On the large-eddy simulation of a fully developed wind-turbine array boundary layer

D.Folch[1], F.X.Trias[1], and A.Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia

Abstract

Direct numerical simulations of the incompressible Navier-Stokes equations at high Reynolds numbers are not yet feasible, so dynamically less complex mathematical formulations such as Large Eddy Simulation (LES) have been developed. **In this work, we will focus on the application and comparative performance of the S3PQR and other LES models on the free boundary layer case and over a fully developed boundary layer wind farm, using a simplified wind turbine model.**

Introduction

LES equations result from applying a spatial filter to the incompressible Navier-Stokes equations that let us prescribe an eddy viscosity ν_e for each LES algorithm. **For most models, this ν_e depends on combinations of invariants of some tensor related to the velocity gradient. This is the case for the Smagorinsky, WALE, Vreman's, σ-model, and all the S3PQR models.**
The different types of S3PQR models (Trias et al. (2015)) were obtained by a combination of two invariants of the tensor GG^T (formally based on the lowest-order approximation of the subgrid stress tensor), that is P_{GG^T}, Q_{GG^T}, R_{GG^T}. Then,

$$\nu_e^{S3PQ} = (C_{s3pq}\Delta)^2 P_{GG^T}^{-5/2} Q_{GG^T}^{3/2}$$
$$\nu_e^{S3PR} = (C_{s3pr}\Delta)^2 P_{GG^T}^{-1} R_{GG^T}^{1/2}$$
$$\nu_e^{S3QR} = (C_{s3qr}\Delta)^2 Q_{GG^T}^{-1} R_{GG^T}^{5/6}$$

where Δ is the subgrid characteristic length.
There are two ways to determine the remaining model constant C_{s3pq}:
1. Imposing numerical stability and less or equal dissipation than Vreman's model.

$$C_{s3pq} = C_{s3pr} = C_{s3qr} = \sqrt{3}C_{Vr} \approx 0.458$$

2. Granting that the averaged dissipation of the models is equal to that of the Smagorinsky model.

$$C_{s3pq} = 0.572,\ C_{s3pr} = 0.709,\ C_{s3qr} = 0.762$$

Therefore, there are six possible combinations S3PQR to test (3 model types x 2 constants, namely PQ1, PQ2 and so on)

