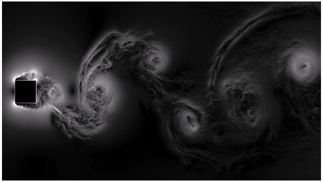
Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

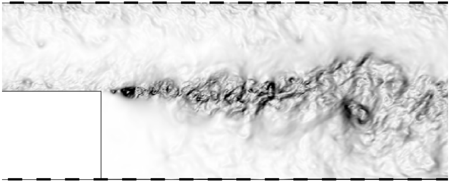
Paving the way for DNS and LES on unstructured grids

F.Xavier Trias[1], Andrey Gorobets[2], Assensi Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia

[2]Keldysh Institute of Applied Mathematics of RAS, Russia





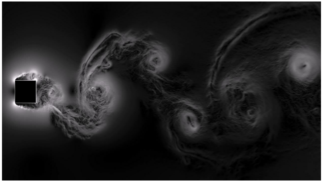
Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

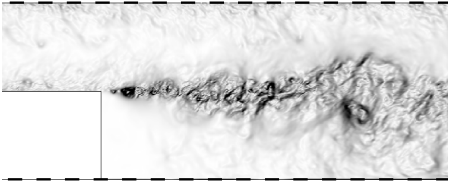
Paving the (right?) way for DNS and LES on unstructured grids

F.Xavier Trias[1], Andrey Gorobets[2], Assensi Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia

[2]Keldysh Institute of Applied Mathematics of RAS, Russia





Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Paving the (right?) way for DNS and LES on unstructured grids: fully conservative collocated/staggered discretizations

F.Xavier Trias[1], Andrey Gorobets[2], Assensi Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia

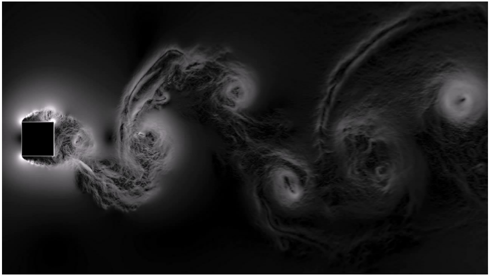
[2]Keldysh Institute of Applied Mathematics of RAS, Russia

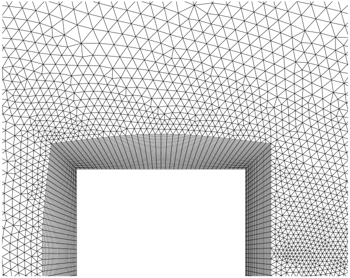
Contents

Motivation

Research question #1:

- Can we construct numerical discretizations of the Navier-Stokes equations suitable for **complex geometries**, such that the **symmetry properties** are exactly preserved?





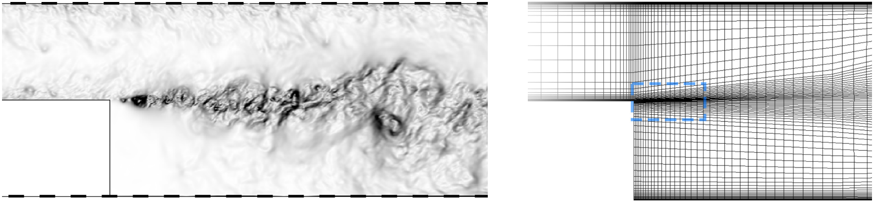
DNS[1] of the turbulent flow around a square cylinder at $Re = 22000$

[1] F.X.Trias, A.Gorobets, A.Oliva. *Turbulent flow around a square cylinder at Reynolds number 22000: a DNS study*, **Computers&Fluids**, 123:87-98, 2015.

Motivation

Research question #1:

- Can we construct numerical discretizations of the Navier-Stokes equations suitable for **complex geometries**, such that the **symmetry properties** are exactly preserved?



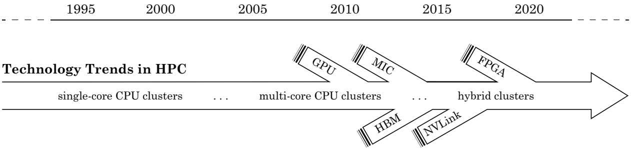
DNS[2] of backward-facing step at $Re_\tau = 395$ and expansion ratio 2

[2] A.Pont-Vílchez, F.X.Trias, A.Gorobets, A.Oliva. *DNS of Backward-Facing Step flow at $Re_\tau = 395$ and expansion ratio 2*. **Journal of Fluid Mechanics**, 863:341-363, 2019.

