An efficient Two-Layer wall model for accurate numerical simulations of aeronautical applications

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Turbulent wall-bounded flows

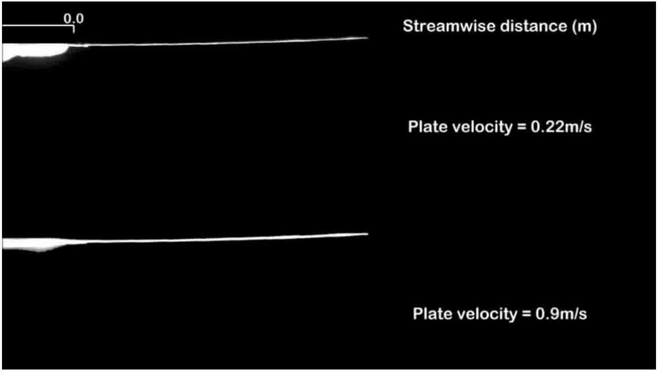


Figure 1: Lee *et al.*, University of Melbourne (2013)

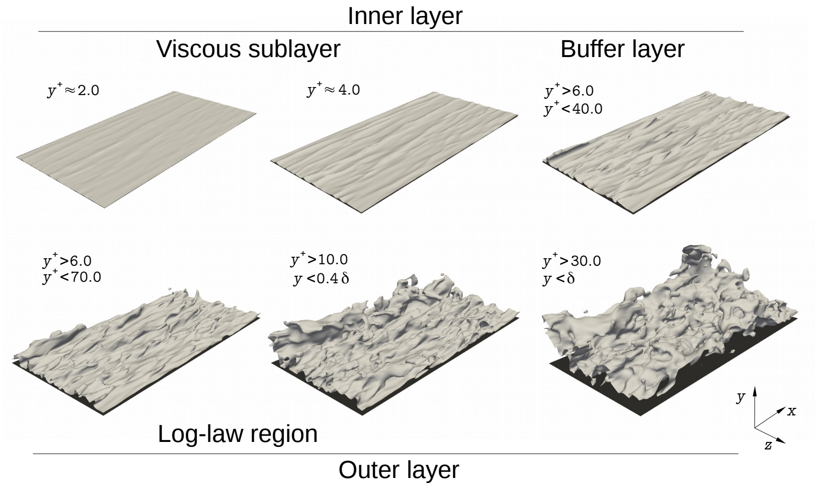
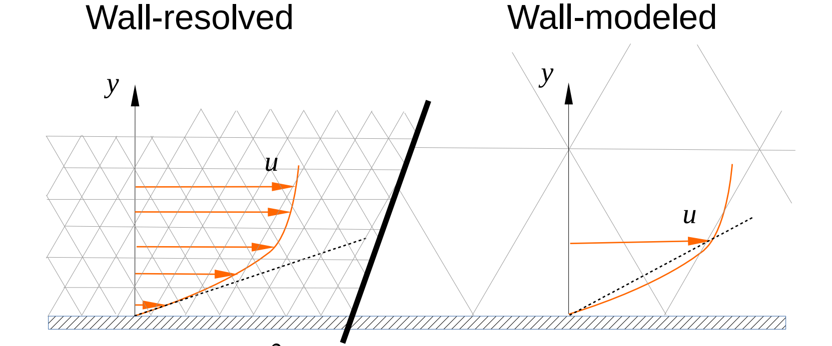


Figure 2: Iso-surfaces of the second invariant of the velocity gradient tensor, Q, at different wall distances of a turbulent Channel flow.

Wall shear stress modeling



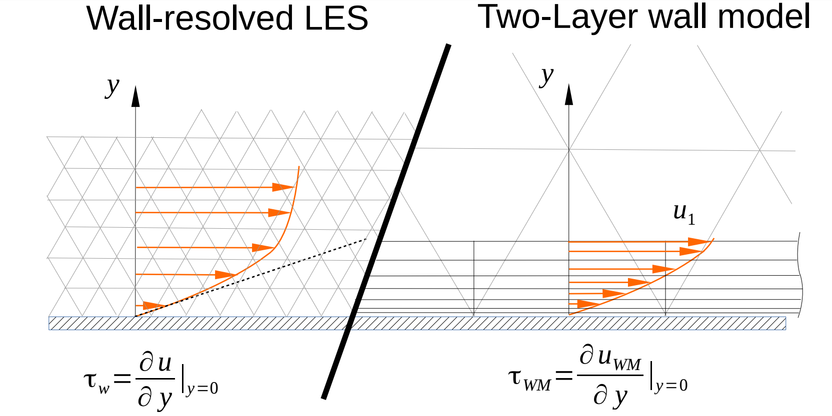
$$\tau_w = \frac{\partial u}{\partial y}\Big|_{y=0} \rightarrow \tau_{WR} > \tau_{WM}$$

Wall shear stress modeling. Computational savings

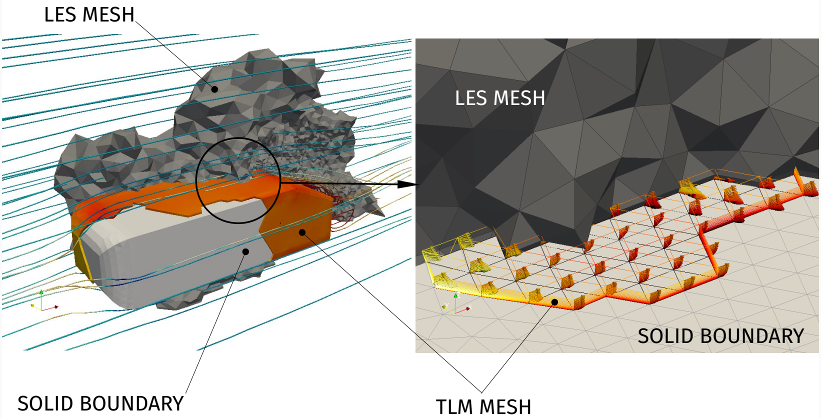
Model	**spatial**[1] considerations		**spatial+temporal**[2] considerations
Wall-Resolved	$T_{cc}^{WR} \sim Re_{L_x}^{\mathbf{1.85}}$	$\rightarrow$	$T_{cc}^{WR} \sim Re_{L_x}^{\mathbf{3.09}}$
Wall-Modeled	$T_{cc}^{WM} \sim Re_{L_x}^{\mathbf{1.0}}$	$\rightarrow$	$T_{cc}^{WM} \sim Re_{L_x}^{\mathbf{1.33}}$

[1] H. Choi and P. Moin. "Grid-point requirements for large eddy simulation: Chapman's estimates revisited." In: *Phys. Fluids* 24 (2012), p. 011702.

[2] J. Calafell et al. "A time-average filtering technique to imporve the efficiency of two-layer wall models for large eddy simulation in complex geometries." In: *Comput. Fluids* 188 (2019), pp. 44–59.


