Assessment and comparison of a recent kinematic sensitive subgrid length scale in Hybrid RANS-LES

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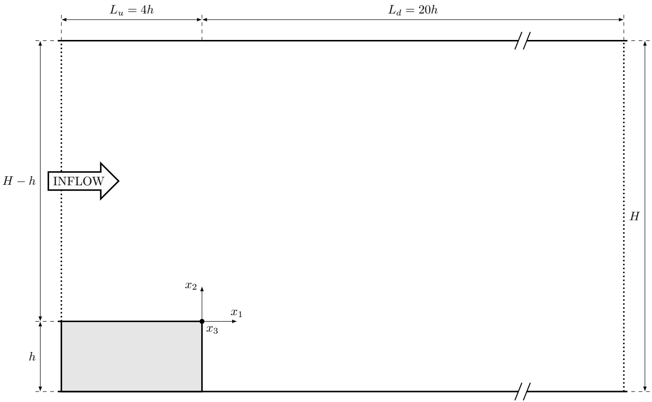
BFS-VE (1985): Case

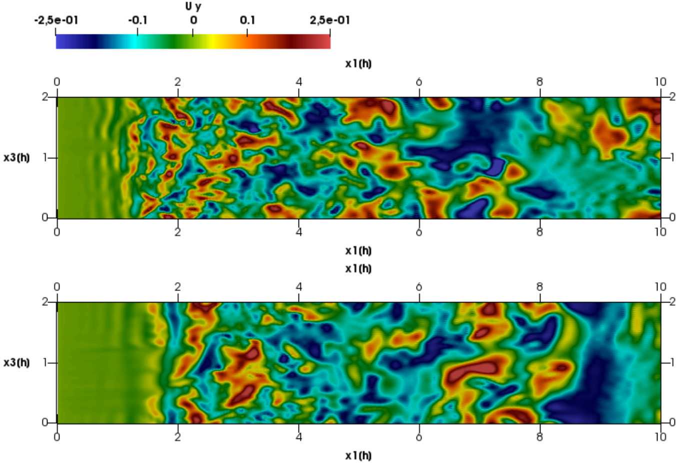
Experimental study: Vogel and Eaton (1985)



BFS-VE (1985): $u_2' = u_2 - \langle u_2 \rangle$

Δ_{lsq} :

$\tilde{\Delta}_{\omega}$:



Subgrid characteristic length for DES: state of the art (II)

- In the context of **LES**, most popular (by far) is:

$$\boxed{\Delta_{\forall} = (\Delta x \Delta y \Delta z)^{1/3}} \Longleftarrow \textbf{Deardorff (1970)}$$

- In the context of **DES**:

$$\boxed{\Delta_{\max} = \max(\Delta x, \Delta y, \Delta z)} \Longleftarrow \textbf{Sparlart et al. (1997)}$$

Recent flow-dependant definitions

$$\Delta_{\omega} = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\omega|^2} \Longleftarrow \textbf{Chauvet et al. (2007)}$$

$$\tilde{\Delta}_{\omega} = \frac{1}{\sqrt{3}} \max_{n,m=1,\dots,8} |l_n - l_m| \Longleftarrow \textbf{Mockett et al. (2015)}$$

$$\Delta_{\mathrm{SLA}} = \tilde{\Delta}_{\omega} F_{\mathrm{KH}}(\langle VTM \rangle) \Longleftarrow \textbf{Shur et al. (2015)}$$

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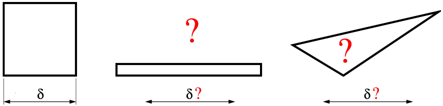
A new subgrid characteristic length for LES/DES

Research question:

- Can we find a **simple and robust** definition of Δ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?



A new subgrid characteristic length for LES/DES

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$$\underbrace{G \equiv \nabla \overline{u}}_{\text{physical space}} \qquad \underbrace{G_\Delta \equiv G\Delta}_{\text{computational space}}$$

$$\underbrace{\tau(\overline{u}) = \frac{\Delta^2}{12} G G^T + \mathcal{O}(\Delta^4)}_{\text{physical space}} \qquad \underbrace{\tau(\overline{u}) = \frac{1}{12} G_\Delta G_\Delta^T + \mathcal{O}(\Delta^4)}_{\text{computational space}}$$

A new subgrid characteristic length for LES/DES

Research question:

- Can we find a **simple and robust** definition of Δ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?

$$\Delta_{\mathrm{lsq}} = \sqrt{\frac{G_\Delta G_\Delta^T : GG^T}{GG^T : GG^T}}$$

A new subgrid characteristic length for LES/DES

Research question:

- Can we find a **simple and robust** definition of Δ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?

$$\Delta_{lsq} = \sqrt{\frac{J G^T G : J G^T G}{G^T G : G^T G}}, J = \begin{pmatrix} \Delta x_1 & & \\ & \Delta x_2 & \\ & & \Delta x_3 \end{pmatrix}$$

Subgrid Length Scale candidates

Trias et al. , PoF 26, (2017)

$$\Delta_{lsq} = \sqrt{\frac{JG^T G : JG^T G}{G^T G : G^T G}}$$

Mockett et al. , HRLM5, (2015)

$$\tilde{\Delta}_\omega = \frac{1}{\sqrt{3}} \max_{n,m=1,\dots,8} |l_n - l_m|$$

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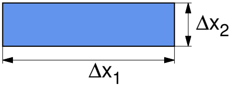
2D Simple Flow: Mesh

Delta (Δ):

$$\Delta = \begin{pmatrix} \Delta x_1 = \beta & \\ & \Delta x_2 = \beta^{-1} \end{pmatrix}$$

