospectral algorithm, two steps Adams-Bashforth method. The Reynolds number is fixed along the simulation to $Re_\delta = 1000$, where δ is the displacement thickness.

To compare the LES models and the Spalart results [5], we can list three main parameters: u_τ as the friction velocity, H as the ratio of the displacement thickness to the momentum thickness, and κ as the Von Kármán constant (see table).

Case:	**Spalart**	No model	Vreman	WALE	PQ1	PR1	QR1	PQ2	PR2	QR2
u_τ	0.049	0.048	0.047	0.046	0.048	0.049	0.049	0.047	0.047	0.048
H	1.52	1.57	1.57	1.55	1.57	1.55	1.56	1.56	1.57	1.56
κ	0.39	0.36	0.45	0.44	0.36	0.40	0.37	0.36	0.36	0.39

The Smagorinsky method did not yield meaningful results with the current algorithm. The rest of LES models give reasonable results, with PQ2 standing by now as the best in the global analysis. As an example of the performance of PQ2, we plot the velocity profile (figure [1]) and the root mean square of the velocities (figure [2]).

Thus so far we have obtained meaningful results with low computational effort for the free boundary layer. The disk actuator model selected can be straightforwardly applied over the algorithm and the results will be shown at the congress.

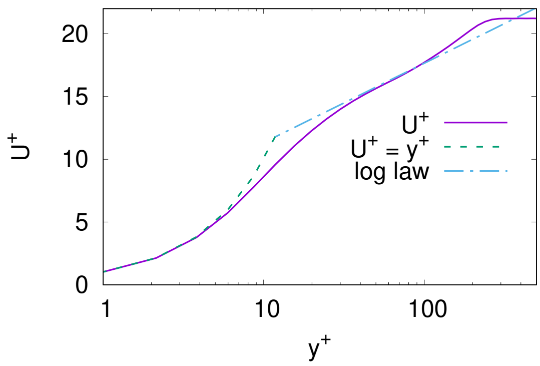

Figure 1: Normalized velocity profile. Case PQ2. Present results: — U^+ ; -·-· log law ; ···· $U^+ = y^+$

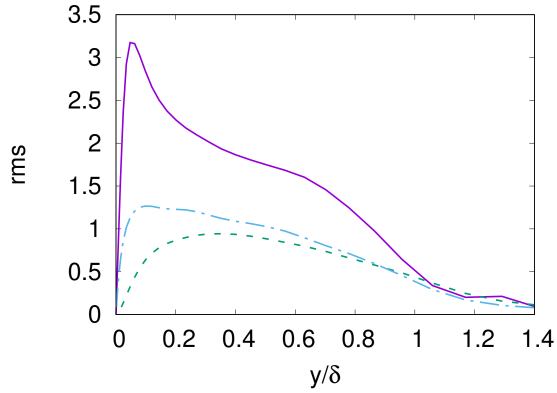

Figure 2: rms profiles. Case PQ2. Present results: — rms u; ··· rms v; -·-· rms w

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