Motivation

Research question #2:

- How can we develop **portable** and **efficient** CFD codes for large-scale simulations on modern supercomputers?



[3] X.Álvarez, A.Gorobets, F.X.Trias, R.Borrell, G.Oyarzun. *HPC^2 - a fully portable algebra-dominant framework for heterogeneous computing. Application to CFD*. **Computers & Fluids**, 173:285-292, 2018.

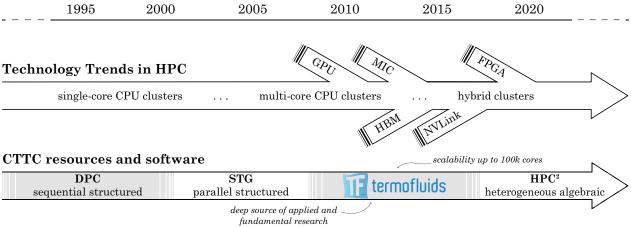
[4] X.Álvarez, A.Gorobets, F.X.Trias. *A hierarchical parallel implementation for heterogeneous computing. Application to algebra-based CFD simulations on hybrid supercomputers*. **Computers & Fluids**, 214:104768, 2021.

[5] X.Álvarez, A.Gorobets, F.X.Trias, A.Oliva. *NUMA-aware strategies for the efficient execution of CFD simulations on CPU supercomputers* **ParCFD2021**. Don't miss it!

Motivation

Research question #2:

- How can we develop **portable** and **efficient** CFD codes for large-scale simulations on modern supercomputers?



HPC^2: portable, algebra-based framework[3] for heterogeneous computing is being developed[4]. Traditional stencil-based data and sweeps are replaced by algebraic structures (sparse matrices and vectors) and kernels. NUMA-aware execution strategies for CFD are presented in this conference[5].

[3] X.Álvarez, A.Gorobets, F.X.Trias, R.Borrell, G.Oyarzun. *HPC^2 - a fully portable algebra-dominant framework for heterogeneous computing. Application to CFD*. **Computers & Fluids**, 173:285-292, 2018.

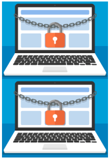
[4] X.Álvarez, A.Gorobets, F.X.Trias. *A hierarchical parallel implementation for heterogeneous computing. Application to algebra-based CFD simulations on hybrid supercomputers*. **Computers & Fluids**, 214:104768, 2021.

[5] X.Álvarez, A.Gorobets, F.X.Trias, A.Oliva. *NUMA-aware strategies for the efficient execution of CFD simulations on CPU supercomputers* **ParCFD2021**. Don't miss it!

Motivation

Frequently used general purpose CFD codes:

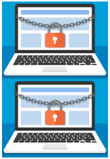
- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM



Motivation

Frequently used general purpose CFD codes:

- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM

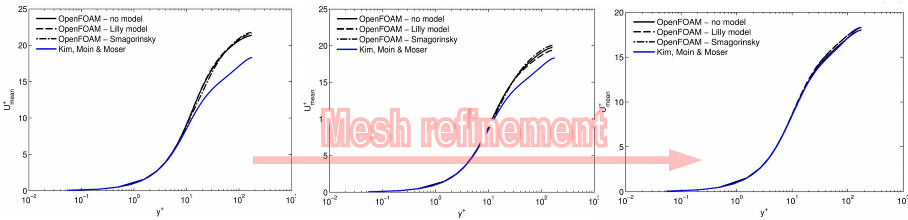


Main common characteristics of LES in such codes:

- **Unstructured finite volume** method, **collocated** grid
- Second-order spatial and temporal discretisation
- Eddy-viscosity type LES models

Motivation

OpenFOAM® LES[6] results of a turbulent channel for at $Re_\tau = 180$

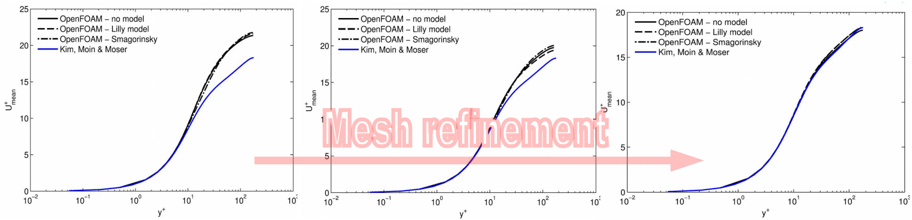


$13\times76\times20$	$19\times78\times28$	$38\times78\times57$
$\Delta x^+=90, \Delta y^+_{wall}=0.5, \Delta z^+=30$	$\Delta x^+=60, \Delta y^+_{wall}=0.5, \Delta z^+=20$	$\Delta x^+=30, \Delta y^+_{wall}=0.5, \Delta z^+=10$

[6] E.M.J.Komen, L.H.Camilo, A.Shams, B.J.Geurts, B.Koren. *A quantification method for numerical dissipation in quasi-DNS and under-resolved DNS, and effects of numerical dissipation in quasi-DNS and under-resolved DNS of turbulent channel flows*, **Journal of Computational Physics**, 345, 565-595, 2017.