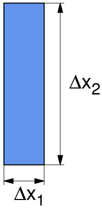
$$\boxed{\forall = \Delta x_1 \Delta x_2 = \quad \beta\beta^{-1} = 1}$$

Possible Cells:



$$\begin{pmatrix} 2 & 0 \\ 0 & 0.5 \end{pmatrix}$$

β=2



$$\begin{pmatrix} 0.5 & 0 \\ 0 & 2 \end{pmatrix}$$

β=2^{-1}=0.5

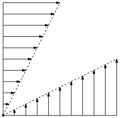
2D Simple Flow: Kinematics

Flow Dynamics:

$$G = \begin{pmatrix} 0 & 1 \\ 1-2\omega & 0 \end{pmatrix}$$

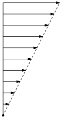
$$\omega \in [0,1]$$

Flow Cases:



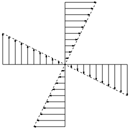
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Pure Shear ($\omega = 0$)



$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Simple Shear (ω=0.5)

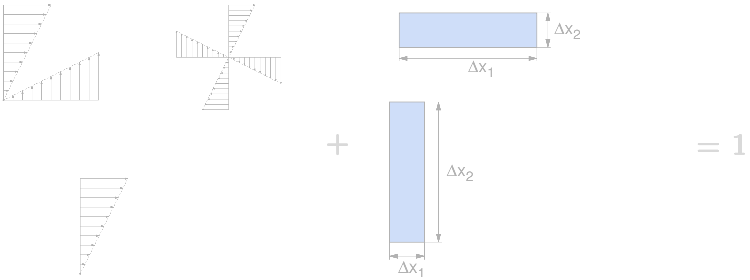


$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Pure Rotation (ω=1)

2D Simple Flow: Delta, $\Delta_\forall$

$$\Delta_\forall = \sqrt{\Delta x_1 \Delta x_2} = \sqrt{\beta \beta^{-1}} = 1$$



Insensitive to the **kinematics** and the **mesh aspect ratio** (β).

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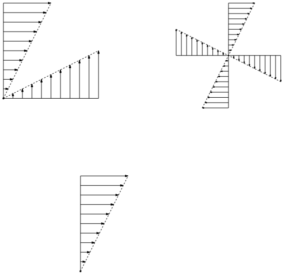

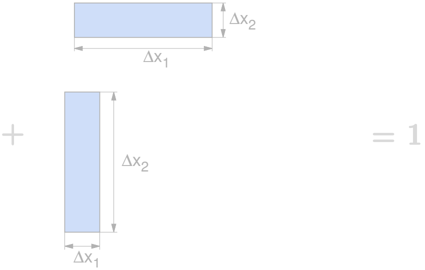

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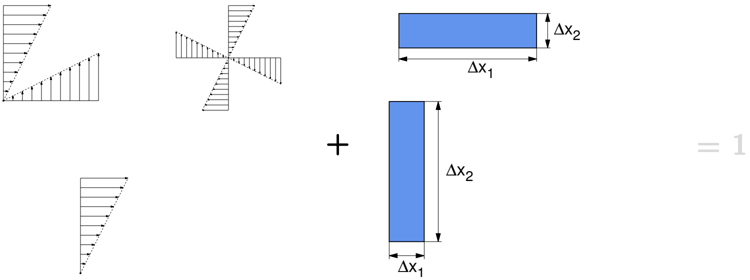

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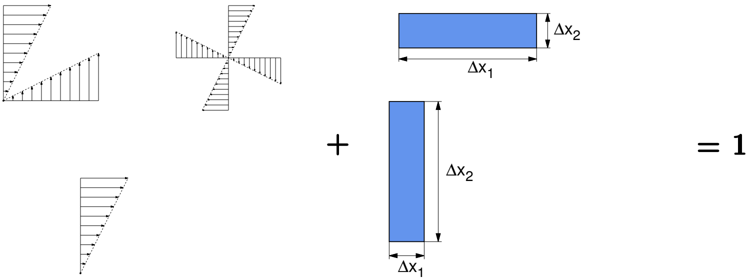

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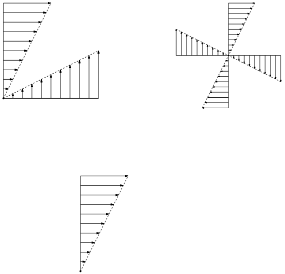
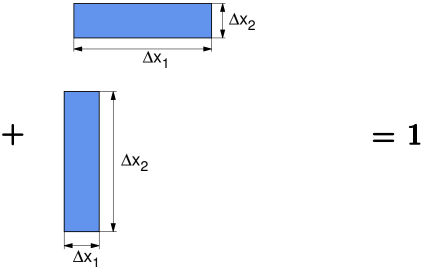
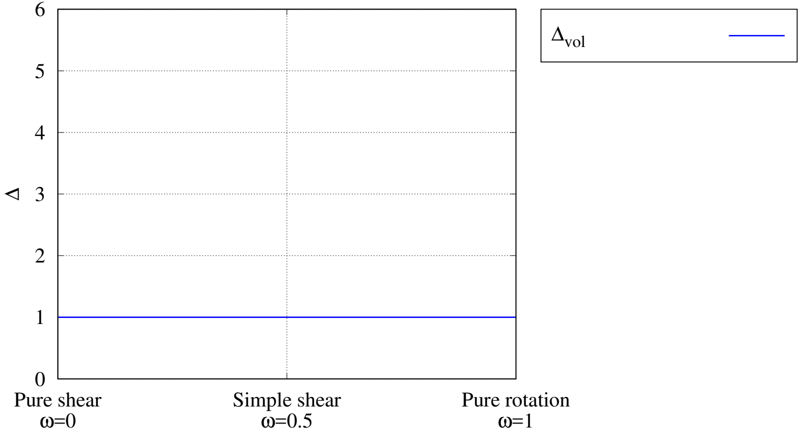




Insensitive to the **kinematics** and the **mesh aspect ratio** (β).

