HiFiLeD 2021
2^{nd} High-Fidelity Industrial LES/DNS Symposium
22-24 September 2021, Toulouse, France

Preserving operator symmetries on unstructured grids: paving the way for DNS and LES simulations on complex geometries

F.Xavier Trias[1], Xavier Álvarez-Farré[1], Andrey Gorobets[2] and Assensi Oliva[1]

[1] Heat and Mass Transfer Technological Center,
Technical University of Catalonia, C/Colom 11, 08222 Terrassa (Barcelona)
[2] Keldysh Institute of Applied Mathematics, 4A, Miusskaya Sq., Moscow 125047, Russia

Key words: DNS, LES, symmetry-preserving, Unstructured grids, collocated, staggered

Abstract. The essence of turbulence are the smallest scales of motion [1]. They result from a subtle balance between convective transport and diffusive dissipation. Mathematically, these terms are governed by two differential operators differing in symmetry: the convective operator is skew-symmetric, whereas the diffusive is symmetric and negative-definite. At discrete level, operator symmetries must be retained to preserve the analogous (invariant) properties of the continuous equations [2, 3]: namely, the convective operator is represented by a skew-symmetric coefficient matrix, the diffusive operator by a symmetric, negative-definite matrix and the divergence is minus the transpose of the gradient operator. Therefore, even for coarse grids, the energy of the resolved scales of motion is convected in a stable manner, *i.e.* the discrete convective operator transports energy from a resolved scale of motion to other resolved scales without dissipating any energy, as it should be from a physical point-of-view. Furthermore, high-order symmetry-preserving discretizatons can be constructed for Cartesian staggered grids [2]. It is noteworthy to mention that in the last decade, many DNS reference results have been successfully generated using this type of discretization (see Figure 1, for example).

However, for unstructured meshes, it is (still) a common argument that accuracy should take precedence over the properties of the operators. Contrary to this, our philosophy is that operator symmetries are critical to the dynamics of turbulence and must be preserved. With this in mind, a fully-conservative discretization method for general unstructured grids was proposed in Ref. [3]: it exactly preserves the symmetries of the underlying differential operators on a collocated mesh. In summary, and following the same notation than in Ref. [3], the method is based on a set of five basic operators: the cell-centered and staggered control volumes (diagonal matrices), Ω_c and Ω_s, the matrix containing the face normal vectors, N_s, the cell-to-face scalar field interpolation, $\Pi_{c\to s}$ and the cell-to-face divergence operator, M. Once these operators are constructed, the rest follows straightforwardly from them. Therefore, the proposed method constitutes a robust and easy-to-implement approach to solve incompressible turbulent flows in complex configurations that can be easily implemented in already existing codes such as OpenFOAM® [4]. However, any pressure-correction method on collocated grids suffer from the same drawbacks: the cell-centered velocity field is not exactly incompressible and some artificial dissipation is inevitable introduced. On the other hand, for staggered velocity fields, the projection onto a divergence-free space is a well-posed problem: given a velocity field, it can be uniquely decomposed into a solenoidal vector and the gradient of a scalar (pressure) field. Regarding these issues, in this work, we address (i) the possibility to build up staggered formulations based on the above-explained reduced set of discrete operators and (ii) how to keep the artificial dissipation introduced in the pressure-correction for collocated grids minimals. The latter is a critical issue not only for DNS but also for LES simulations since in standard approaches this artificial dissipation can reach values higher than the subgrid scale (SGS) dissipation blurring the effect of SGS models [5]. In our

