A mimetic Finite Volume Method on Collocated Grids for incompressible flows: Application to thermal magnetohydrodynamics

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UNIVERSITAT POLITÈCNICA DE CATALUNYA

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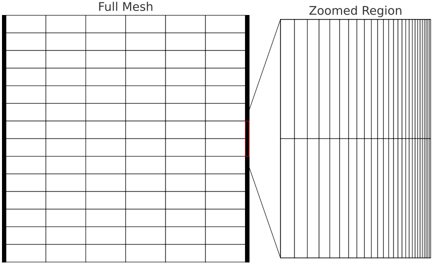

1. Definition of basic collocated operators

Let us suppose we have n control volumes and m faces.

Finite volume discretization of incompressible NS equations on an arbitrary collocated mesh

$$\Omega \frac{d\mathbf{u}_c}{dt} + C(\mathbf{u}_s)\mathbf{u}_c = -D\mathbf{u}_c - \Omega G_c \mathbf{p}_c, \quad (1)$$

$$M\mathbf{u}_s = \mathbf{0}_c. \quad (2)$$

- $\mathbf{p}_c = (p_1, ..., p_n)^T \in \mathbb{R}^n$ is the cell-centered pressure.
- $\mathbf{u}_c = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)^T \in \mathbb{R}^{3n}$, where $\mathbf{u}_i = ((u_i)_1, ..., (u_i)_n)^T$ are the vectors containing the velocity components corresponding to the x_i-spatial direction.
- $\mathbf{u}_s = ((u_s)_1, ..., (u_s)_m)^T \in \mathbb{R}^m$ is the staggered velocity.
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- $\boxed{\Omega_c \in \mathbb{R}^{n\times n}}$ is a diagonal matrix with the cell-centered volumes $\implies \quad \Omega = I_3 \otimes \Omega_c$.
- $\boxed{C_c(\mathbf{u}_s) \in \mathbb{R}^{n\times n}}$ is the cell-centered convective operator for a discrete scalar field $\implies \quad C(\mathbf{u}_s) = I_3 \otimes C_c(\mathbf{u}_s)$.
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Finally,

- $G_c \in \mathbb{R}^{3n\times n}$ represents the discrete collocated gradient.
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Mimicking Hilbert adjointness in L^2 inner product $\rightarrow$

$$\boxed{G = -\Omega_s^{-1} M^T,}$$

Mimicking continuous Laplacian $\rightarrow$

$$L = MG = -M\Omega_s^{-1} M^T,$$

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Metric-consistency of L^2 inner product $\rightarrow$

$$\Gamma_{s\rightarrow c} = \Omega^{-1}\Gamma_{c\rightarrow s}^T \Omega_s. \quad (3)$$

where G is the center-to-face staggered gradient, L is the Laplacian operator, L_c is the collocated-Laplacian operator and $\Gamma_{s\rightarrow c}$ is the face-to-cell interpolator.

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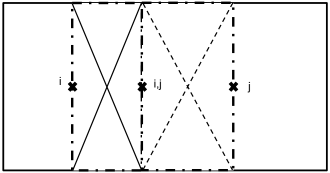
Volume-weighted interpolator

- Momentum is conserved when interpolated by the volume-weighted:

$$(\mathbf{u}_c, \mathbf{1}_c)_\Omega = (\mathbf{u}_s, \mathbf{1}_s)_{\Omega_s} \to \phi_f = \frac{\tilde{V}_{1,f}}{\tilde{V}_{1,f} + \tilde{V}_{2,f}} \phi_{c1} + \frac{\tilde{V}_{2,f}}{\tilde{V}_{1,f} + \tilde{V}_{2,f}} \phi_{c2}$$



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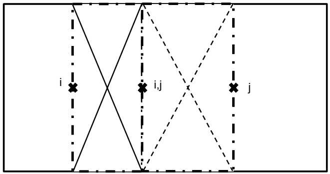

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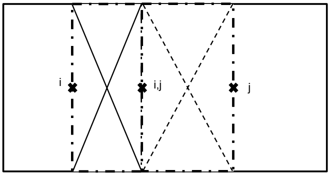

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- MidPoint interpolation is required for the convective term.