Wall-resolved LES
Two-Layer wall model
y
y
u_1
$\tau_w = \frac{\partial u}{\partial y}|_{y=0}$
$\tau_{WM} = \frac{\partial u_{WM}}{\partial y}|_{y=0}$

Two-Layer wall models



Efficient Two-Layer Wall model: Governing equations

- **Momentum Equations: RANS**

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{U} = \nabla \cdot [2(\nu + \nu_{T,wm})S(\mathbf{U})] - \nabla P$$

$$\nabla \cdot \mathbf{U} = 0$$

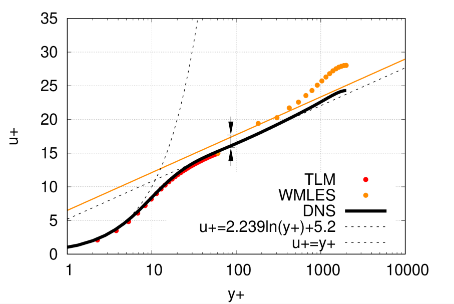
- **RANS Model: Algebraic Mixing-length-based**[3,4,5]

$$\nu_{T,wm} = (\kappa y)^2 |S| \left[1 - exp\left(-y^+/A^+\right)\right]^2$$

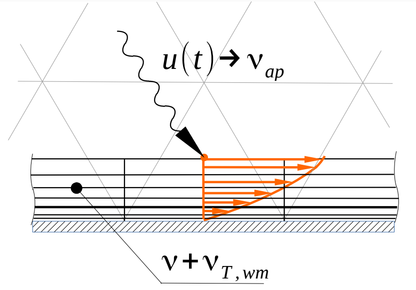
[3] E. Balaras, C. Benocci, and U. Piomelli. *AIAA J.* 34 (6) (1996), pp. 1111–1119.
[4] S. Kawai and J. Larsson. *Phys. Fluids* 24 (2012), p. 015105.
[5] G. I. Park and P. Moin. *Phys. Fluids* 26 (2014), p. 015108.

Two-Layer Wall model errors

- **Log-layer mismatch**



- **Resolved Reynolds stresses inflow**

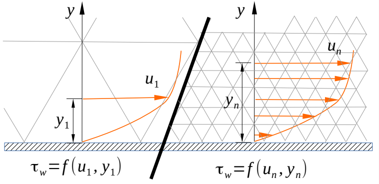


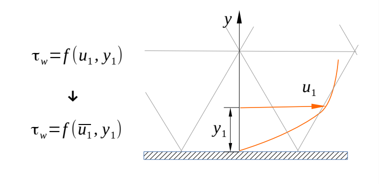
Log-layer mismatch

Error sources:

- Numerical and subgrid errors at near-wall nodes
- Unphysical coupling between τ_w and u_1

Proposed solutions:

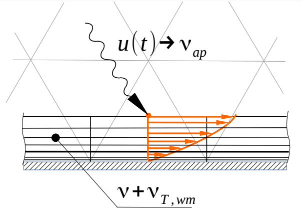




proposed solutions references[6],[7]

[6] S. Kawai and J. Larsson. *Phys. Fluids* 24 (2012), p. 015105.
[7] X. I. A. Yang, G. I. Park, and P. Moin. *Phys. Rev. Fluids* 2 (2017), p. 104601.

$$\nu + \nu_{T,wm} \rightarrow \nu + \nu_{T,wm} + \nu_{ap}$$



Proposed solutions:

- Reducing the modeled contribution[8]:

$$\nu_{T,wm} = (\kappa y)^2 \, |S| \left[1 - exp\left(-y^+ / A^+\right)\right]^2$$

- Subtracting the apparent diffusion[9]:

$$-\nu_{ap} = \frac{R(\mathbf{U}) S^d(\mathbf{U})}{2 S^d(\mathbf{U}) S^d(\mathbf{U})}$$

[8] S. Kawai and J. Larsson. *Phys. Fluids* 24 (2012), p. 015105.
[9] G. I. Park and P. Moin. *Phys. Fluids* 26 (2014), p. 015108.

Time-filtering approach

Applied for the first time to a TLM model

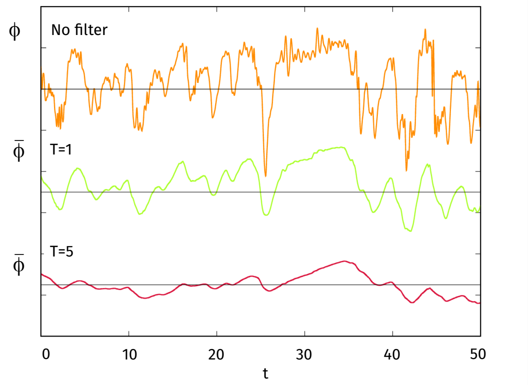
- **Filter behavior:**

$$\frac{\partial\overline{\phi}}{\partial t} = \frac{\phi - \overline{\phi}}{T}$$

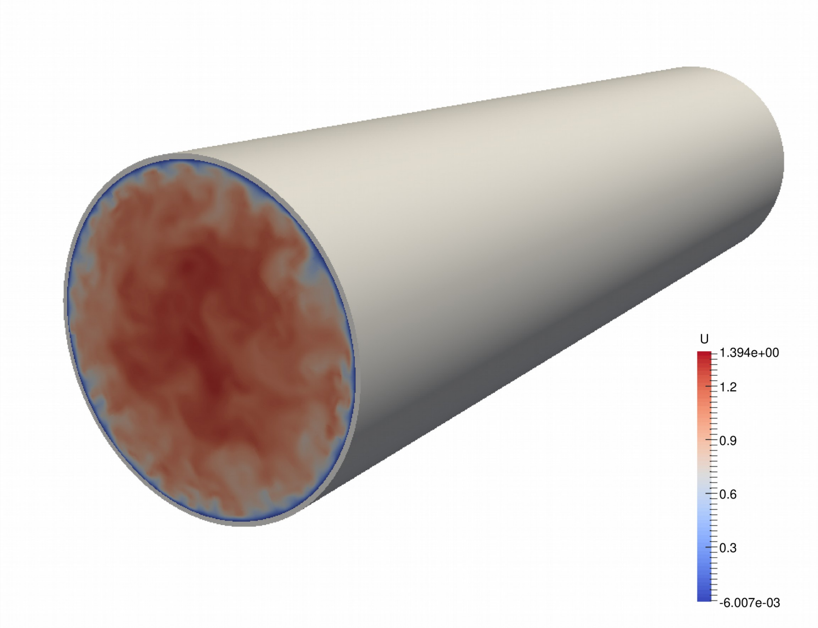
- **Discrete solution:**

$$\overline{\phi}^n = (1-\epsilon)\overline{\phi}^{n-1} + \epsilon\phi^n$$

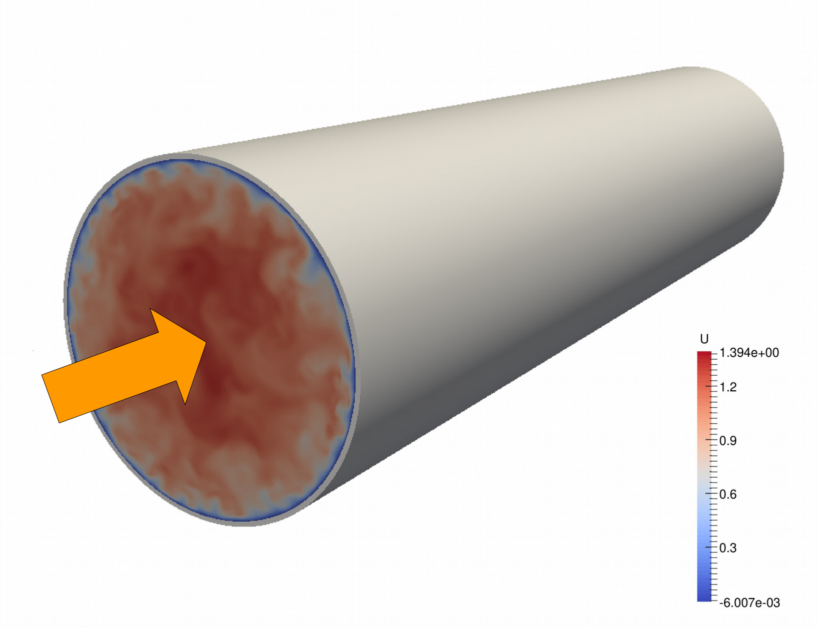
$$\epsilon = \frac{\Delta t/T}{1+\Delta t/T}$$