2D Simple Flow: Graph, $\Delta_\forall$

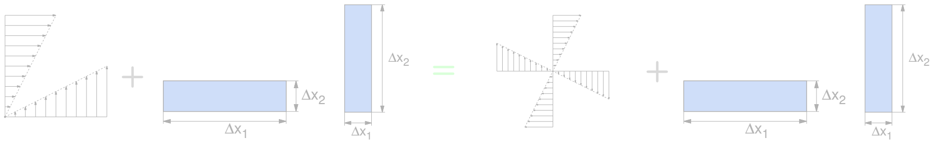


2D Simple Flow: Delta, $\tilde{\Delta}_\omega$ (I)

$$\tilde{\Delta}_\omega(\beta) = \sqrt{\left(\Delta x_1^2 + \Delta x_2^2\right)/3} = \sqrt{\left(\beta^2 + \beta^{-2}\right)/3}$$

(Skew-)symmetric Kinematics:

$$\tilde{\Delta}_\omega(\beta = \beta) = \tilde{\Delta}_\omega\left(\beta = \beta^{-1}\right)$$



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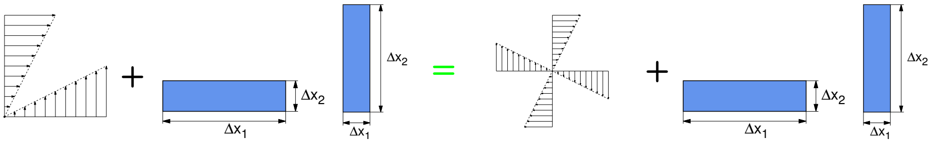
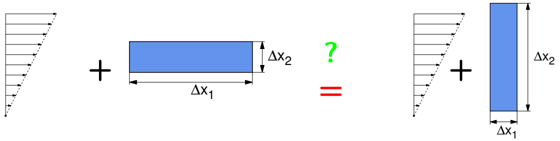


2D Simple Flow: Delta, $\tilde{\Delta}_\omega$ (II)

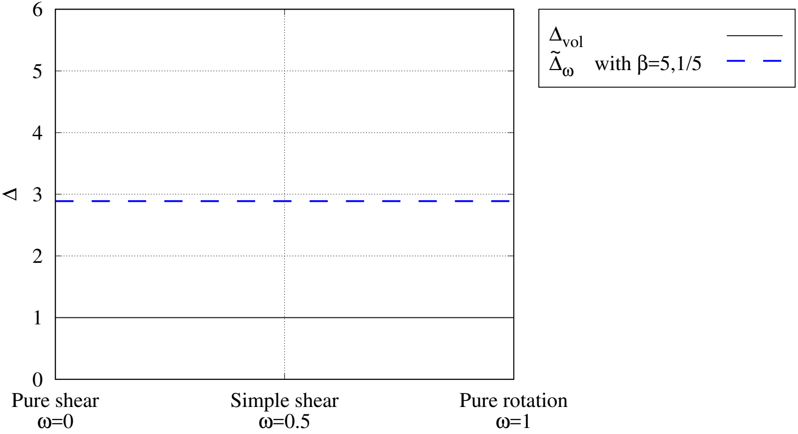
$$\tilde{\Delta}_\omega(\beta) = \sqrt{\left(\Delta x_1^2 + \Delta x_2^2\right)/3} = \sqrt{\left(\beta^2 + \beta^{-2}\right)/3}$$

Non-symmetric Kinematics:

$$\tilde{\Delta}_\omega(\beta = \beta) = \tilde{\Delta}_\omega\left(\beta = \beta^{-1}\right) \left\{ \frac{\text{Unsensitive Mesh Rotation}}{\tilde{\Delta}_\omega(\beta = 5, 1/5) \sim 2.9} \right.$$



2D Simple Flow: Graph, $\tilde{\Delta}_{\omega}$



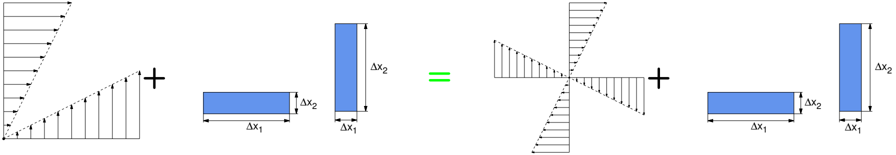
2D Simple Flow: Delta, Δ_{lsq} (I)

$$\Delta_{lsq}(\omega,\beta) = \sqrt{\frac{\Delta x_1^2 (1-2\omega)^4 + \Delta x_2^{-2}}{(1-2\omega)^4+1}} = \sqrt{\frac{\beta^2 (1-2\omega)^4 + \beta^{-2}}{(1-2\omega)^4+1}}$$

(Skew-)Symmetric Kinematics:

$$\Delta_{lsq}(\omega = \{0,1\},\beta) = \sqrt{\left(\beta^2+\beta^{-2}\right)/2} = \sqrt{\frac{3}{2}}\tilde{\Delta}_{\omega}(\beta)$$

$$\Delta_{lsq}(\omega = \{0,1\},\beta = \{5,1/5\}) \sim 3.5$$



2D Simple Flow: Delta, Δ_{lsq} (II)

