Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

JOHANNES KEPLER UNIVERSITY LINZ

On a physically-consistent nonlinear subgrid-scale heat flux model for LES of buoyancy driven flows

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[3]Keldysh Institute of Applied Mathematics of RAS, Russia

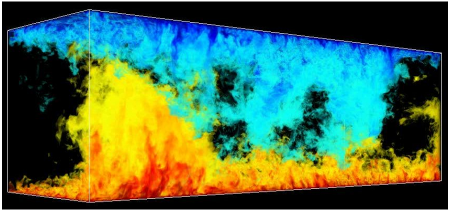
ICCFD10
INTERNATIONAL CONFERENCE ON COMPUTATIONAL FLUID DYNAMICS

Contents

Motivation

Research question:

- Can we find a nonlinear SGS heat flux model with **good physical** and **numerical properties**, such that we can obtain satisfactory predictions for a turbulent Rayleigh-Bénard convection?



DNS of an air-filled Rayleigh-Bénard convection at $Ra = 10^{10}$

[1] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *On the evolution of flow topology in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 28:115105, 2016.

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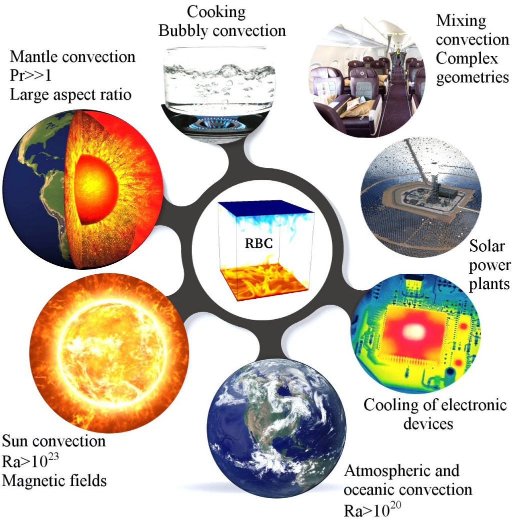
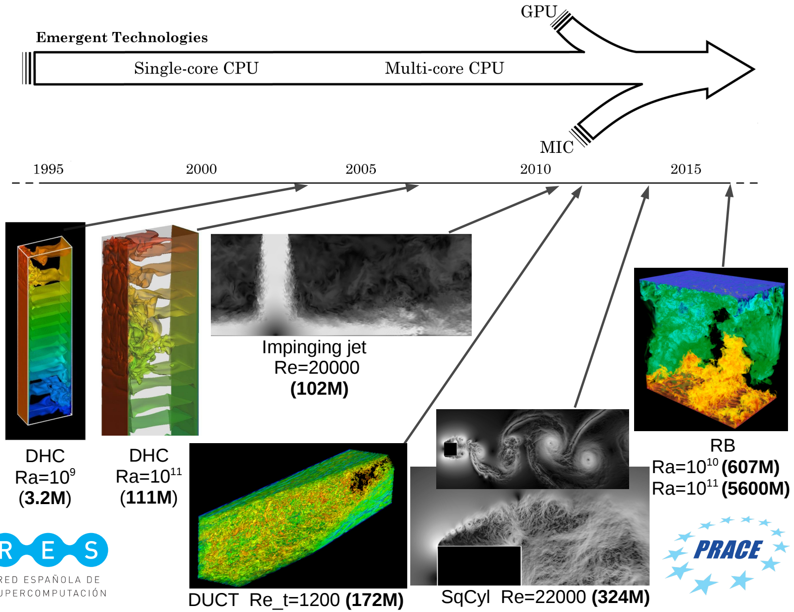

Cooking
Bubbly convection
Mixing
convection
Complex
geometries
Mantle convection
Pr>>1
Large aspect ratio
RBC
Solar
power
plants
Cooling of electronic
devices
Sun convection
Ra>10^23
Magnetic fields
Atmospheric and
oceanic convection
Ra>10^20

Motivation

And of course... saving the planet!



