On the conservative discretization of surface tension.

N. Valle *
Delft University of Technology

R.W.C.P. Verstappen
University of Groningen

F.X. Trias
Technical University of Catalonia

The Direct Numerical Simulation (DNS) of multiphase flows is a tool of uttermost importance to analyse the complex interplay of phenomena in multiphase flows. The presence of multiple phases introduces a level of complexity on top of the already challenging fluid dynamics governed by the (incompressible) Navier-Stokes equations. There are, for example, considerable complications in dealing with discontinuities at the interface between two thermodynamic phases as well as in discretizing the capillary forces at the interface. In such a complex system, the accurate discretization of all components is critical. Therefore we aim to obtain a physics-compatible discretization.

However, to date, there is no discretization for surface tension that preserves mass, momentum and energy simultaneously, preventing physics-compatible DNS. Instead, numerical pathologies arise, hindering the accurate simulation of multiphase flows in relevant scientific and technological applications.

Our approach is inspired on the well-known symmetry-preserving strategy for the DNS of single-phase turbulent flows [1]. It stems from the idea that as far as a physical system is stable, a mimetic discretization of such a system will also be numerically stable by the physical mechanisms as well, without the need of including additional numerical artefacts or stabilisation terms. While there has been some recent advances [2], in general the introduction of surface tension force does not preserve linear momentum (for closed surfaces), which is an essential geometric identity.

In this work, we summarize the conditions for conservation of both linear momentum and mechanical energy at the continuum level, extend these into the regularized and discrete level, and discuss our recent results in this direction. We adopt a Conservative Level Set [3] approach for the regularization of the interface and the classical Continuum Surface Force [4], and extend our previous results for an energy-preserving discretization of surface tension [5, 6] into momentum-preserving discretization.

Our results highlight the relevance of a consistent reconstruction of the interface, namely the calculation of the interface normal. We subsequently discuss alternative definitions of the normal vector and its impact both in accuracy, consistency and computational cost.

References

[1] R. W. C. P. Verstappen and A. E. P. Veldman. Symmetry-preserving discretization of turbulent flow. *J. Comput. Phys.*, 187(1):343–368, 2003.

[2] Shahab Mirjalili and Ali Mani. Consistent, energy-conserving momentum transport for simulations of two-phase flows using the phase field equations. *J. Comput. Phys.*, 426:109918, 2021.

[3] Elin Olsson and Gunilla Kreiss. A conservative level set method for two phase flow. *J. Comput. Phys.*, 210(1):225–246, 2005.

[4] J.U Brackbill, D.B Kothe, and C Zemach. A continuum method for modeling surface tension. *J. Comput. Phys.*, 100(2):335–354, 1992.

[5] N. Valle, F. X. Trias, and J. Castro. An energy-preserving level set method for multiphase flows. *J. Comput. Phys.*, 400:108991, 2020.

[6] N. Valle and R. W. C. P. Verstappen. Conservation of energy in the DNS of interface-resolved multiphase fows. *Exp. Comput. Multiph. Flow*, 4:1–15, 2023.

*Email: n.vallemarchante@tudelft.nl