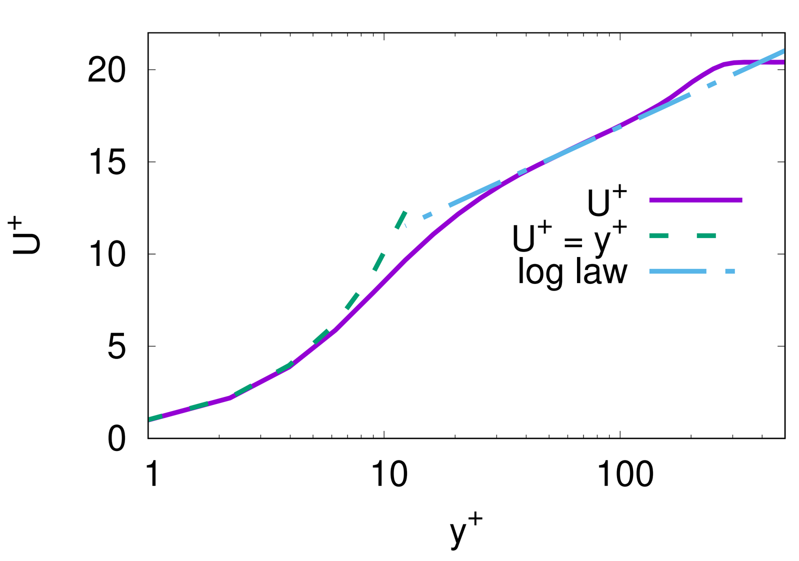
Case especifications

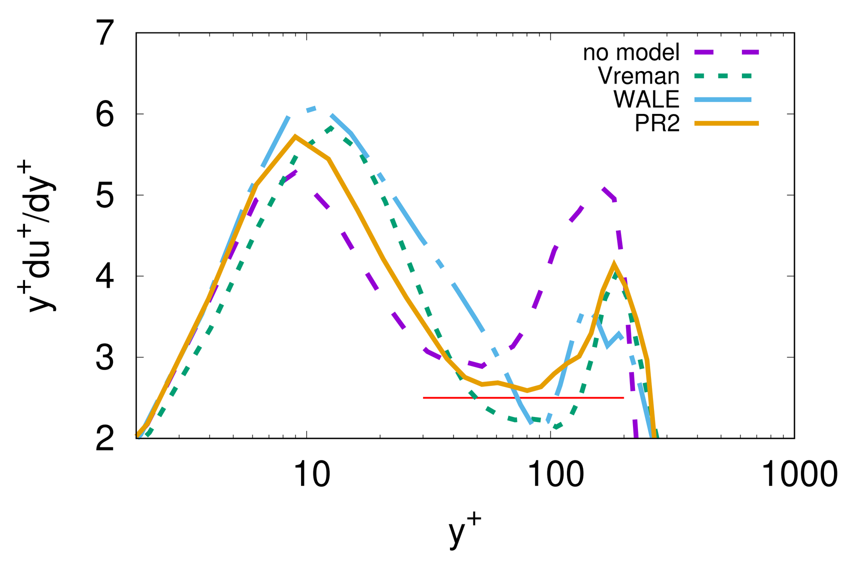
For all the current computations of this work, the grid size of the domain is $Nx = 32$, $Ny = 64$, and $Nz = 32$ points, where x, y, and z, are the streamwise, wall-normal, and spanwise directions. The Reynolds number is fixed along the simulation to $Re_{\delta^*} = 1000$, where δ^* is the displacement thickness. We will test the zero mean pressure gradient case.

Boundary layer results

We will follow the general method proposed by Spalart and Leonard (1987) for the boundary layer. We will compare three main parameters: u_τ as the friction velocity, H as the ratio of the displacement thickness to the momentum thickness, and κ as the Von Kármán constant ("SL" stands for the reference values and "Vr." as Vreman's model).

Case:	SL	No mod.	Vr.	WALE	PQ1	PR1	QR1	PQ2	PR2	QR2
u_τ	**0.049**	0.049	0.050	0.046	0.048	0.050	0.049	0.046	0.049	0.048
H	**1.52**	1.61	1.51	1.54	1.58	1.54	1.57	1.57	1.53	1.57
κ	**0.39**	0.35	0.47	0.47	0.35	0.44	0.35	0.42	0.39	0.32





The main differences between the models can be seen in plotting the derivative of the velocity profile y^+du^+/dy^+. The logarithmic layer corresponds to the minimum (red line, $\kappa = 0.4$). PR2 model (plotted on the left) seems to perform the best. Smagorinsky, in this configuration, fails to reproduce the behavior near the wall.

Wind farm results

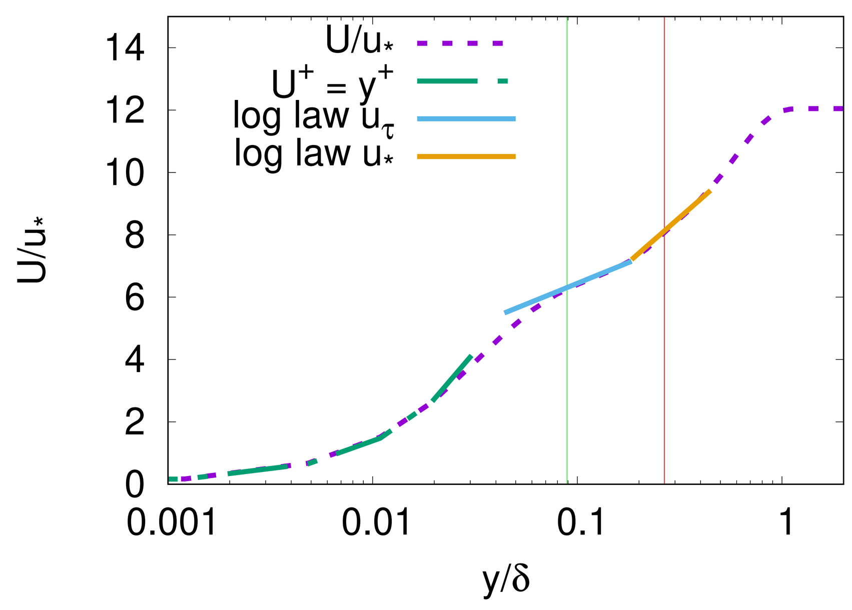
We will follow Calaf, Meneveau, and Meyers (2010) using a disk actuator for every wind turbine. The force of the turbine (per unit mass), in the streamwise direction, at a given grid point i, j, k, is given by