Emergent Technologies
Single-core CPU
Multi-core CPU
GPU
MIC
1995
2000
2005
2010
2015
DHC
Ra=10^9
(3.2M)
DHC
Ra=10^11
(111M)
Impinging jet
Re=20000
(102M)
DUCT Re_t=1200 (172M)
SqCyl Re=22000 (324M)
RB
Ra=10^10 (607M)
Ra=10^11 (5600M)
RES
RED ESPAÑOLA DE SUPERCOMPUTACIÓN
PRACE

Eddy-viscosity models for LES

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nabla^2 \overline{u} - \nabla \overline{p} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_t S(\overline{u})$$

[2] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

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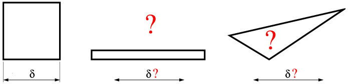

δ?

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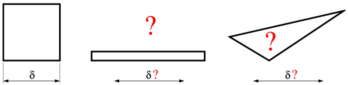

δ

[3]F.X.Trias, A.Gorobets, M.Silvis, R.Verstappen, A.Oliva. *A new subgrid characteristic length for turbulence simulations on anisotropic grids*, **Phys.Fluids**, 26:115109, 2017.

How to model the subgrid heat flux in LES?

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$$Pr_t?$$

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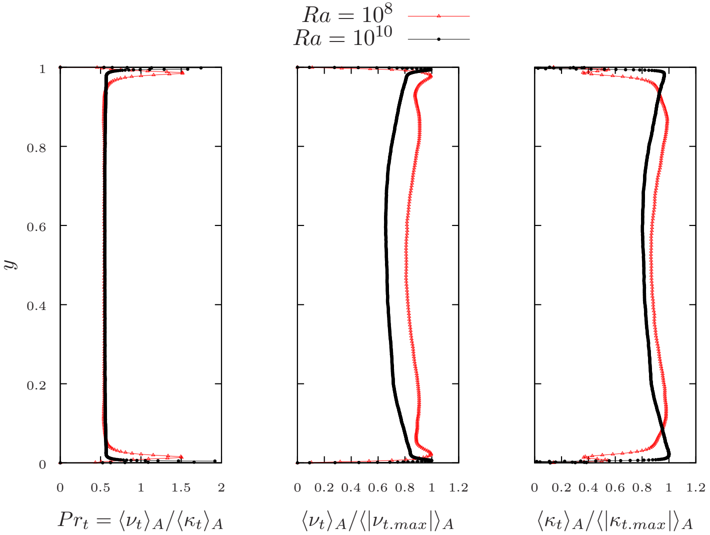

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$$G \equiv \nabla \overline{u} \qquad\qquad q = -\frac{\delta^2}{12} G \nabla \overline{T} + \mathcal{O}(\delta^4)$$

A priori alignment trends[4]

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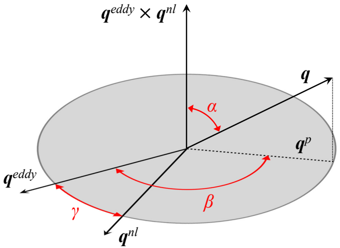

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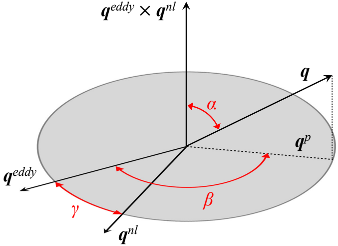

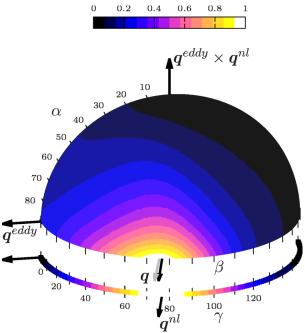

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A priori alignment trends

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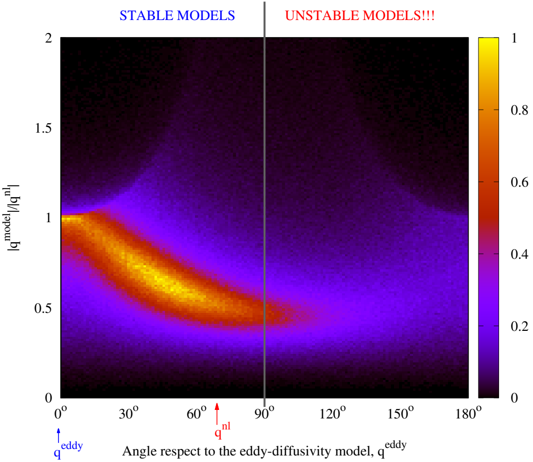