Conservation of global kinetic energy

Global kinetic energy equation

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Conservation of global kinetic energy

Global kinetic energy equation with skew-symmetric convective operator

$$\frac{d||\mathbf{u}_c||^2}{dt} = -\mathbf{u}_c^T(D + D^T)\mathbf{u}_c - \mathbf{u}_c^T \Omega G_c \mathbf{p}_c - \mathbf{p}_c^T G_c^T \Omega^T \mathbf{u}_c.$$

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In collocated framework, we either solve:

$$M\mathbf{u}_s = 0 \rightarrow Lp_c = M\Gamma_{c\rightarrow s}\mathbf{u}_c^p \rightarrow \text{Kinetic Energy Error} \tag{5}$$

$$M\Gamma_{c\rightarrow s}\mathbf{u}_c = 0 \rightarrow L_c p_c = M\Gamma_{c\rightarrow s}\mathbf{u}_c^p \rightarrow \text{Checkerboard} \tag{6}$$

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Global kinetic energy equation with skew-symmetric convective operator

$$\frac{d||\mathbf{u}_c||^2}{dt} = -\mathbf{u}_c^T(D + D^T)\mathbf{u}_c - \mathbf{u}_c^T\Omega G_c\mathbf{p}_c - \mathbf{p}_c^T G_c^T\Omega^T\mathbf{u}_c.$$

In collocated framework and explicit time integration, the (artificial) kinetic energy added is given by:

$$-\mathbf{p}_c^T G_c^T\Omega^T\mathbf{u}_c = \mathbf{p}_c^T(L - L_c)\mathbf{p}_c\Delta t \tag{7}$$

This term is strictly dissipative iff the volume-weighted interpolator is used.

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2. A mimetic FVM Thermal MHD solver

Equations for thermal MHD buoyant flow under low magnetic Re:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{u}) = -\frac{\nabla p}{\rho} + \nu \nabla^2 \boldsymbol{u} + \frac{\mathbf{j} \times \mathbf{B}_0}{\rho} + \mathbf{g}\beta(T - T_{ref}),$$

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\mathbf{j} = \sigma_m(-\nabla\phi + \boldsymbol{u} \times \mathbf{B}_0),$$

$$\nabla \cdot \mathbf{j} = 0,$$

$$\frac{\partial T}{\partial t} + \nabla \cdot (\boldsymbol{u} T) = \alpha \nabla^2 T,$$

Poisson equation for the electric potential:

$$\nabla^2 \phi = \nabla \cdot (\boldsymbol{u} \times \mathbf{B}_0).$$

Steps of the FSM (explicit first-order time-integration scheme):

$$\boldsymbol{u}_c^p = \boldsymbol{u}_c^n - \Delta t \Omega^{-1}[\mathrm{C}(\boldsymbol{u}_s) + \mathrm{D}]\boldsymbol{u}_c^n + \frac{\mathbf{j}_c^n \times \mathbf{B}_0}{\rho} + \mathbf{g}\beta(T_c^n - T_{ref}),$$

$$\boldsymbol{u}_s^p = \Gamma_{c \to s} \boldsymbol{u}_c^p,$$

$$\mathrm{L}\boldsymbol{p}_c^{n+1} = \mathrm{M}\boldsymbol{u}_s^p \rightarrow \boldsymbol{p}_c^{n+1},$$

$$\boldsymbol{u}_s^{n+1} = \boldsymbol{u}_s^p - \mathrm{G}\boldsymbol{p}_c^{n+1},$$

$$\boldsymbol{u}_c^{n+1} = \boldsymbol{u}_c^p - \Gamma_{s \to c}\mathrm{G}\boldsymbol{p}_c^{n+1},$$

$$\mathbf{j}_c^p = \boldsymbol{u}_c^n \times \mathbf{B}_0,$$

$$\mathbf{j}_s^p = \Gamma_{c \to s}\mathbf{j}_c^p,$$

$$\mathrm{L}\phi^{n+1} = \mathrm{M}\mathbf{j}_s^p, \rightarrow \phi_c^{n+1}$$

$$\mathbf{j}_c^{n+1} = \sigma_m \Gamma_{s \to c}(\mathbf{j}_s^p - \mathrm{G}\phi^{n+1}),$$

$$T_c^{n+1} = T_c^n + \Delta t(-\mathrm{C}(\boldsymbol{u}_s) + \alpha \mathrm{L}) T_c^n$$