Motivation

Open∇FOAM® LES[6] results of a turbulent channel for at $Re_\tau = 180$



$13 \times 76 \times 20$ — $\Delta x^+ = 90, \Delta y^+_{wall} = 0.5, \Delta z^+ = 30$

$19 \times 78 \times 28$ — $\Delta x^+ = 60, \Delta y^+_{wall} = 0.5, \Delta z^+ = 20$

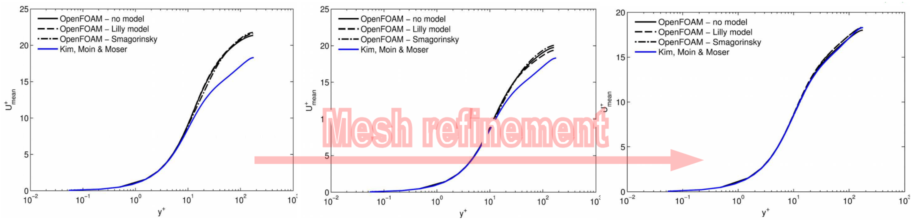
$38 \times 78 \times 57$ — $\Delta x^+ = 30, \Delta y^+_{wall} = 0.5, \Delta z^+ = 10$

- Are LES results are merit of the SGS model?

[6] E.M.J.Komen, L.H.Camilo, A.Shams, B.J.Geurts, B.Koren. *A quantification method for numerical dissipation in quasi-DNS and under-resolved DNS, and effects of numerical dissipation in quasi-DNS and under-resolved DNS of turbulent channel flows*, **Journal of Computational Physics**, 345, 565-595, 2017.

Motivation

OpenFOAM® LES[6] results of a turbulent channel for at $Re_\tau = 180$



$13 \times 76 \times 20$ | $19 \times 78 \times 28$ | $38 \times 78 \times 57$

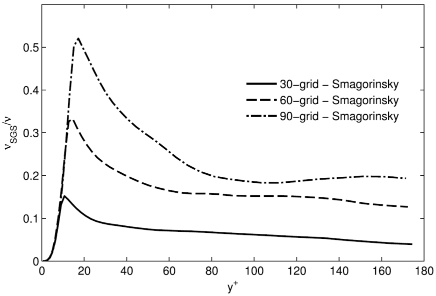
$\Delta x^+ = 90, \Delta y^+_{wall} = 0.5, \Delta z^+ = 30$ | $\Delta x^+ = 60, \Delta y^+_{wall} = 0.5, \Delta z^+ = 20$ | $\Delta x^+ = 30, \Delta y^+_{wall} = 0.5, \Delta z^+ = 10$

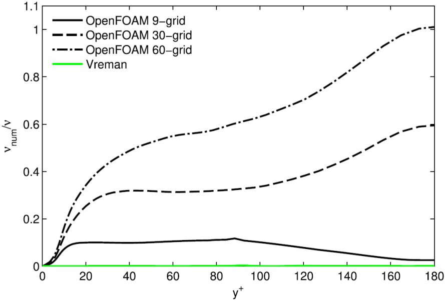
- Are LES results are merit of the SGS model? Apparently **NOT**!!! ✗

[6] E.M.J.Komen, L.H.Camilo, A.Shams, B.J.Geurts, B.Koren. *A quantification method for numerical dissipation in quasi-DNS and under-resolved DNS, and effects of numerical dissipation in quasi-DNS and under-resolved DNS of turbulent channel flows*, **Journal of Computational Physics**, 345, 565-595, 2017.