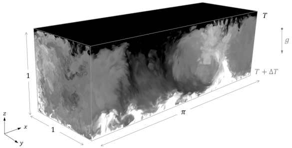

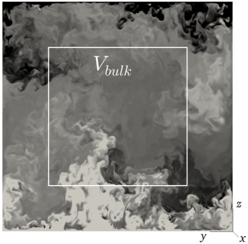

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$$\mathcal{T}_{SGS} = 1/|S|$$

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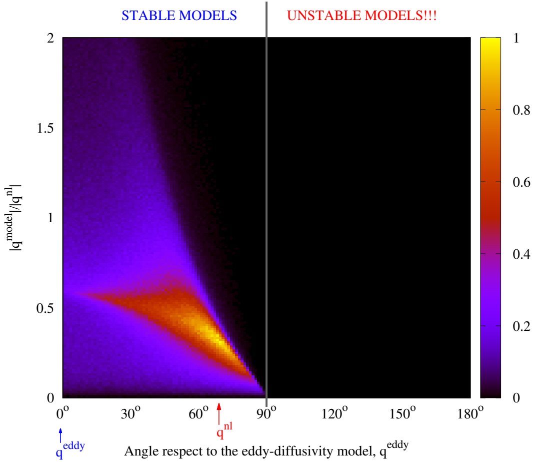
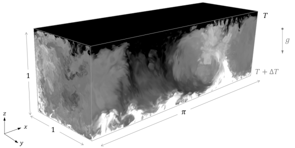
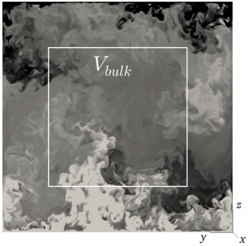
A priori alignment trends

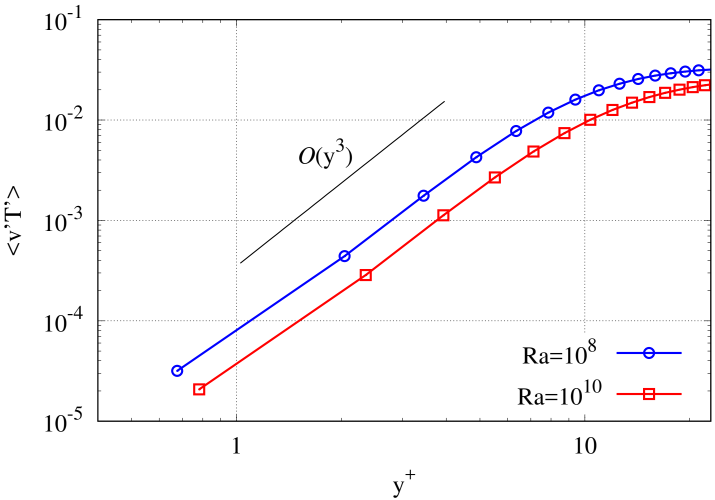
$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T} \qquad \boxed{q^{DH} \equiv -\mathcal{T}_{SGS} \frac{\delta^2}{12} G G^T \nabla \overline{T}}$$







Near-wall scaling



Near-wall scaling

$$\boxed{q^{DH} \equiv -\mathcal{T}_{SGS}\frac{\delta^2}{12}GG^T\nabla\overline{T}\;; \qquad \mathcal{T}_{SGS} = 1/|S|}$$

$$u = ay + \mathcal{O}(y^2); \quad v = by^2 + \mathcal{O}(y^3); \quad w = cy + \mathcal{O}(y^2); \quad T = dy + \mathcal{O}(y^2)$$

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$$G = \begin{pmatrix} y & 1 & y \\ y^2 & y & y^2 \\ y & 1 & y \end{pmatrix}; \; \nabla\overline{T} = \begin{pmatrix} y \\ 1 \\ y \end{pmatrix}$$

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$$\boxed{\mathcal{T}_{SGS} = 1/|S| = \mathcal{O}(y^0)}$$

Idea: build a $\mathcal{T}_{SGS}$ with the proper $\mathcal{O}(y^2)$ scaling!!!