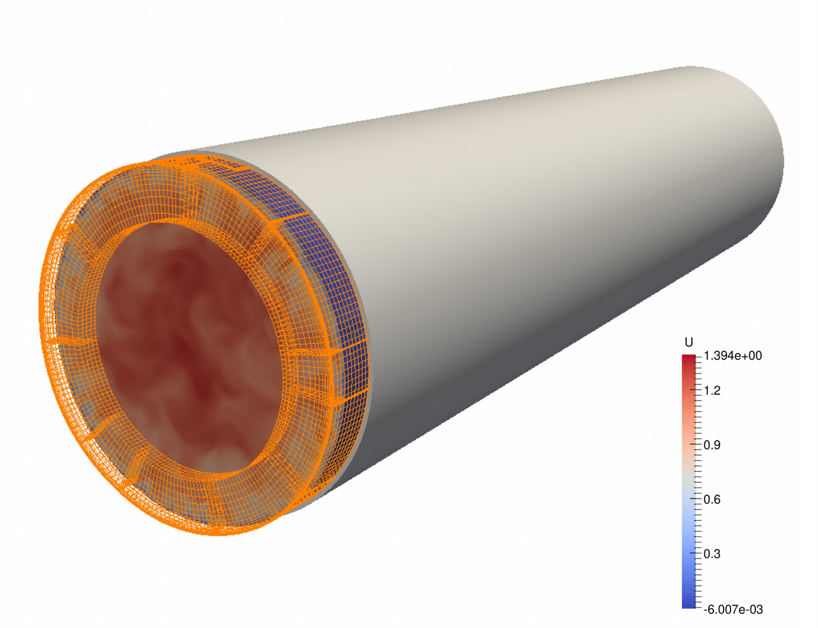
Time filter performance test: Pipe flow $Re_\tau \approx 500$



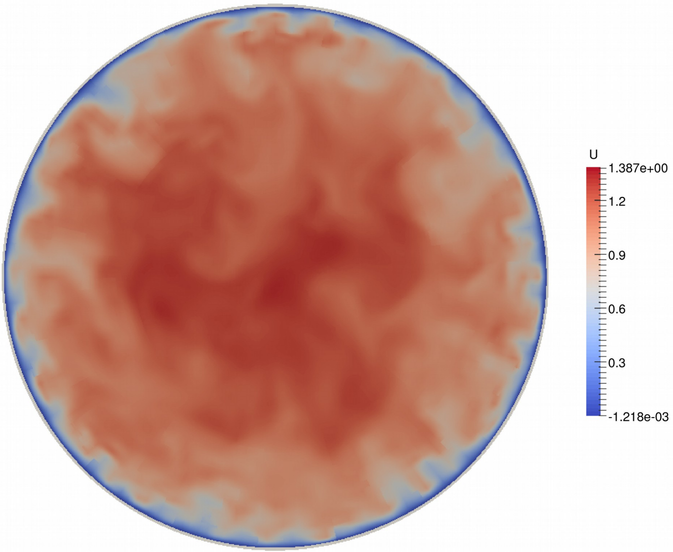
Time filter performance test: Pipe flow $Re_\tau \approx 500$



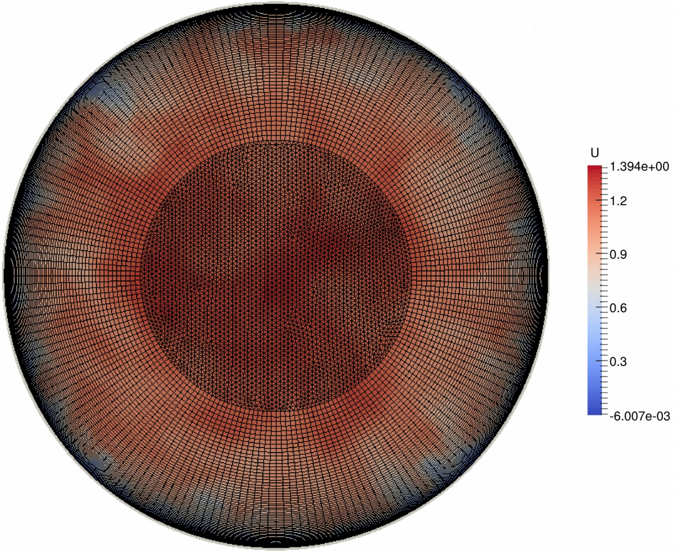
Time filter performance test: Pipe flow $Re_\tau \approx 500$



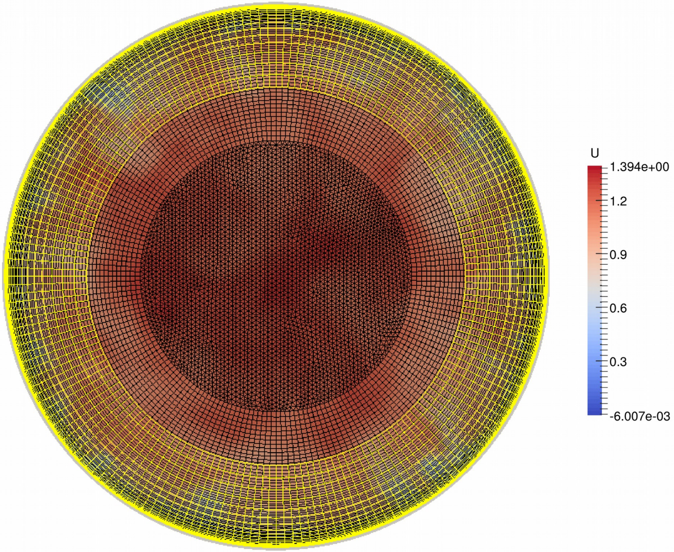
Time filter performance test: Pipe flow $Re_\tau \approx 500$



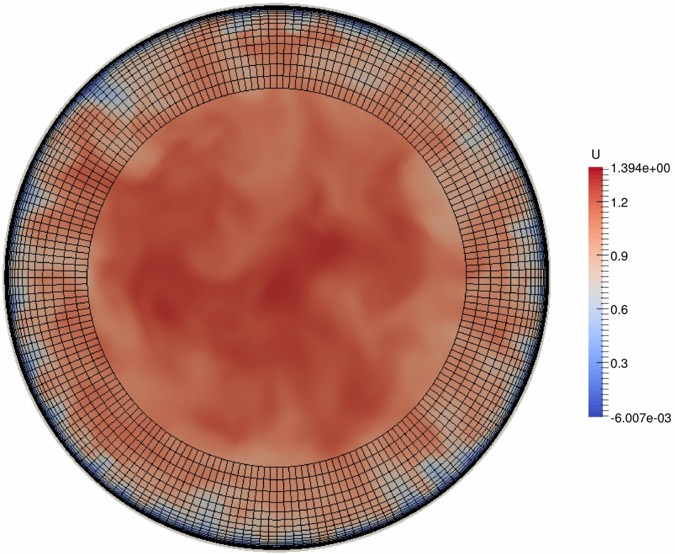
Time filter performance test: Pipe flow $Re_\tau \approx 500$