Motivation

OpenFOAM® LES[7] results of a turbulent channel for at $Re_\tau = 180$



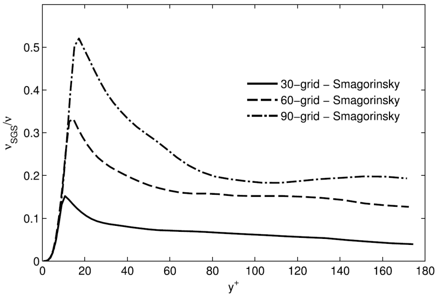


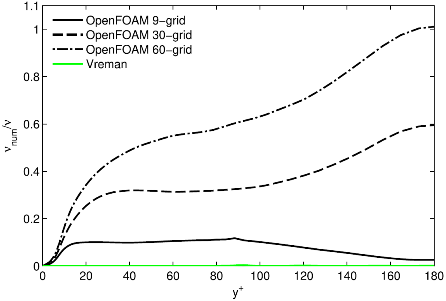
$$\nu_{num} \neq 0$$

[7] E.M.J.Komen, L.H.Camilo, A.Shams, B.J.Geurts, B.Koren. *A quantification method for numerical dissipation in quasi-DNS and under-resolved DNS, and effects of numerical dissipation in quasi-DNS and under-resolved DNS of turbulent channel flows*, **Journal of Computational Physics**, 345, 565-595, 2017.

Motivation

OpenFOAM® LES[7] results of a turbulent channel for at $Re_\tau = 180$





$$\nu_{SGS} < \nu_{num} \neq 0$$

[7] E.M.J.Komen, L.H.Camilo, A.Shams, B.J.Geurts, B.Koren. *A quantification method for numerical dissipation in quasi-DNS and under-resolved DNS, and effects of numerical dissipation in quasi-DNS and under-resolved DNS of turbulent channel flows*, **Journal of Computational Physics**, 345, 565-595, 2017.

Symmetry-preserving discretization

Continuous

$$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$$

$$\nabla \cdot \boldsymbol{u} = 0$$

Symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\,\boldsymbol{u}_h = \mathsf{D}\boldsymbol{u}_h - \mathsf{G}\boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M}\boldsymbol{u}_h = \boldsymbol{0}_h$

Symmetry-preserving discretization

Continuous

$$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \int_\Omega \boldsymbol{a}\boldsymbol{b}\, d\Omega$$

Discrete

$$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\,\boldsymbol{u}_h = \mathsf{D}\boldsymbol{u}_h - \mathsf{G}\boldsymbol{p}_h$$

$$\mathsf{M}\boldsymbol{u}_h = \boldsymbol{0}_h$$

$$\langle \boldsymbol{a}_h, \boldsymbol{b}_h \rangle_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$$

Symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\boldsymbol{u}_h = \mathsf{D}\boldsymbol{u}_h - \mathsf{G}\boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M}\boldsymbol{u}_h = \boldsymbol{0}_h$
$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \int_\Omega \boldsymbol{a}\boldsymbol{b} d\Omega$	$\langle \boldsymbol{a}_h, \boldsymbol{b}_h \rangle_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$
$\langle C(\boldsymbol{u}, \varphi_1), \varphi_2 \rangle = -\langle C(\boldsymbol{u}, \varphi_2), \varphi_1 \rangle$	$\mathsf{C}(\boldsymbol{u}_h) = -\mathsf{C}^T(\boldsymbol{u}_h)$

Symmetry-preserving discretization

Continuous

$$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$$
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \int_\Omega \boldsymbol{a}\boldsymbol{b} d\Omega$$

$$\langle C(\boldsymbol{u}, \varphi_1), \varphi_2 \rangle = - \langle C(\boldsymbol{u}, \varphi_2), \varphi_1 \rangle$$
$$\langle \nabla \cdot \boldsymbol{a}, \varphi \rangle = - \langle \boldsymbol{a}, \nabla \varphi \rangle$$

Discrete

$$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\boldsymbol{u}_h = \mathsf{D}\boldsymbol{u}_h - \mathsf{G}\boldsymbol{p}_h$$
$$\mathsf{M}\boldsymbol{u}_h = \boldsymbol{0}_h$$

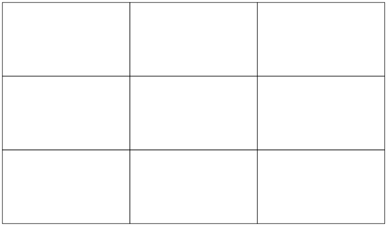
$$\langle \boldsymbol{a}_h, \boldsymbol{b}_h \rangle_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$$

