S3PQR models theory

Based on two invariants of the tensor GG^T, P_{GG^T}, Q_{GG^T}, R_{GG^T}.

$$\begin{aligned}
\nu_e^{S3PQ} &= (C_{s3pq}\Delta)^2 P_{\mathrm{GG}^T}^{-5/2} Q_{\mathrm{GG}^T}^{3/2} \\
\nu_e^{S3PR} &= (C_{s3pr}\Delta)^2 P_{\mathrm{GG}^T}^{-1} R_{\mathrm{GG}^T}^{1/2} \\
\nu_e^{S3QR} &= (C_{s3qr}\Delta)^2 Q_{\mathrm{GG}^T}^{-1} R_{\mathrm{GG}^T}^{5/6}
\end{aligned}$$

Two ways to determine the model constant C_{s3pq}:

1. Less or equal dissipation than Vreman's model.

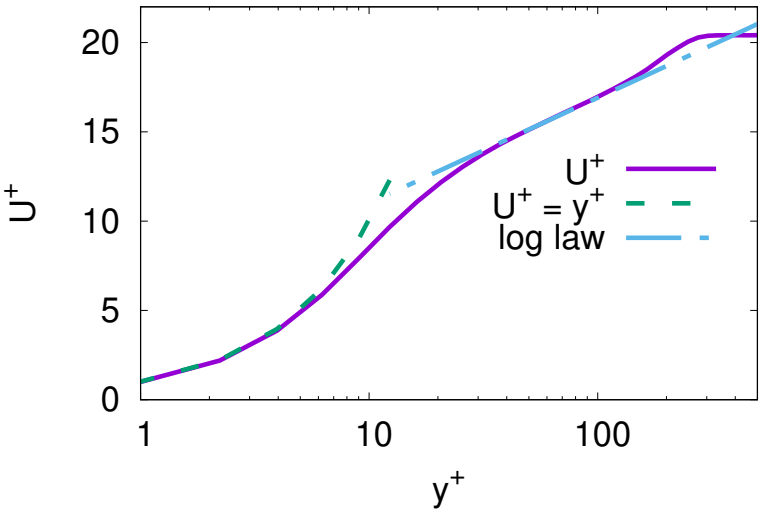
$C_{s3pq} = C_{s3pr} = C_{s3qr} = \sqrt{3} C_{Vr} \approx 0.458$

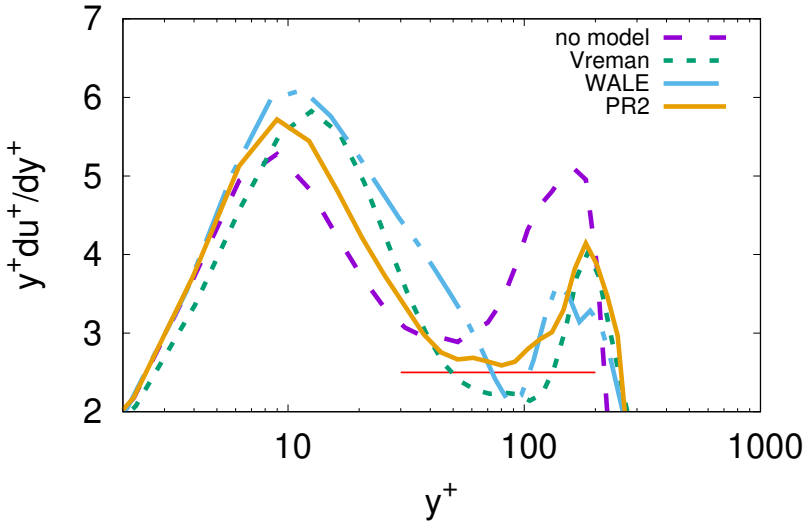
2. Averaged dissipation is equal to Smagorinsky model.

$C_{s3pq} = 0.572,\ C_{s3pr} = 0.709,\ C_{s3qr} = 0.762$

Boundary layer results. 32x64x32. $Re_{\delta*} = 1000$

Case:	SL	No mod.	Vr.	WALE	PR2
u_τ	**0.049**	0.049	0.050	0.046	0.049
H	**1.52**	1.61	1.51	1.54	1.53
κ	**0.39**	0.35	0.47	0.47	0.39





Wind farm results. 32x64x32. $Re_{\delta*} = 1000$. 24 turbines

MODEL	$z0_{Hi}/zH$	u_τ	u_*	u_τ/u_*	$P/\delta\Phi$	$W_t/\delta\Phi$	EB
no model	0.160	0.051	0.109	0.47	0.68	0.81	94%
Vreman	0.072	0.056	0.085	0.66	0.67	0.78	94%
WALE	0.082	0.050	0.089	0.56	0.79	0.90	94%
PR2	0.065	0.052	0.083	0.63	0.77	0.88	97%