Building proper models for the subgrid heat flux

Let us consider models that are based on the invariants of the tensor GG^T

$$q \approx -C_M \left(P_{GG^T}^{p} Q_{GG^T}^{q} R_{GG^T}^{r} \right) \frac{\delta^2}{12} GG^T \nabla \overline{T} \qquad (\equiv q^{S2})$$

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	P_{GG^T}	Q_{GG^T}	R_{GG^T}
Formula	$2(Q_\Omega - Q_S)$	$V^2 + Q_G^2$	R_G^2

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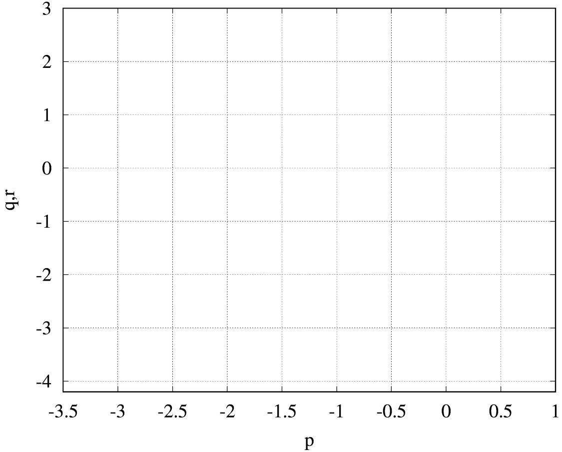
$$-6r - 4q - 2p = 1 \;\; [T]; \qquad 6r + 2q = s,$$

where s is the slope for the asymptotic near-wall behavior, *i.e.* $\mathcal{O}(y^s)$.

Building proper models for the subgrid heat flux

Solutions: $q(p,s) = -(1+s)/2 - p$ and $r(p,s) = (2s+1)/6 + p/3$



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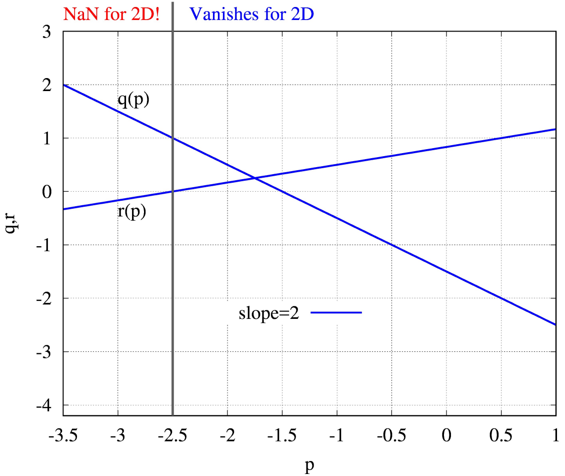

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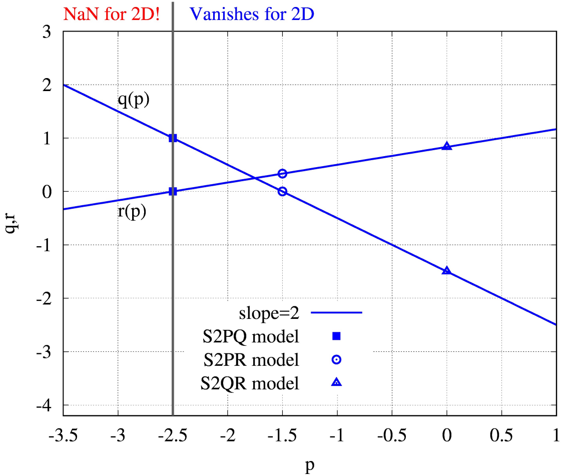