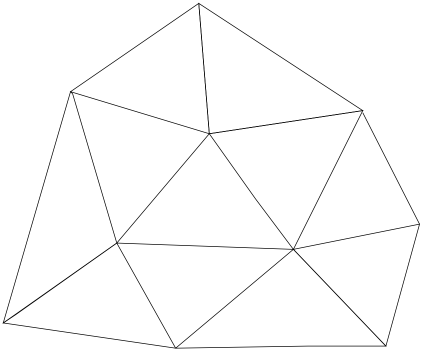
$$\mathsf{C}(\boldsymbol{u}_h) = -\mathsf{C}^T(\boldsymbol{u}_h)$$
$$\Omega \mathsf{G} = -\mathsf{M}^T$$

Symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\, \boldsymbol{u}_h = \mathsf{D}\boldsymbol{u}_h - \mathsf{G}\boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M}\boldsymbol{u}_h = \boldsymbol{0}_h$
$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \int_\Omega \boldsymbol{a}\boldsymbol{b}\, d\Omega$	$\langle \boldsymbol{a}_h, \boldsymbol{b}_h \rangle_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$
$\langle C(\boldsymbol{u}, \varphi_1), \varphi_2 \rangle = -\langle C(\boldsymbol{u}, \varphi_2), \varphi_1 \rangle$	$\mathsf{C}(\boldsymbol{u}_h) = -\mathsf{C}^T(\boldsymbol{u}_h)$
$\langle \nabla \cdot \boldsymbol{a}, \varphi \rangle = -\langle \boldsymbol{a}, \nabla \varphi \rangle$	$\Omega \mathsf{G} = -\mathsf{M}^T$
$\left\langle \nabla^2 \boldsymbol{a}, \boldsymbol{b} \right\rangle = \left\langle \boldsymbol{a}, \nabla^2 \boldsymbol{b} \right\rangle$	$\mathsf{D} = \mathsf{D}^T \quad def-$