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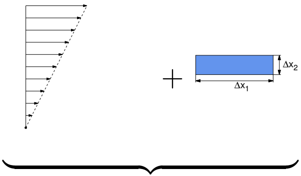
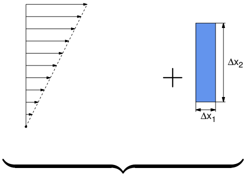
Non-symmetric Kinematics:

$$\Delta_{lsq}(\omega=0.5,\beta) = \Delta x_2 = \beta^{-1}$$



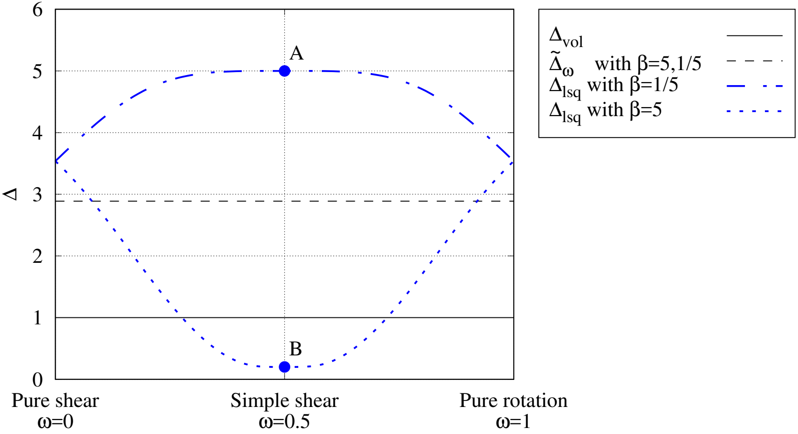
$\Delta_{lsq}(\omega{=}0.5,\beta{=}5){=}1/5$

$\neq$



$\Delta_{lsq}(\omega{=}0.5,\beta{=}1/5){=}5$

2D Simple Flow: Graph, Δ_{lsq}



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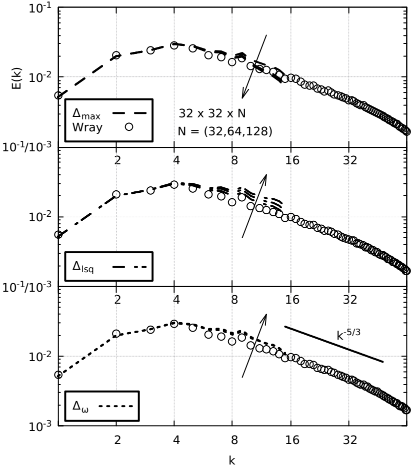
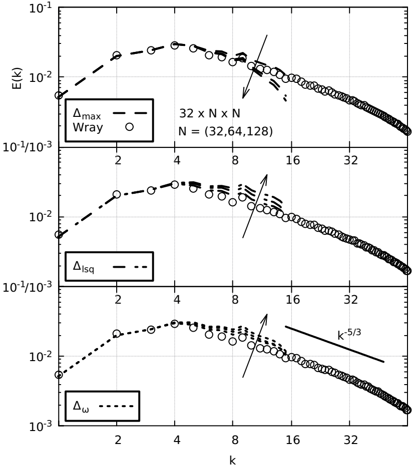
Mesh resilience study in DHIT ($L^3, L = 2\pi$)

Book Cell ($32 \times 32 \times N$)



Pencil Cell ($32 \times N \times N$)



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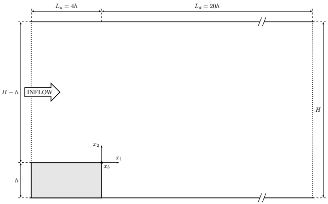
Cases

Experimental study: Vogel and Eaton (1985)



- $ER = H/(H-h) = 5/4$
- $Re_h = 28000,\ Re_\tau = 2500$
- Geom: $(4h + 20h) \times (h + 4h) \times 2h$
- Mesh:$(58 + 242) \times (32 + 46) \times 60 \sim 1.28E6$

DNS: Pont-Vílchez et al (2018, *Under Review, JFM*)

- $ER = H/(H-h) = 2$
- $Re_h \sim 13700,\ Re_\tau = 395$
- Geom: $(6h + 32h) \times (h + h) \times 2h$
- Mesh: $(62 + 270) \times (42x44) \times 60 \sim 1.5E6$

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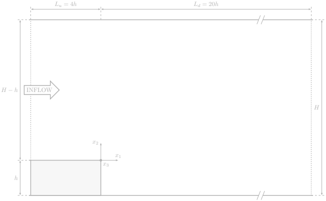

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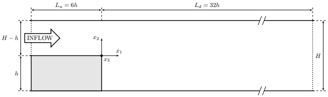

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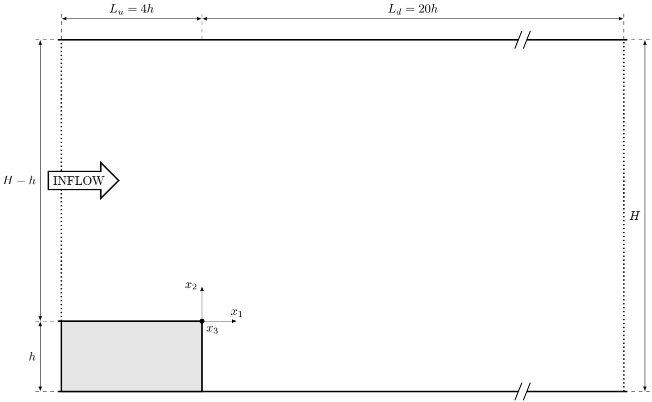
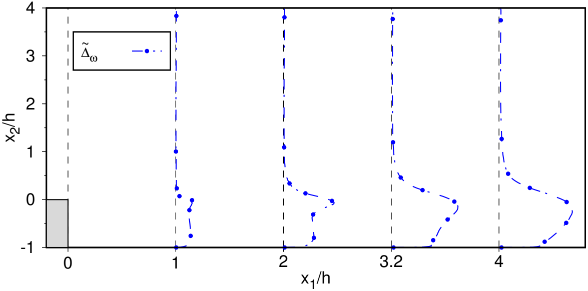


BFS-VE (1985): u_1^{rms} along the stream-wise.