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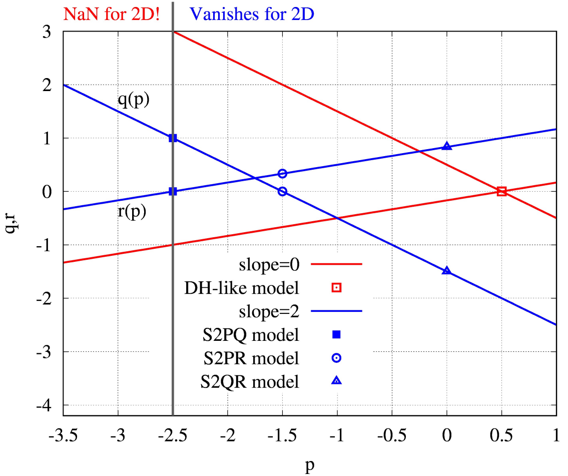
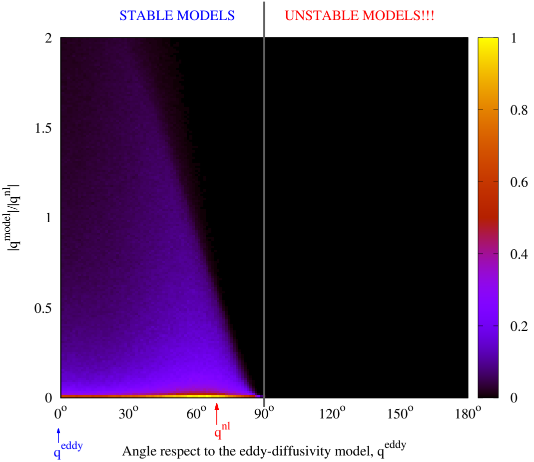
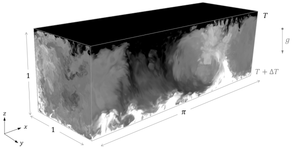


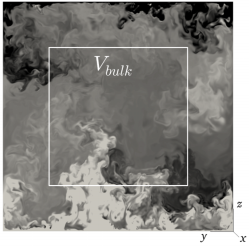
A priori alignment trends of S2QR

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T}$$

$$\boxed{q^{s2QR} \equiv -C_M Q_{GG^T}^{3/2} R_{GG^T}^{5/6} \frac{\delta^2}{12} G G^T \nabla \overline{T}}$$







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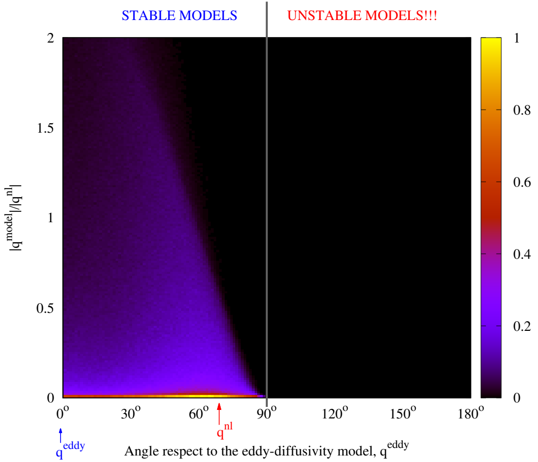