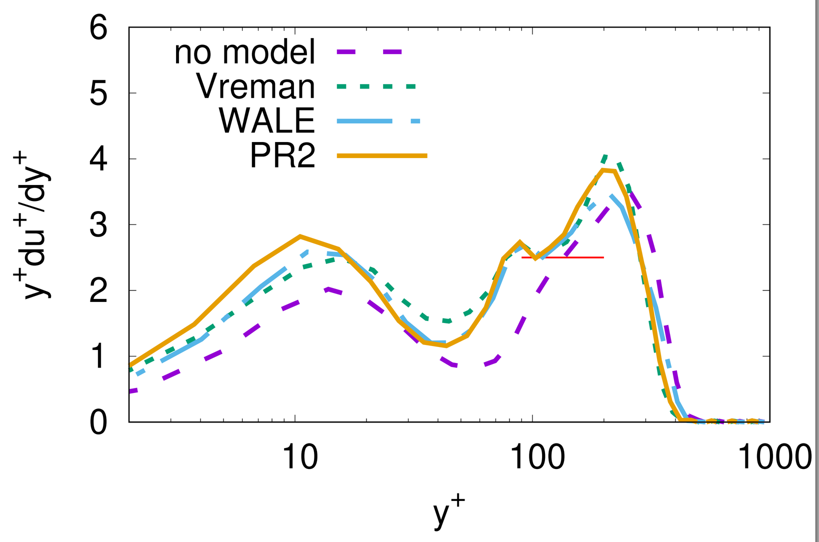
$$F(i,j,k) = -\frac{1}{2}C'_T\langle \overline{\boldsymbol{u}}^T\rangle_d^2 \frac{\gamma_{j,k}}{\Delta x}$$

where C'_T is a thrust coefficient, $\langle \overline{\boldsymbol{u}}^T\rangle_d^2$ is the disk local averaged velocity, $\gamma_{j,k}$ is the fraction area overlap of the disk and Δx is the distance between turbines. The layout geometry is 24 disk actuators evenly distributed in four rows and six columns.
Some of the several quantities that may be of interest are: $z0_{Hi}/zH$, as the ratio of the effective roughness above the turbine hub and the height of the turbines' center; u_τ, the usual friction velocity at the wall; u_*, the computed friction velocity above the hub; P, the time and horizontally averaged power extracted for every turbine; W_t, the time, horizontally, and vertically (along the hub) averaged power; $\delta\Phi$, the vertical flux of kinetic energy; EB, for energy budget.

MODEL	$z0_{Hi}/zH$	u_τ	u_*	u_τ/u_*	$P/\delta\Phi$	$W_t/\delta\Phi$	EB
no model	0.160	0.051	0.109	0.47	0.68	0.81	94%
Vreman	0.072	0.056	0.085	0.66	0.67	0.78	94%
WALE	0.082	0.050	0.089	0.56	0.79	0.90	94%
PQ1	0.096	0.052	0.092	0.57	0.75	0.86	96%
PR1	0.105	0.052	0.094	0.55	0.74	0.85	95%
QR1	0.123	0.052	0.100	0.52	0.73	0.84	95%
PQ2	0.074	0.052	0.085	0.61	0.75	0.86	95%
PR2	0.065	0.052	0.083	0.63	0.77	0.88	97%
QR2	0.098	0.052	0.093	0.56	0.74	0.86	95%

From the column of $P/\delta\Phi$, it seems that it also reproduces the observed behavior that the wind turbines, in a fully developed boundary layer regime, extract kinetic energy through vertical fluxes.





On the left, we can see the PR2 model streamwise velocity profile. The green vertical line is the position of the bottom of the turbine hub. The red line is the top. On the right, the y^+du^+/dy^+ plot. Note the presence of one or two minima corresponding to the log laws. The red horizontal line shows the expected second log law with, by definition, $\kappa = 0.4$

Conclusions

We have shown that S3PQR models yield good performance for the boundary layer and wind farm cases. The accuracy is similar to other LES methods in the case of the boundary layer, with perhaps S3PR standing as the best of all. For the wind farm, most of the S3PQR give the expected two log law profile, the right magnitude values, and the correct energy flux balance, with, again, PR as the most reliable.
Therefore, we can confidently say that at least the S3PR models are well-suited for both free boundary layer and wind farm simulations.

Further readings

- Calaf, M., C. Meneveau, and J. Meyers (2010). "Large eddy simulation study of fully developed wind-turbine array boundary layers". In: *Physics of Fluids* 22, p. 015110.
- Spalart, P.R. and A. Leonard (1987). *Direct Numerical Simulation of Equilibrium Turbulent Boundary Layers.* Berlin: Springer-Verlag, pp. 234–252.
- Trias, F. X. et al. (2015). "Building proper invariants for eddy-viscosity subgrid-scale models". In: *Physics of Fluids* 27.6, p. 065103.