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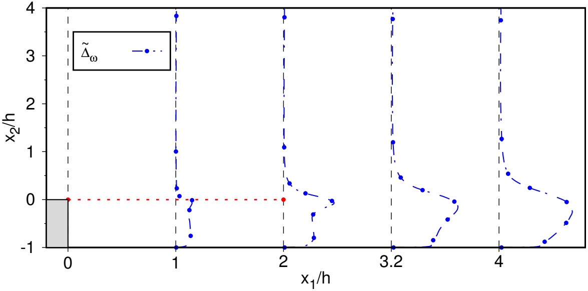

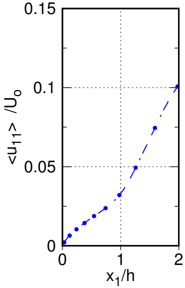

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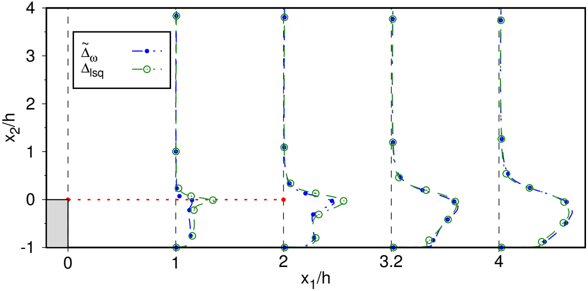

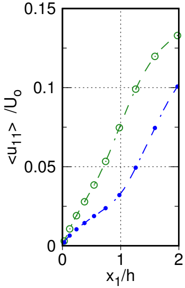

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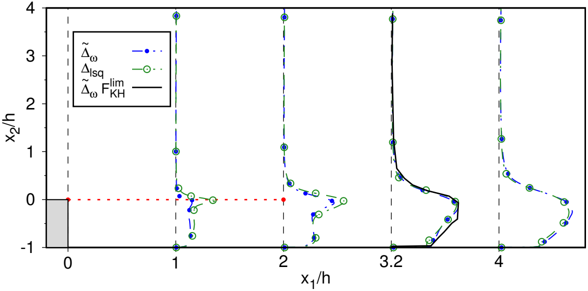
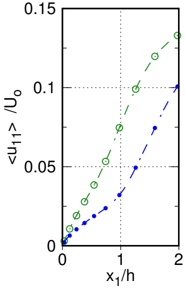
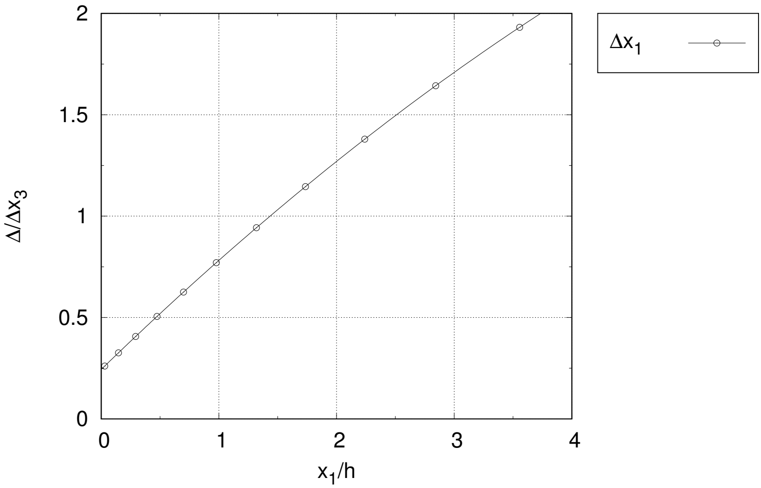




BFS-VE (1985): Δ along the stream-wise at $x_2 = 0$.



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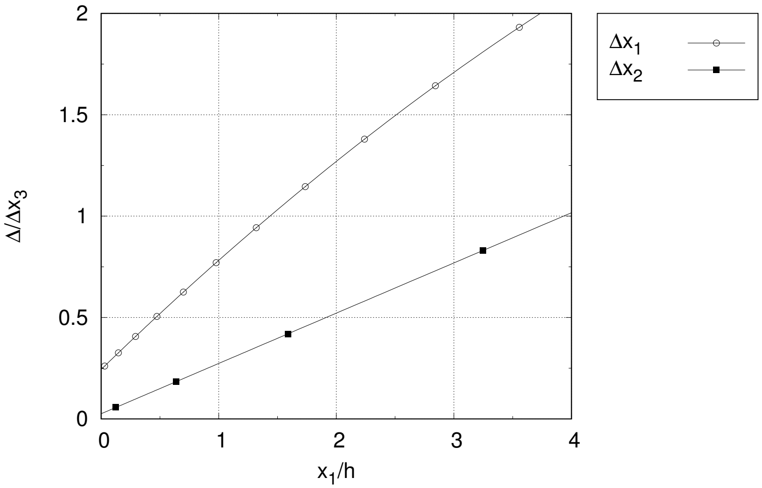