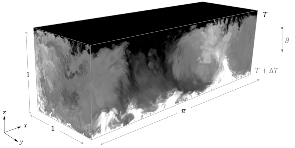

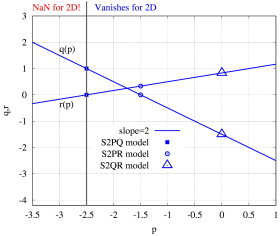

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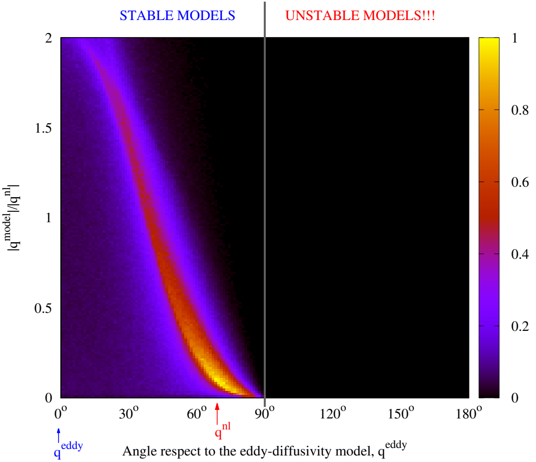

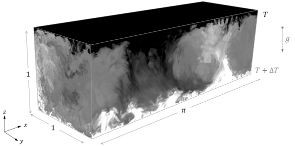

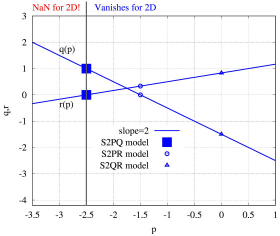

A priori alignment trends of S2PR

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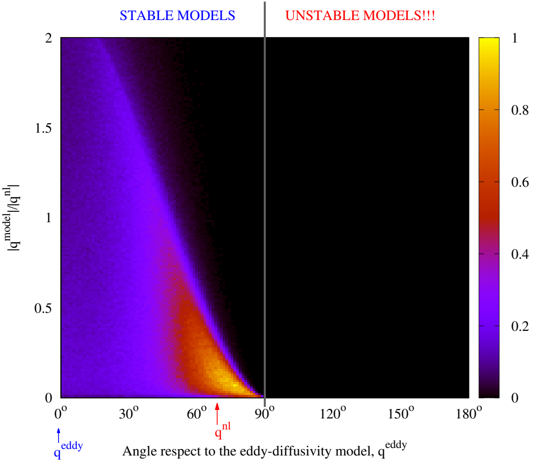

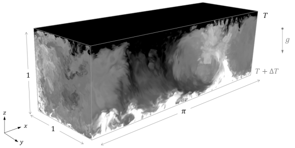

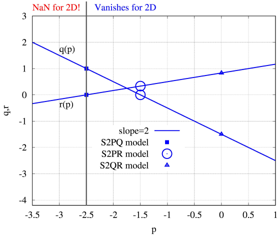

A priori alignment trends of S2PR in the near-wall region

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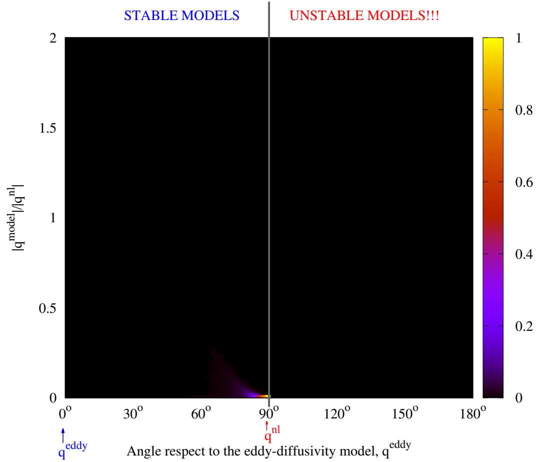

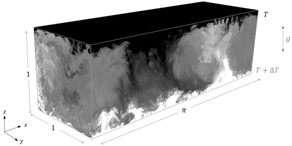

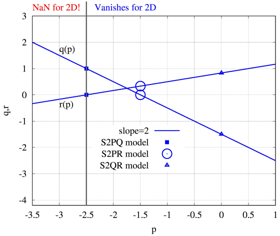

A priori alignment trends of DH in the near-wall region

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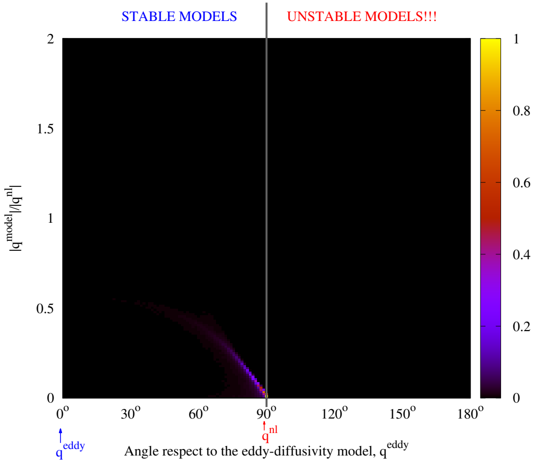

