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Algebraic implementation of a flux limiter for heterogeneous computing

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1 Introduction

The presence of flow shocks, or discontinuities, in industrial applications has fostered the research in shock capturing schemes for many years. Such discontinuities appear in both compressible and multiphase. Applications range from high speed aerodynamics to multiphase flows.

As a consequence of Godunov's theorem, there is no linear scheme able to provide Total Variation Diminishing (TVD) solutions, and consequently the use of non-linear becomes mandatory. Among them, flux limiter schemes are a mature and robust method, which has been implemented in a diversity of applications. Several authors have developed techniques for both structured [1] and unstructured meshes [2].

The advent of heterogeneous architectures has motivated a new demand for portability. From this perspective, the use of a unified approach is desired in order to simplify the architecture-oriented efficient implementations to a more global, portable solution without a significant lack of performance. This provides an opportunity for High Performance Computing (HPC) optimization, parallelization and portability [3].

In this work, an operator-based implementation of a Flux Limiter (FL) is deployed. By casting discrete operators into algebraic forms (i.e., matrices and vectors) it has been shown that nearly 90% of the operations comprised in a typical CFD algorithm are comprised of the following: Sparse Matrix-Vector multiplication (SpMV), generalized vector addition (AXPY) and dot product (DOT) [4].

Oyarzun et al. [5] have implemented a Conjugate Gradient (CG) method following such an operator-based approach. Borrell et al. [6] followed the same approach with the aim of pursuing petascale simulations. In all cases the use of an operator-based formulation has provided with robust, portable and optimized implementations.

Under this conditions, the design of operator-based algorithms for its use in massively parallel architectures is a smart strategy towards the efficient solution of both industrial and academic scale problems [7].

2 Problem Statement

The solution of hyperbolic problems in finite volume methods can be turned in the calculation of flows at the faces problem. Prone to introduce numerical instabilities, this requires of an appropriate flux reconstruction strategy to guarantee a Total Variation Diminishing (TVD) behavior.

Well-established in 1984 by Sweby [1] for cartesian grids, flux limiters have been widely used by the community and proven to be useful. Forthcoming authors adapted this for unstructured meshes in a diversity of approaches [2].

The most common form of a flux limiter is as follows:

$$\theta_f = \theta_U + \Psi(r)\left(\frac{\theta_D - \theta_U}{2}\right) \tag{1}$$

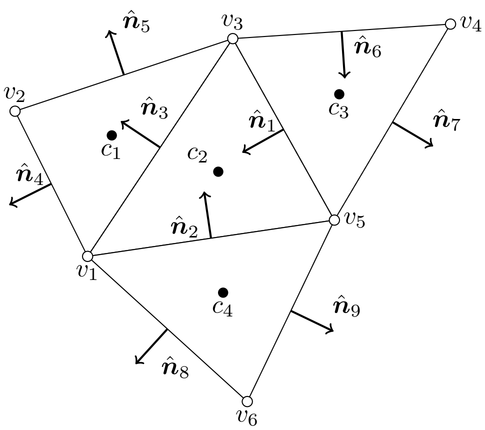
Where θ_U and θ_D stand for the upwind and downwind values and $\Psi(r)$ is the flux limiter function. This can be recast in the less common form:

$$\theta_f = \frac{\theta_U + \theta_D}{2} + \frac{\Psi(r) - 1}{2}\left(\theta_D - \theta_U\right) \tag{2}$$

Which can be transformed into a global approach as:

$$\theta_f = \left(\Pi_{c\to f} + \Omega_{c\to f}(r)\right)\theta_c \tag{3}$$

Where $\Pi_{c\to f}$ is the standard cell-to-face interpolation and $\Omega_{c\to f}(r)$ absorbs the right term of equation 2. Conceiving the mesh as a graph, incidence matrices can be used to construct both $\Pi_{c\to f}$ and $\Omega_{c\to f}(r)$. The proper construction of $\Omega_{c\to f}(r)$ will be discussed in this work.



$$T_{\mathcal{FC}} = \begin{pmatrix} 0 & -1 & +1 & 0 \\ 0 & -1 & 0 & +1 \\ -1 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

Figure 1: Incidence matrix

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