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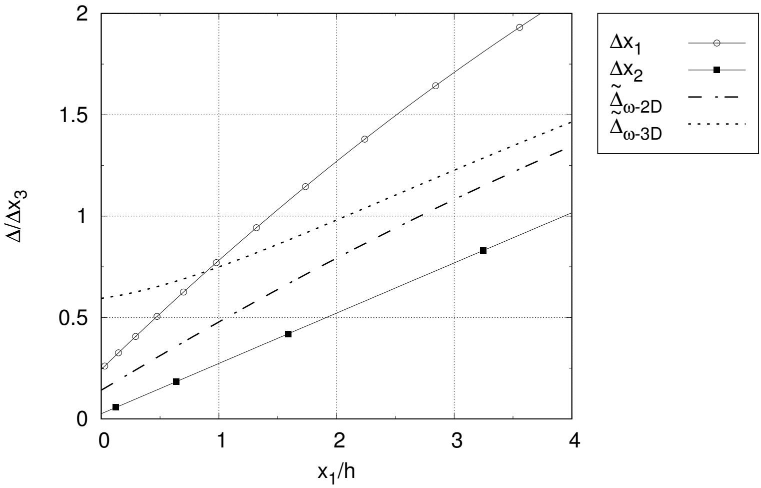

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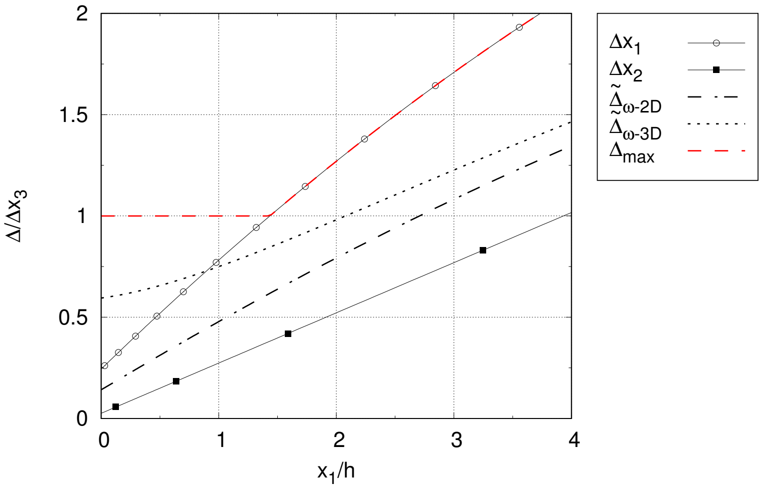

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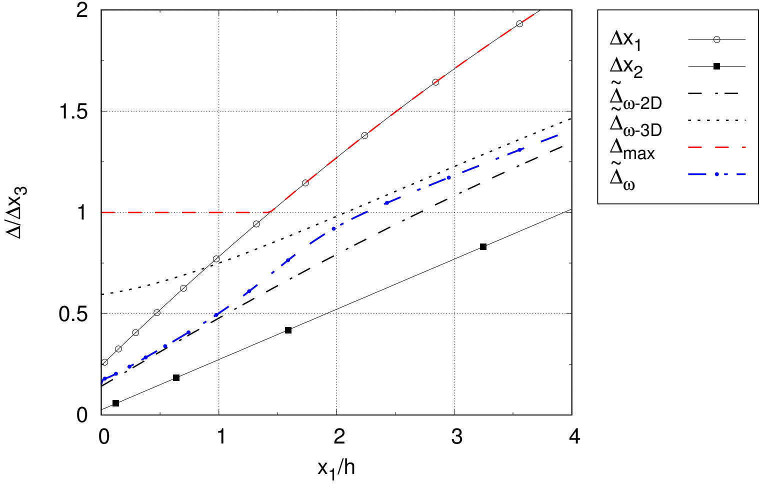

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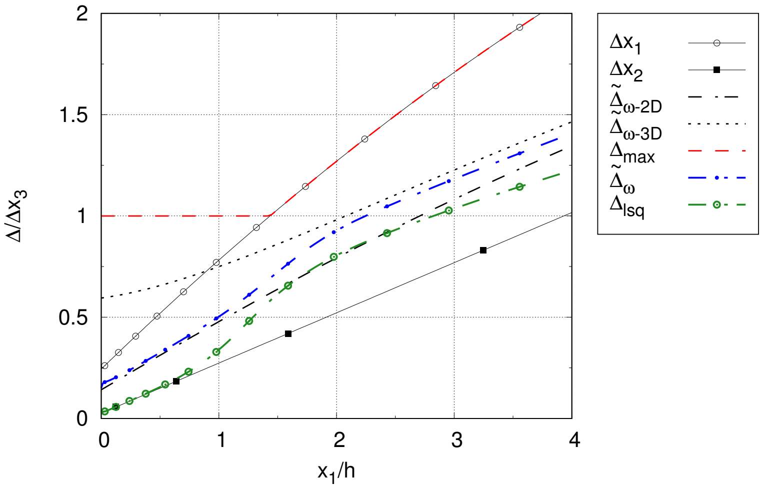
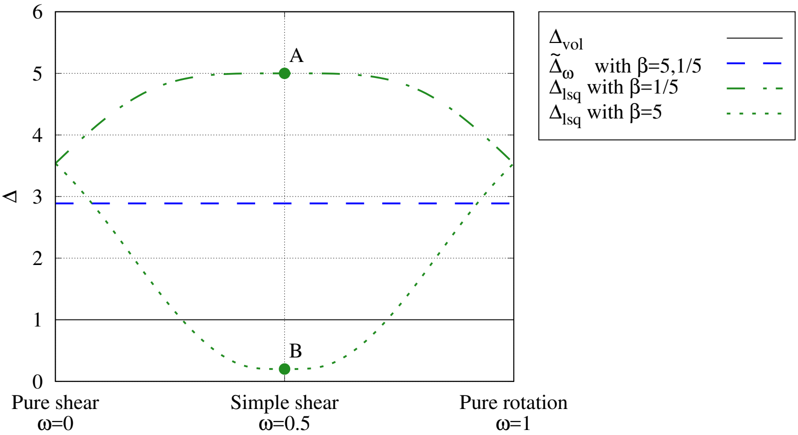
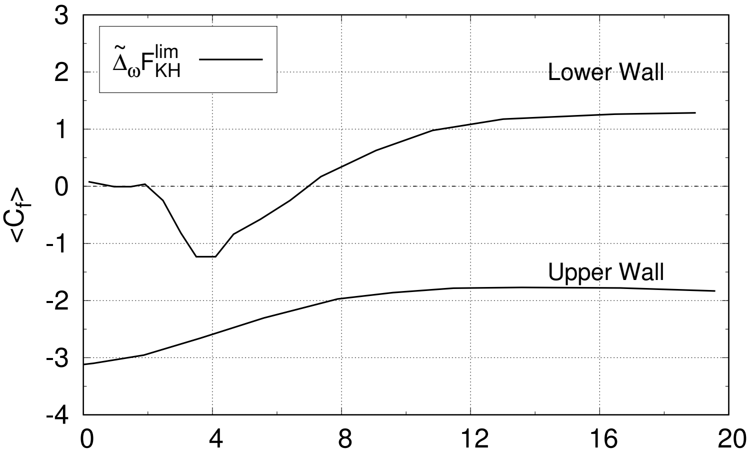