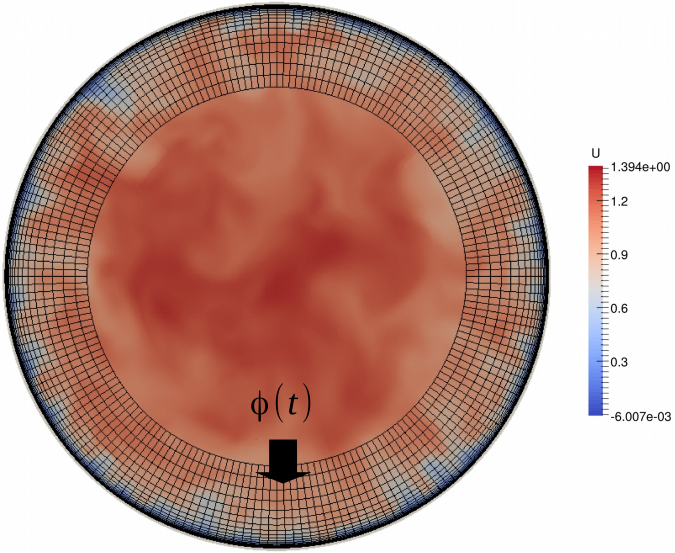
Time filter performance test: Pipe flow $Re_\tau \approx 500$



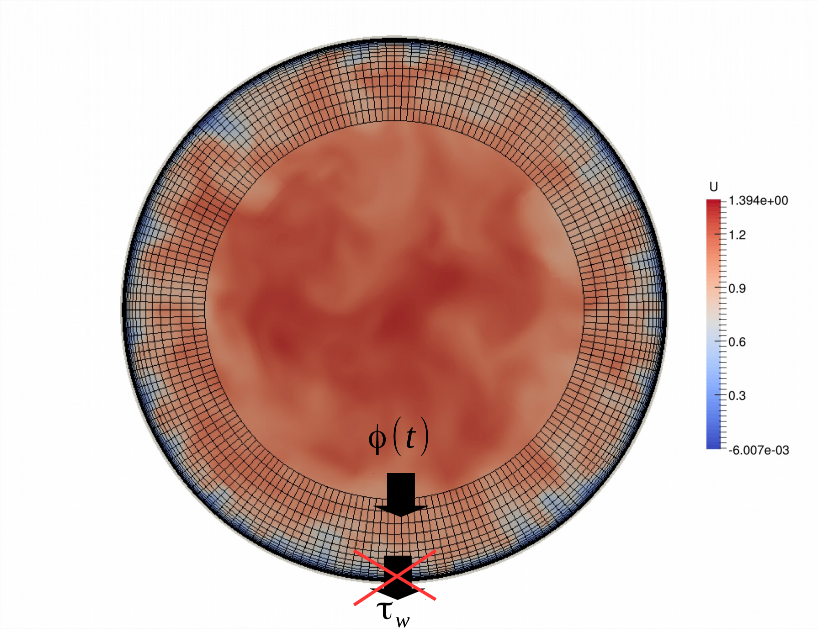
Time filter performance test: Pipe flow $Re_\tau \approx 500$



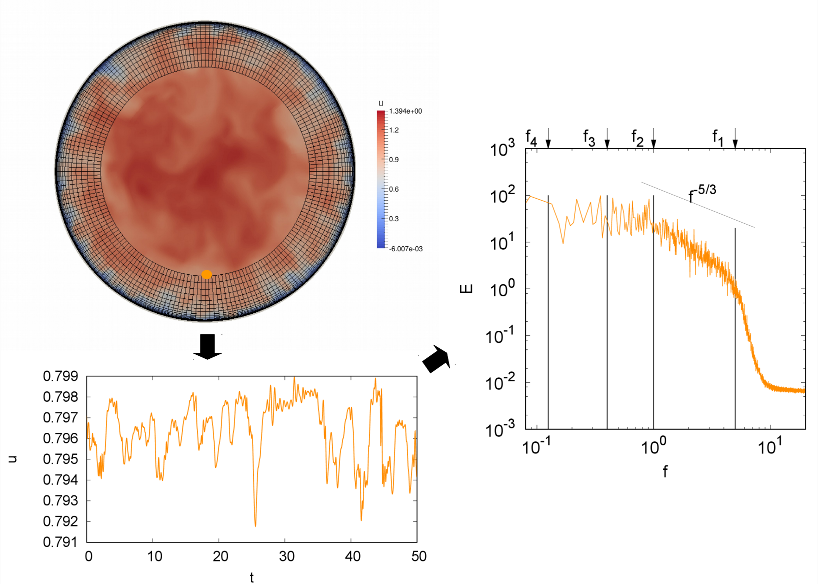
Time filter performance test: Pipe flow $Re_\tau \approx 500$



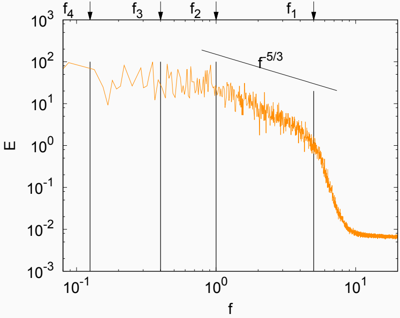
Time filter performance test: Pipe flow $Re_\tau \approx 500$