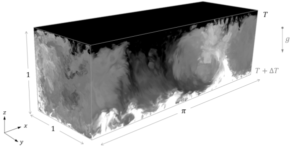

A posteriori results?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} \qquad - \nabla \cdot \tau(\overline{u}) \;\; ; \qquad \nabla \cdot \overline{u} = 0$$

eddy-viscosity $\longrightarrow$ $\tau\ (\overline{u}) = -2\nu_t S(\overline{u})$

$$\boxed{\nu_t \approx (C_m \delta)^2 D_m(\overline{u})}$$

A posteriori results?

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$$\partial_t \overline{T} + (\overline{u} \cdot \nabla)\overline{T} = \alpha \nabla^2 \overline{T} - \nabla \cdot q \quad \text{where} \quad q = \overline{uT} - \overline{u}\,\overline{T}$$

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⚠ But first we need to answer the following **research question**:

- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

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- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

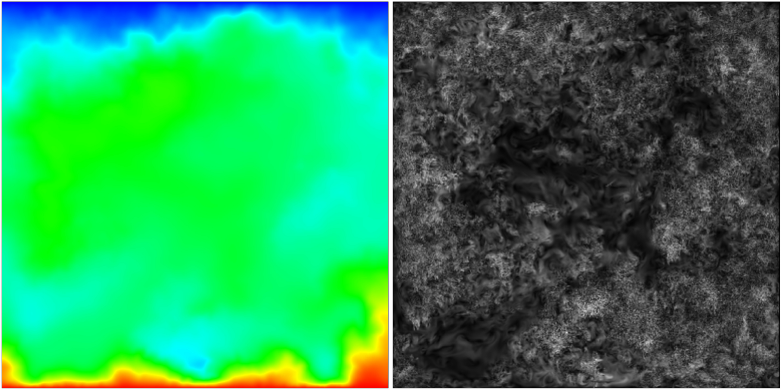
Idea: let's do an LES for momentum and a DNS for temperature!

DNS at very low Pr number

Why? scale separation scales with $Pr^{0.5}$

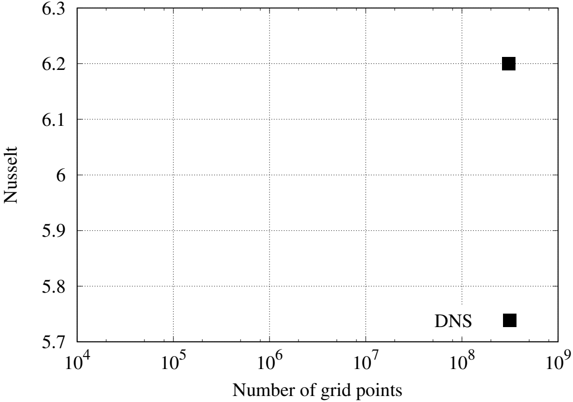
DNS at very low *Pr* number

Why? scale separation scales with $Pr^{0.5}$ (≈ 0.07 is our case)



DNS of a RB at $Ra = 7.14 \times 10^6$ and $Pr = 0.005$ (liquid sodium)

LES[8] results at very low Pr number



[8] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

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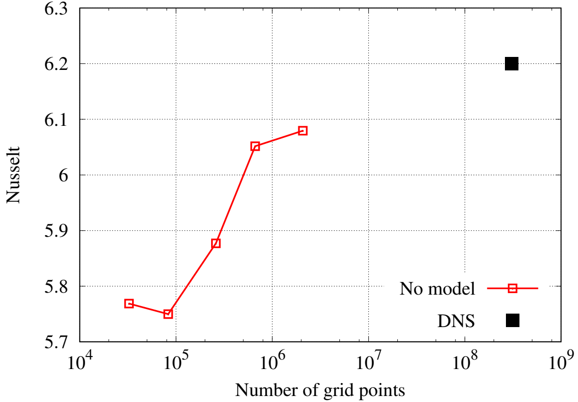