2D Simple Flow: Graph, Δ_{lsq}



BFS-VE (1985): Skin friction, $\langle C_f \rangle$, along the stream-wise.



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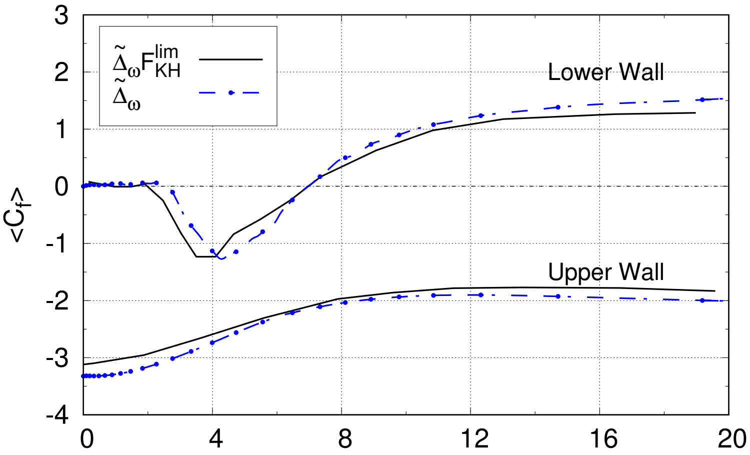

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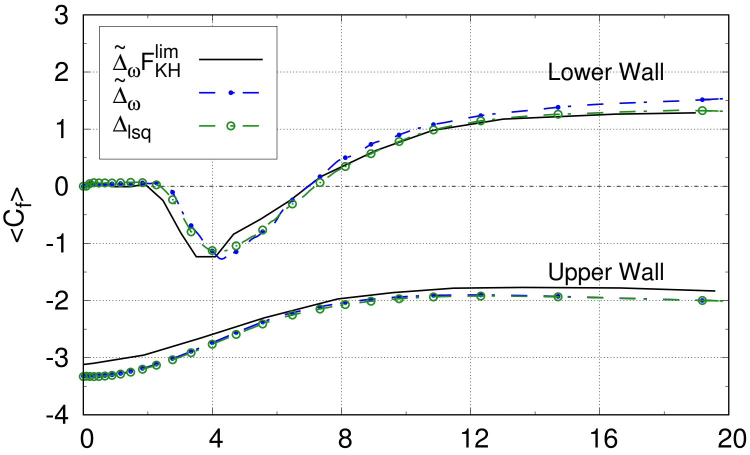
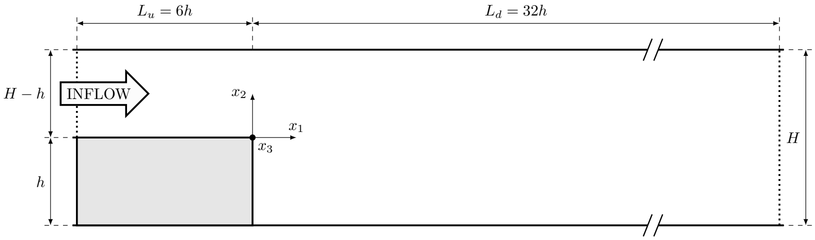
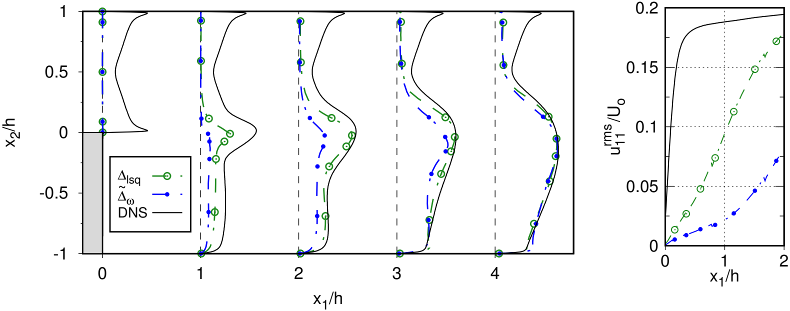


BFS-DNS (2018): Case

DNS: Pont-Vílchez et al (2018,*Under Review, JFM*)



BFS-DNS (2018): u_1^{rms} along the stream-wise.