LES characteristic frequencies: Power spectrum

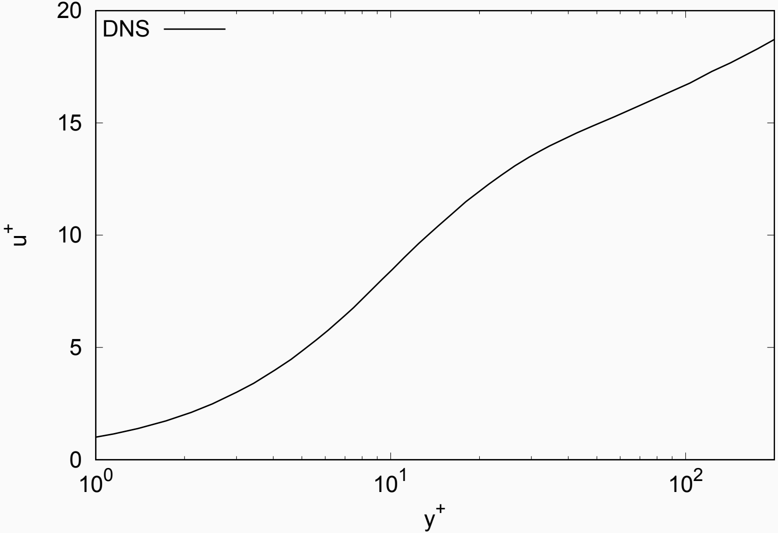


Filtering period setup

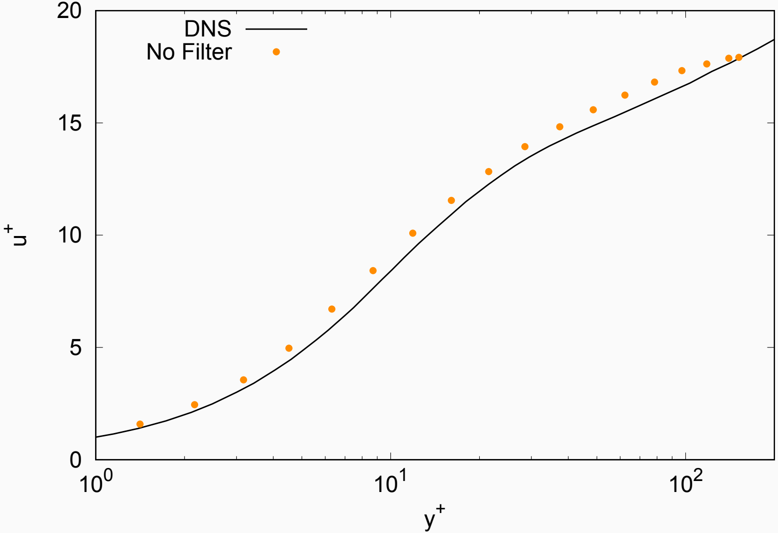


TAF Config.(n)	f_n	Filter length $T_n = 1/f_n$	Energy spectrum range
0	no filter	no filter	N/A
1	5.0	0.2	inertial/dissipation range limit
2	1.0	1.0	inertial/energy-containing range limit
3	0.4	2.5	within the energy-containing range
4	0.125	8.0	flow-through period, largest flow scales

Time filter performance test results



Time filter performance test results



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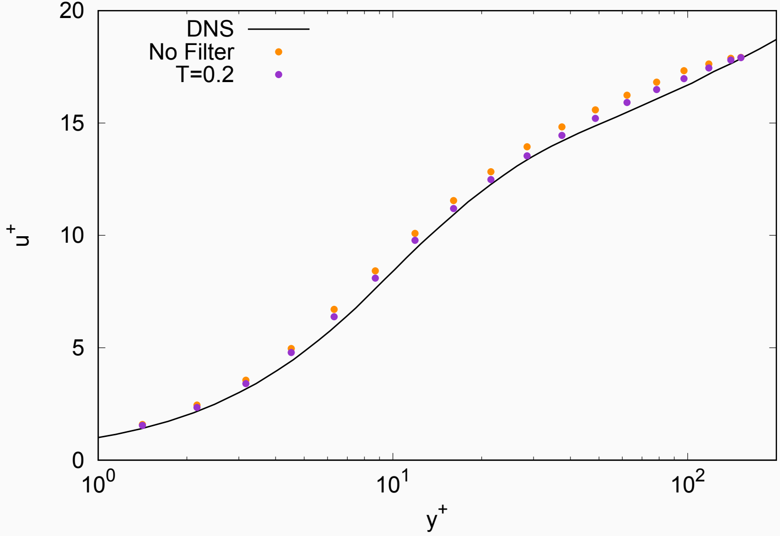
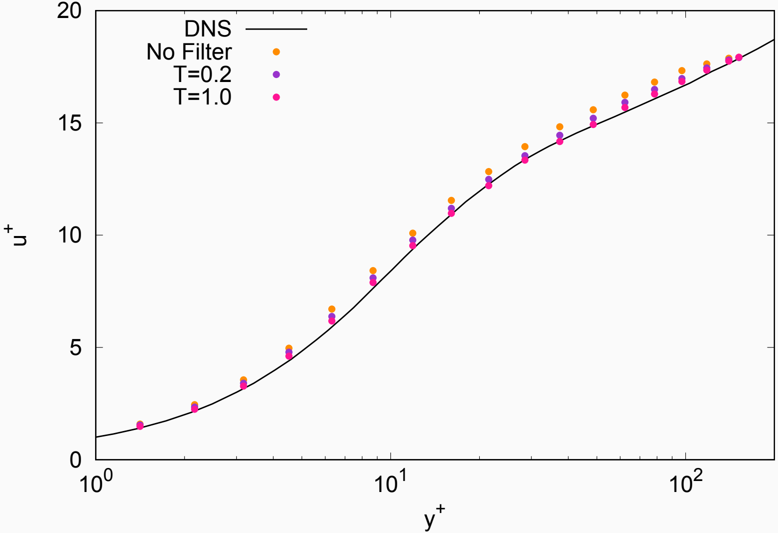
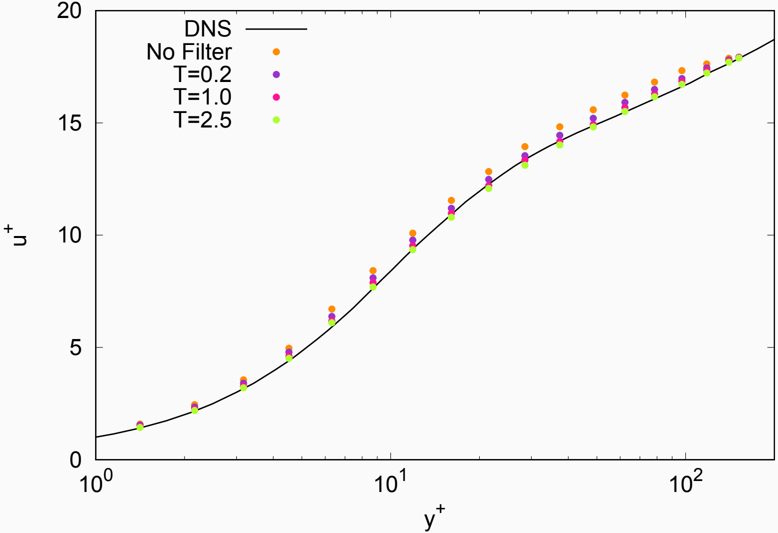