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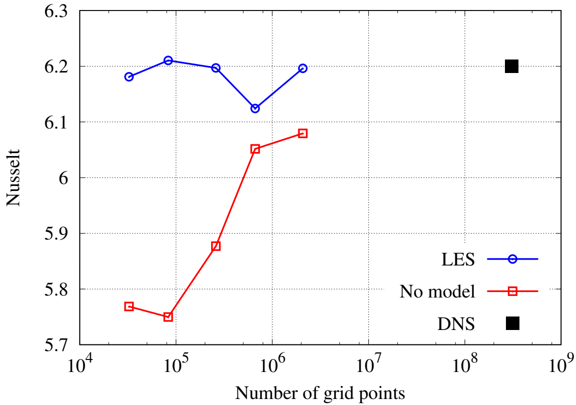

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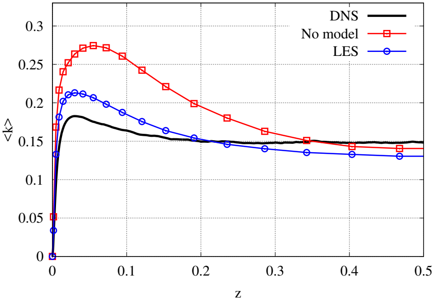
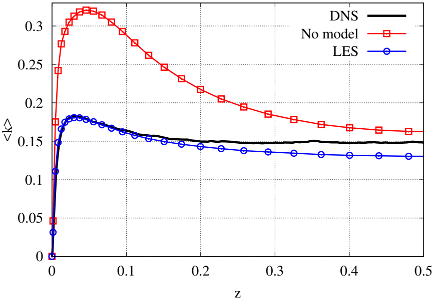
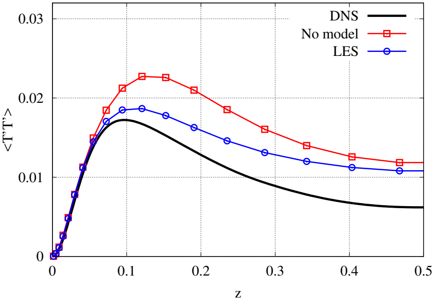


$64 \times 32 \times 32$



$96 \times 52 \times 52$

LES results at very low *Pr* number



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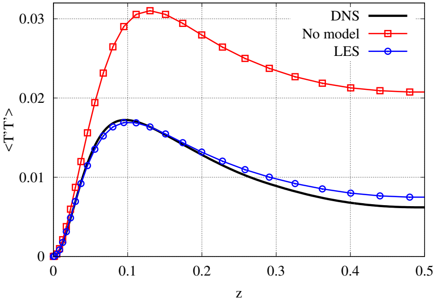

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Concluding remarks

- A new tensor-diffusivity model has been proposed

$$\boxed{q^{s2PR} \equiv -C_M P_{GG^T}^{-3/2} R_{GG^T}^{1/3} \frac{\delta^2}{12} GG^T \nabla \overline{T}}$$

- Locally defined, unconditionally stable and vanishes for 2D flows ✓
- Good *a priori* alignment trends ✓
- Proper near-wall scaling ✓
- Eddy-viscosity models work for RB ✓

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Future:

- *A posteriori* tests using q^{s2PR} for Rayleigh-Bénard convection.

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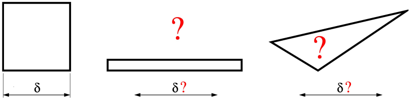
Future:

- *A posteriori* tests using q^{s2PR} for Rayleigh-Bénard convection.
- How δ should be defined for highly anisotropic grids?



Thank you for your attention