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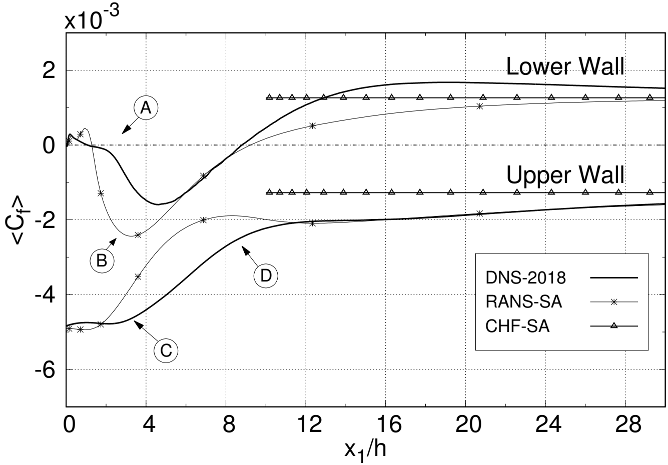

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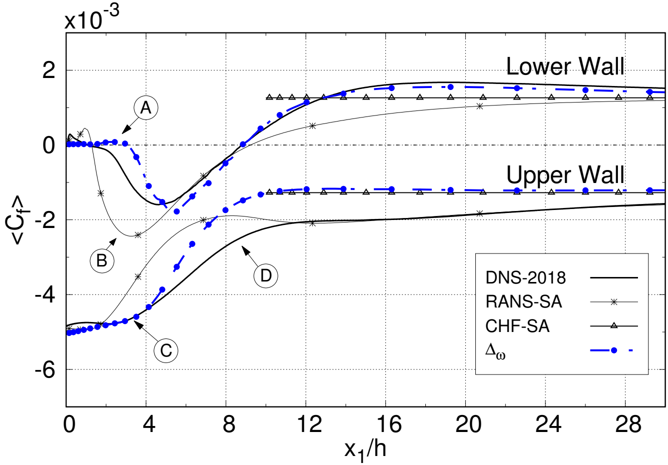

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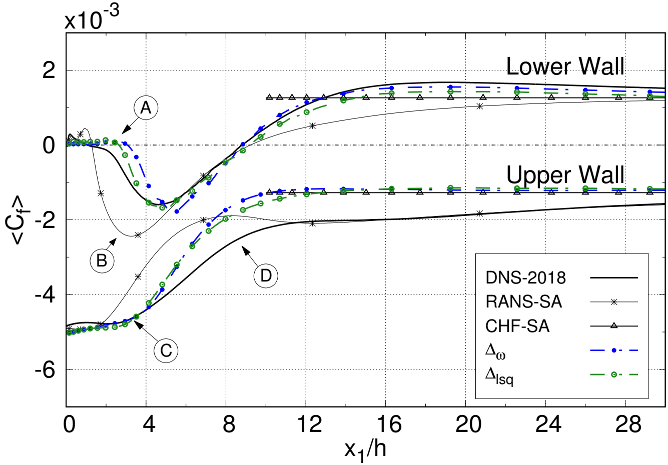

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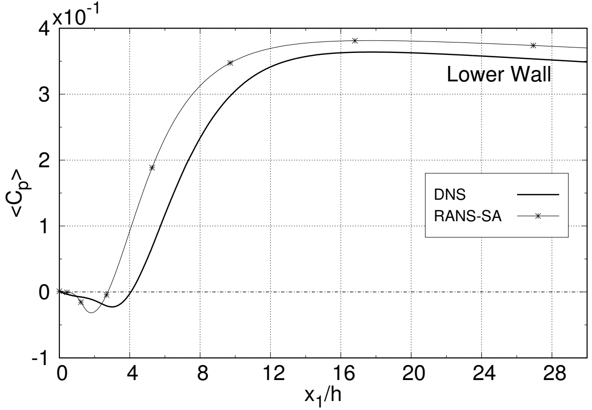

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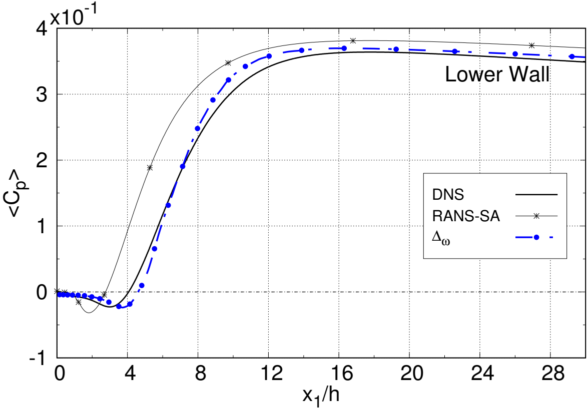

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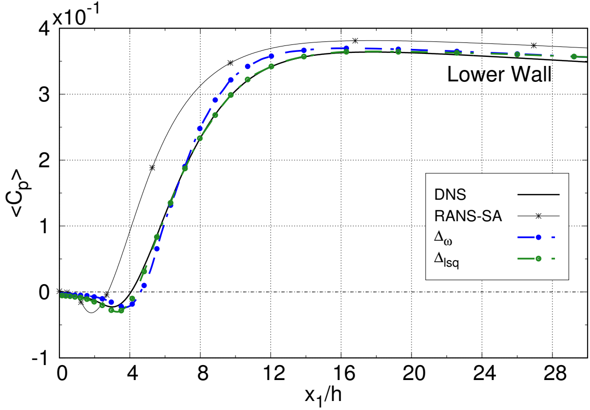

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Conclusions

Take away Message

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Further work

- Δ_{lsq} has to be tested with more challenging flow configurations and unstructured meshes.
- Computational performance analysis is desirable.
- Coupling Δ_{lsq} with more suitable *LES* models such as $WALE-DES$, $Vreman-DES$ and $\sigma-DES$.

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Thank you for your attention.