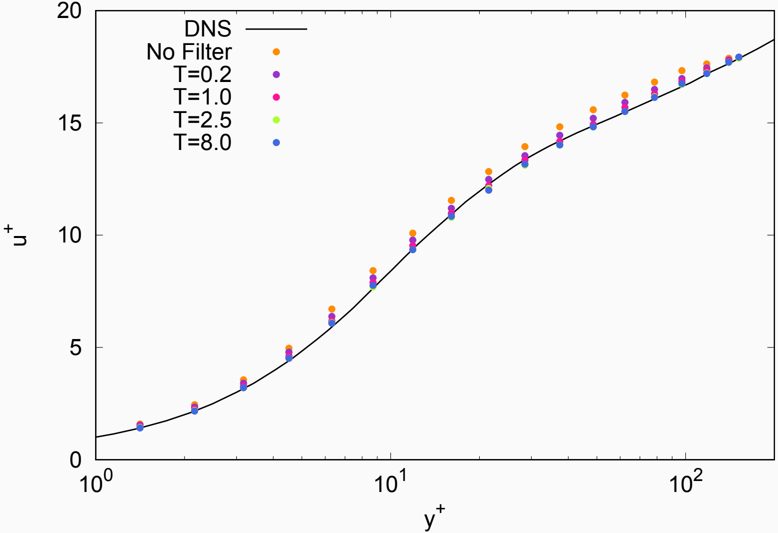
Time filter performance test results



Time filter performance test results



Time filter performance test results

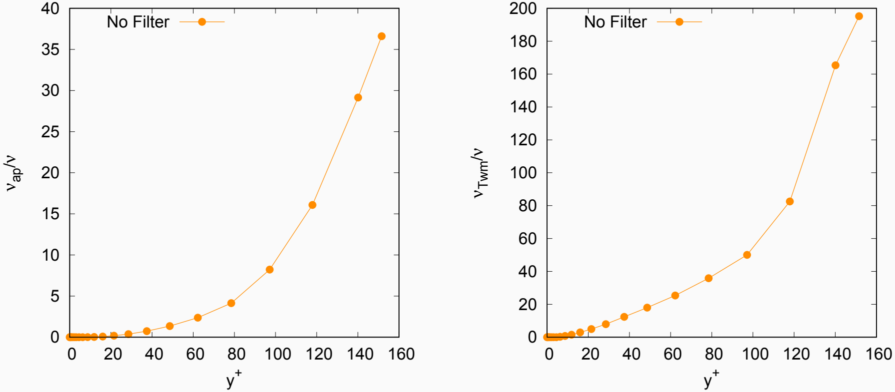


Time filter performance test results

- **Wall model output assessment**

Filter length T_n	Computed Re_τ	rel. err. [%]	Energy spectrum range
no filter	528.70	5.74	N/A
0.2	515.66	3.13	inertial/dissipation range limit
1.0	506.81	1.36	inertial/energy-containing range limit
2.5	502.06	0.41	within the energy-containing range
8.0	502.18	0.43	flow-through period

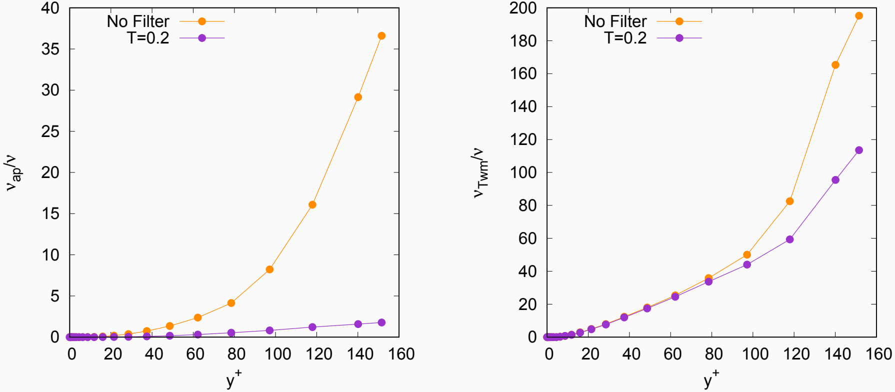
Time filter performance test: Diffusivity analysis



- No Filter.
- Output shear stress relative error: 5.74%

$$\nu + \nu_{Twm} + \nu_{ap}$$

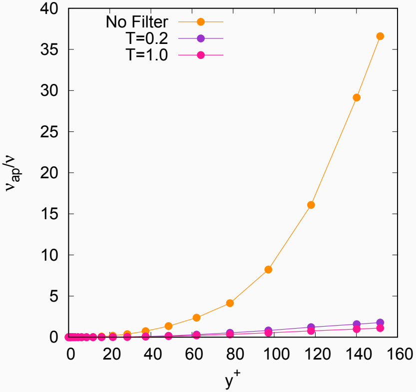
Time filter performance test: Diffusivity analysis

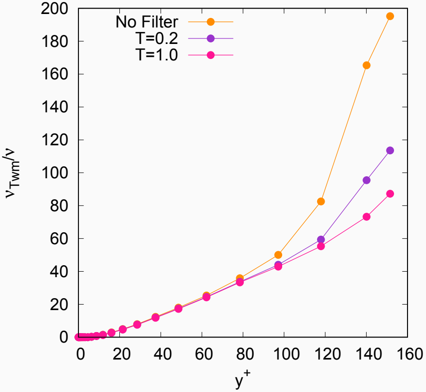


- Filter cut-off frequency: Intertial/dissipation range limit.
- Output shear stress relative error: 3.13%

$$\nu + \nu_{Twm} + \cancel{\nu_{ap}}$$

Time filter performance test: Diffusivity analysis

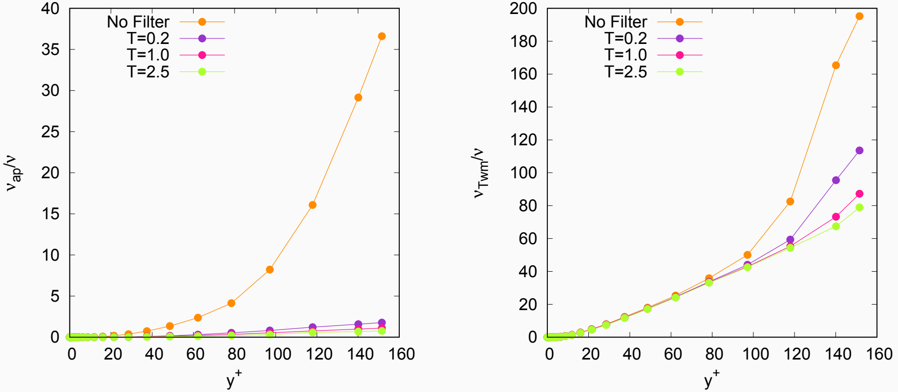




- Filter cut-off frequency: Energy-containing/Inertial range limit.
- Output shear stress relative error: 1.36%

$$\nu + \nu_{Twm} + \cancel{\nu_{ap}}$$

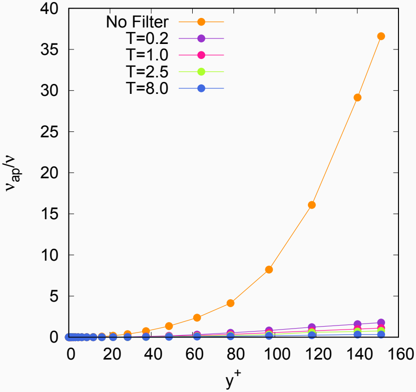
Time filter performance test: Diffusivity analysis