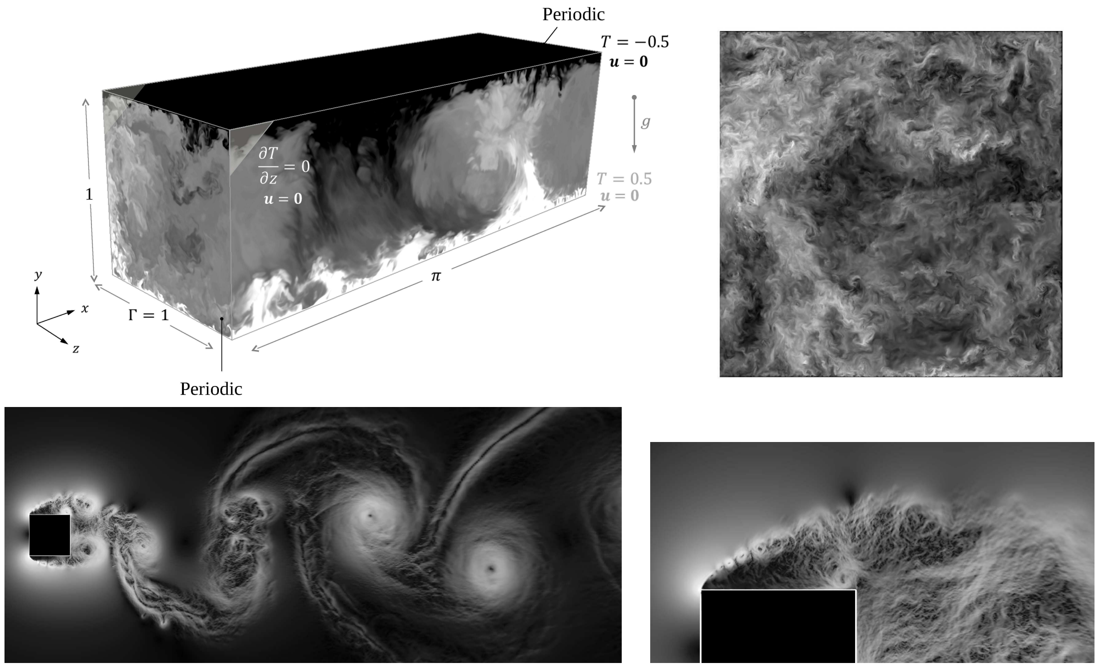

Figure 1: Examples of DNSs computed using symmetry-preserving discretizations. Top: air-filled ($Pr = 0.7$) Rayleigh-Bénard configuration studied in Ref. [1]. Instantaneous temperature field at $Ra = 10^{10}$ (left) and instantaneous velocity magnitude at $Ra = 10^{11}$ (right) for a span-wise cross section are shown. The latter was computed on 8192 CPU cores of the MareNostrum 4 supercomputer on a mesh of 5.7 billion grid points. Bottom: DNS of the turbulent flow around a square cylinder at $Re = 22000$ computed on 784 CPU cores of the MareNostrum 3 supercomputer on a mesh of 323 million grid points [6] .

opinion, losing control of the actual amount of dissipation is incompatible with the *high-fidelity* concept. Apart from this, other relevant issues such as the time-integration method or the portability challenges imposed by current HPC trends will be discussed during the Symposium.

Acknowledgments. F.X.T. and A.O. are supported by the *Ministerio de Economía y Competitividad*, Spain (ENE2017-88697-R). Simulations were carried out on the MareNostrum supercomputer at the BSC. The authors thankfully acknowledge these institutions.

REFERENCES

[1] F. Dabbagh, F. X. Trias, A. Gorobets, and A. Oliva. Flow topology dynamics in a three-dimensional phase space for turbulent Rayleigh-Bénard convection. *Physical Review Fluids*, 5:024603, 2020.

[2] R. W. C. P. Verstappen and A. E. P. Veldman. Symmetry-Preserving Discretization of Turbulent Flow. *Journal of Computational Physics*, 187:343–368, 2003.

[3] F. X. Trias, O. Lehmkuhl, A. Oliva, C.D. Pérez-Segarra, and R.W.C.P. Verstappen. Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured meshes. *Journal of Computational Physics*, 258:246–267, 2014.

[4] E. Komen, J. A. Hopman, E. M. A. Frederix, F. X. Trias, and R. W. C. P. Verstappen. A symmetry-preserving second-order time-accurate PISO-based method. *Computers & Fluids*, 225:104979, 2021.

[5] E. M. J. Komen, L. H. Camilo, A. Shams, B. J. Geurts, and B. Koren. A quantification method for numerical dissipation in quasi-DNS and under-resolved DNS, and effects of numerical dissipation in quasi-DNS and under-resolved DNS of turbulent channel flows. *Journal of Computational Physics*, 345:565–595, 2017.

[6] F. X. Trias, A. Gorobets, and A. Oliva. Turbulent flow around a square cylinder at Reynolds number 22000: a DNS study. *Computers & Fluids*, 123:87–98, 2015.