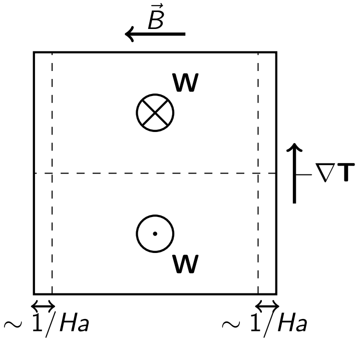

Figure 2: 2D enclosed buoyant cavity with conductive walls full of liquid metal with a strong horizontal magnetic field. Hadamard boundary layers are also depicted.

Analytical solution:

$$\frac{dW}{dX}_{\perp} = \frac{-Gr}{2Ha} \qquad \frac{dW}{dX}_{\parallel} = \frac{-Gr}{Ha^2} \tag{10}$$

2D MHD Cavity

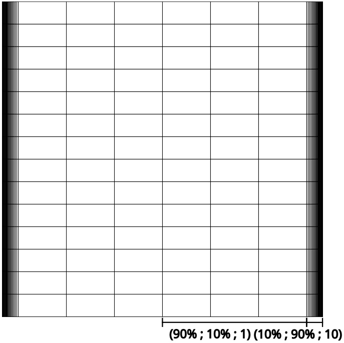

Figure 3: Test mesh (90-10;10-90;10). The aspect ratio between the bulk and the wall control volumes is 37.5.

Time integration: RK3

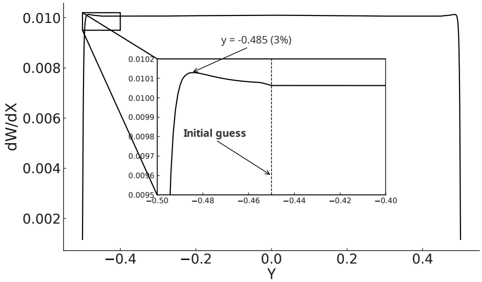

Figure 4: Plot of dW/dX with the adimensional position. The initial estimate for the boundary layer thickness was 0.05, while the simulation results indicate a refined value of approximately 0.015.

2D MHD Cavity

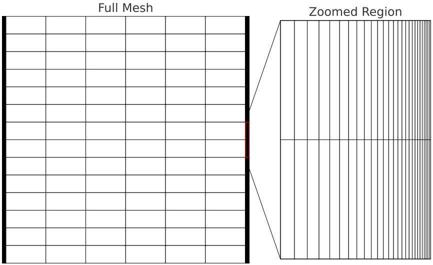

Figure 5: Test mesh (97-3;10-90;10). A zoomed region is shown close to the wall. The aspect ratio between the bulk and the wall control volumes is 108.

Ha = 500	**Mesh**	**Analytical: 10.00**
15x60 (A)	(90-10; 10-90; 10)	10.10
15x60 (B)	(96-4; 10-90; 10)	10.03
15x60 (C)	(97-3; 10-90; 10)	10.02
15x60 (D)	(97-3; 10-90; 4)	10.03
15x60 (E)	(97-3; 10-90; 2)	10.05
15x60 (F)	(97-3; 10-90; 1)	10.09
15x30 (G)	(97-3; 10-90; 10)	10.07
15x30 (H)	(97-3; 10-90; 20)	10.05

Table 1: Comparison of numerical results of dW/dX at the center of the plane for $Ha = 500$ for different meshes.

- Better results with 8 times smaller meshes than literature.
- 27 nodes located at the Hadamard BL vs 4 nodes in the literature.

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Ongoing work

- Currently working on DCLL breeding blanket conditions: 3D simulation, $Ha = 6500$, $Gr = 7 \times 10^9$.