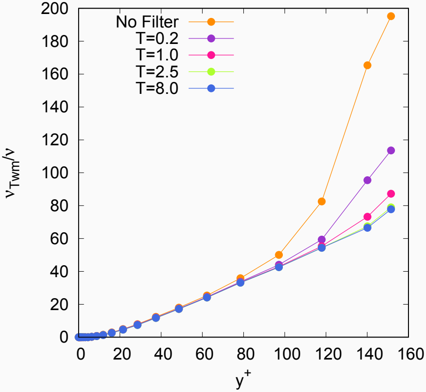


- Filter cut-off frequency: Within Energy-containing range.
- Output shear stress relative error: 0.41%

$$\nu + \nu_{Twm} + \cancel{\nu_{ap}}$$

Time filter performance test: Diffusivity analysis

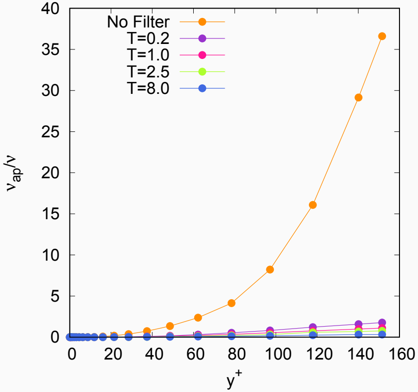


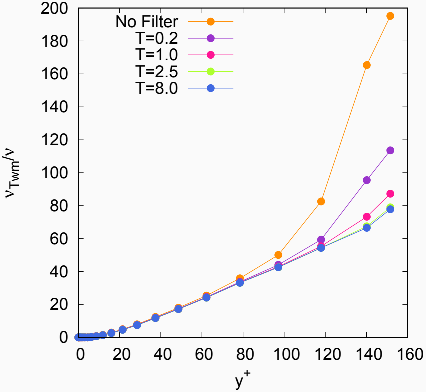


- Filter cut-off frequency: Laregest flow scale frequency.
- Output shear stress relative error: 0.43%

$$\nu + \nu_{Twm} + \cancel{\nu_{ap}}$$

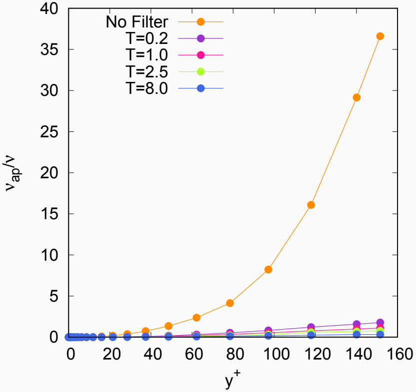
Time filter performance test: Diffusivity analysis

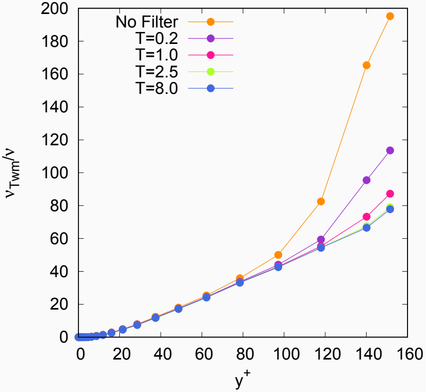




$$\nu_{T,wm} = (\kappa y)^2 \left|S\right| \left[1 - exp\left(-y^+/A^+\right)\right]^2$$

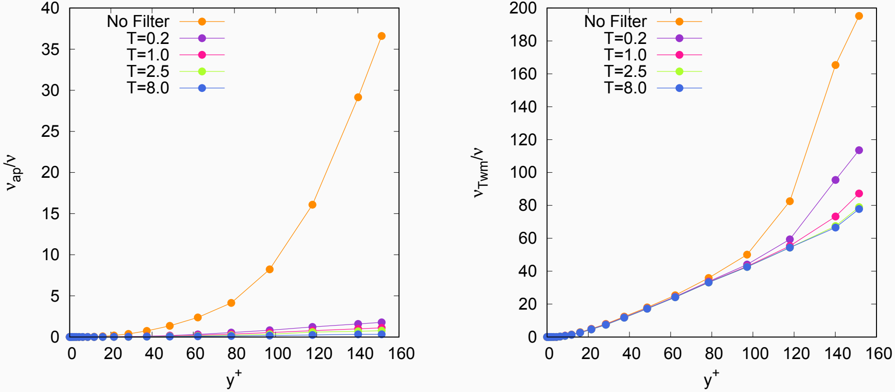
Time filter performance test: Diffusivity analysis





$$\nu_{T,wm} = (\kappa y)^2 \, |S| \left[1 - exp\left(-y^+/A^+\right)\right]^2$$

Time filter performance test: Diffusivity analysis



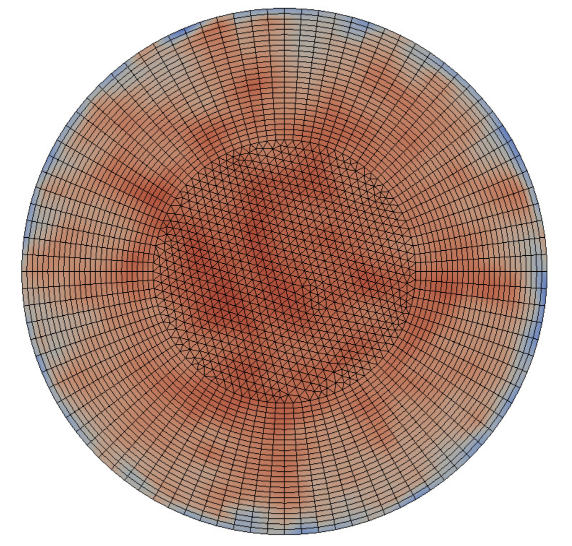
$$\nu_{T,wm} = (\kappa y)^2 \, |S| \left[1 - exp\left(-y^+/A^+\right)\right]^2$$

$$|S| = \sqrt{2S_{ij}S_{ij}} \rightarrow S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Equilibrium conditions: Pipe flow at $Re_\tau \approx 3000$

LES domain parameters:

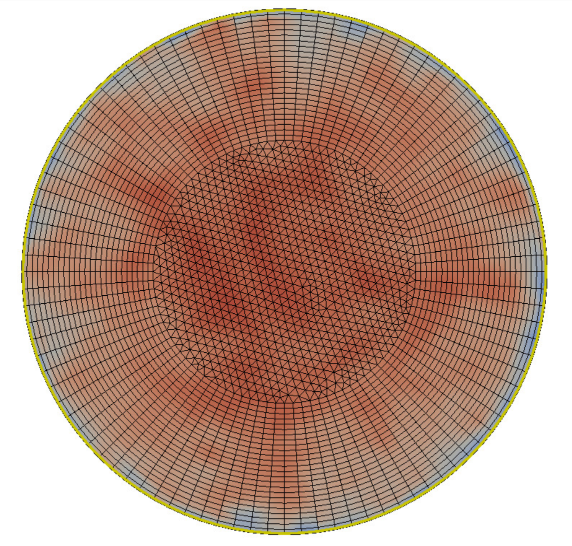
- Reynolds Number:
  - $Re_\tau \approx 3000$
- SGS model:
  - Dyn. Smagorinsky
- Domain size:
  - $R = 1$
  - $L_z = 10$
- Grid resolution:
  - $\Delta r_w^+ \approx 60$
  - $\Delta r\theta_w^+ \approx 198$
  - $\Delta_z^+ \approx 236$
- First off-wall node:
  - $\Delta y_{1w}^+ \approx 30$



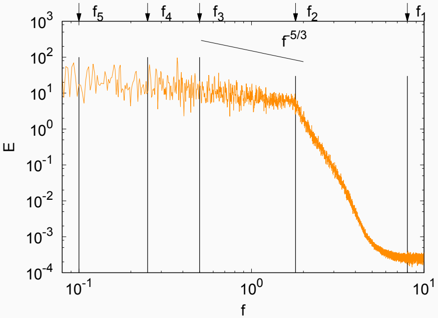
Equilibrium conditions: Pipe flow at $Re_\tau \approx 3000$

WMLES parameters:

- Reynolds Number:
  - $Re_\tau \approx 3000$
- WM extrusion height:
  - $h^+_{wm} \approx 30$
    First off-wall node

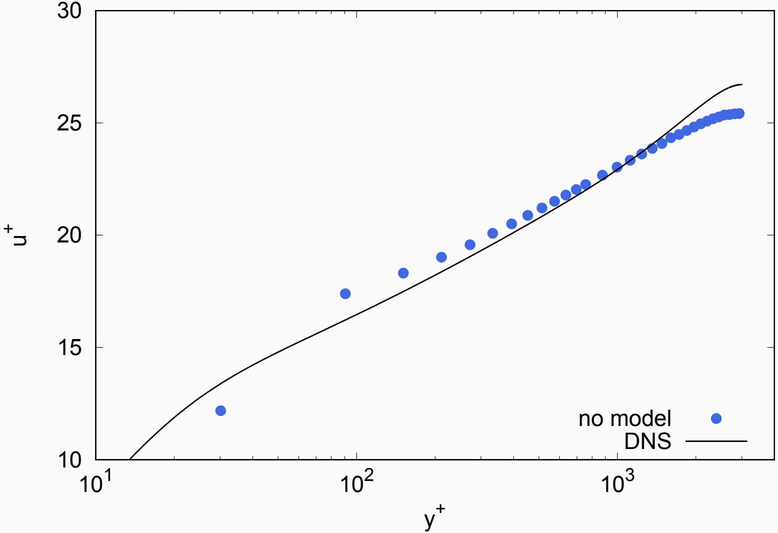


Pipe flow at $Re_\tau \approx 3000$: Temporal filter setup



TAF Config.(n)	$f_n = 1/T_n$	Filter length T_n	Energy spectrum range
0	no wall model	no wall model	N/A
1	no filter	no filter	N/A
2	1.8	0.55	inertial/dissipation range limit
3	0.5	2.0	inertial/energy-containing range limit
4	0.25	4.0	within the energy-containing range
5	0.1	10.0	flow-through period, largest flow scales

Pipe flow at $Re_\tau \approx 3000$ results: Filter width effects



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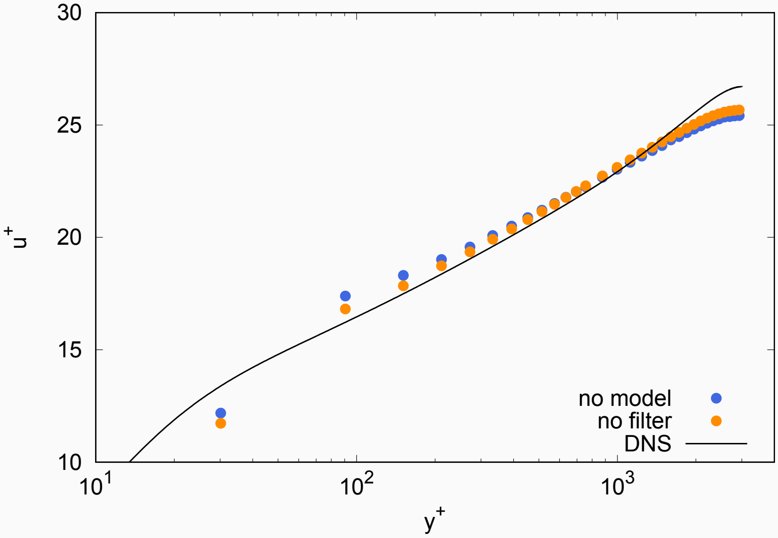

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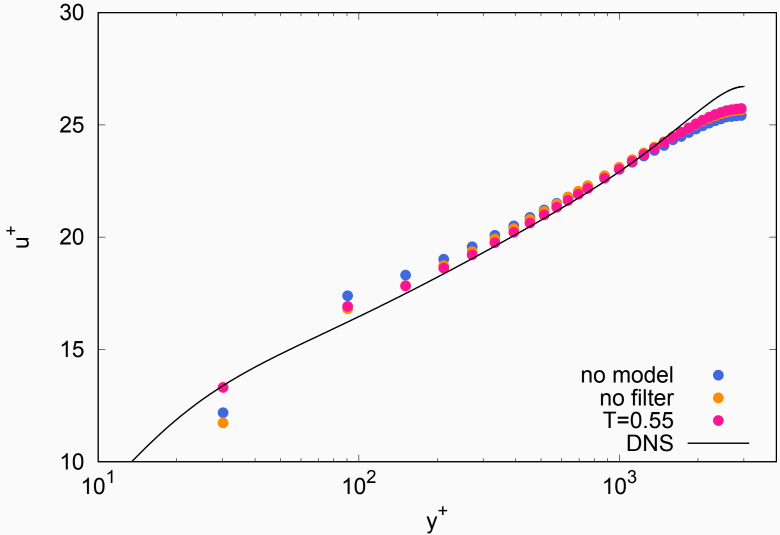
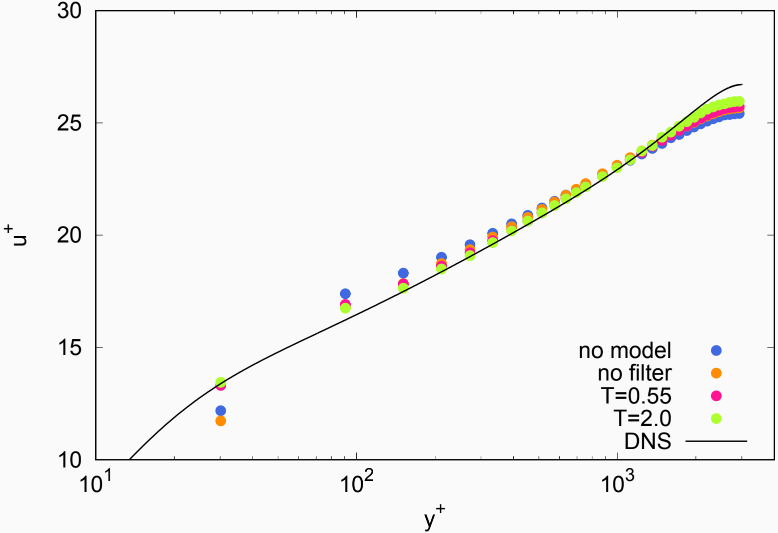


Pipe flow at $Re_\tau \approx 3000$ results: Filter width effects



Pipe flow at $Re_\tau \approx 3000$ results: Filter width effects

- **Wall shear stress**

Test (n)	Filter length T_n	Computed Re_τ	rel. err. [%]
0	no wall model	1923.6	36.40
1	no filter	3409.2	12.66
2	0.55	3201.1	5.78
3	2.0	3141.3	3.81
4	4.0	3138.0	3.70
5	10.0	3135.6	3.62

Conclusions

- A new Two-Layer wall model has been proposed.
- A time-averaging filter (TAF) is applied for the first time in the LES/WM interface.
- The TAF suppresses the log-layer mismatch and the Reynolds stresses inflow problems at once.
- The Reynolds stresses inflow not only causes an apparent diffusivity excess but also makes the RANS model to work out of range.
- A methodology based on the velocity power spectrum is proposed to determine an appropriate filter size.
- The frequencies higher than the Energy-containing/intertial range limit must be supressed.

Thank you for your attention!