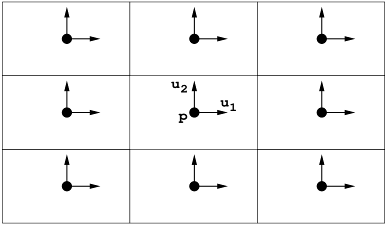
Collocated vs staggered

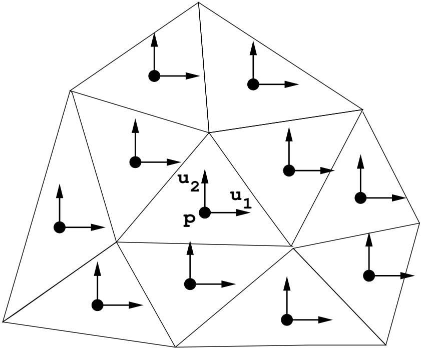




Collocated vs staggered

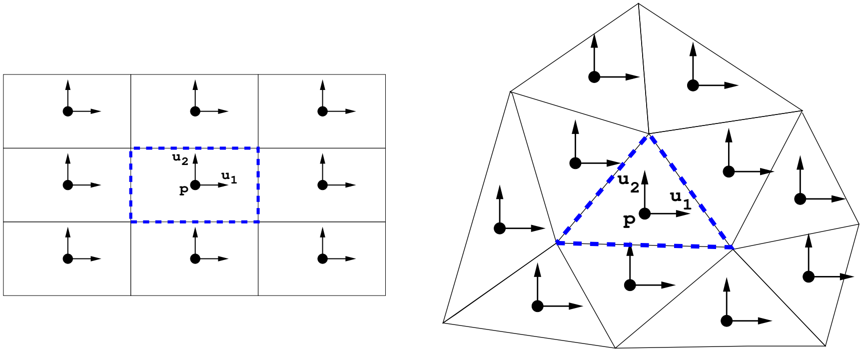
Collocated



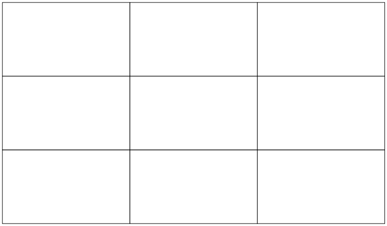


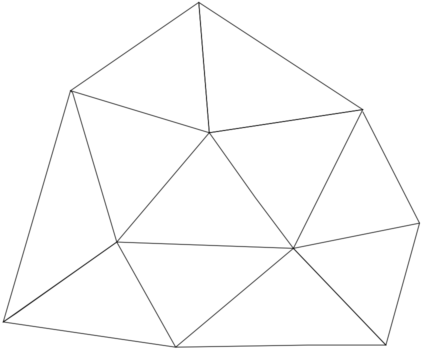
Collocated vs staggered

Collocated



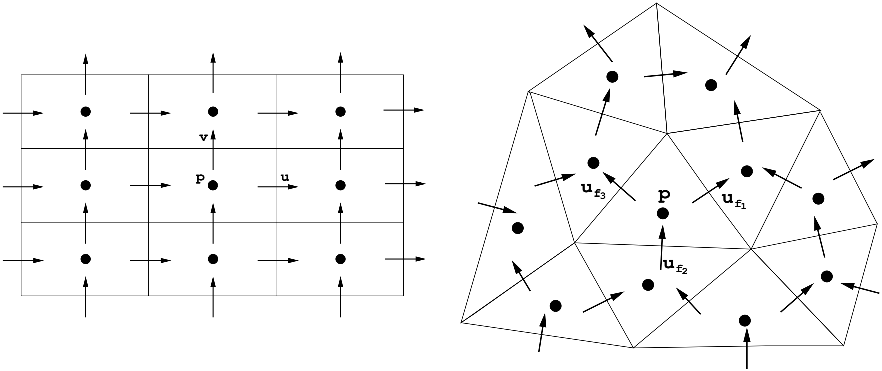
Collocated vs staggered





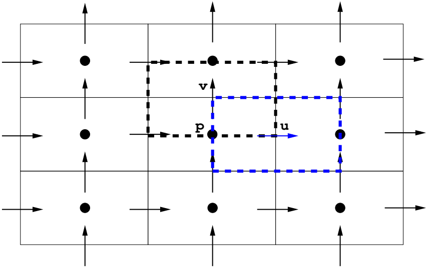
Collocated vs staggered

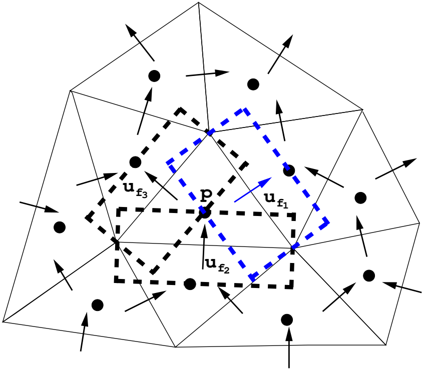
Staggered



Collocated vs staggered

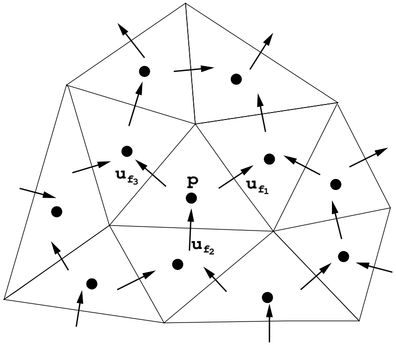
Staggered





Why staggered?

$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}(\boldsymbol{u}_s)\,\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c$$

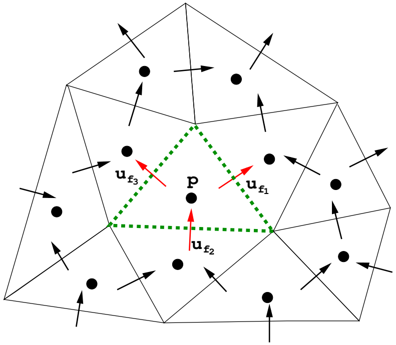


Why staggered?

$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}(\boldsymbol{u}_s)\,\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c$$

Let's consider we have $\boldsymbol{u}_s$ such as

$$\mathsf{M}\boldsymbol{u}_s \neq \mathbf{0}_c$$



Why staggered?

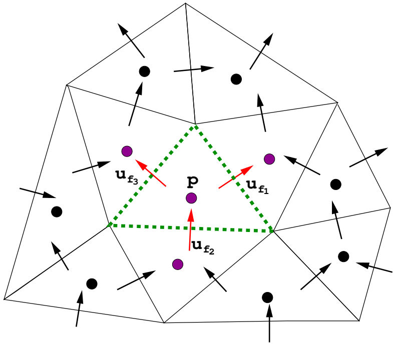
$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}(\boldsymbol{u}_s)\,\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \boldsymbol{0}_c$$

Let's consider we have $\boldsymbol{u}_s$ such as

$$\mathsf{M}\boldsymbol{u}_s \neq \boldsymbol{0}_c$$

then, we can easily project $\boldsymbol{u}_s$

$$\boldsymbol{u}_s = \quad \boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c$$



Why staggered?

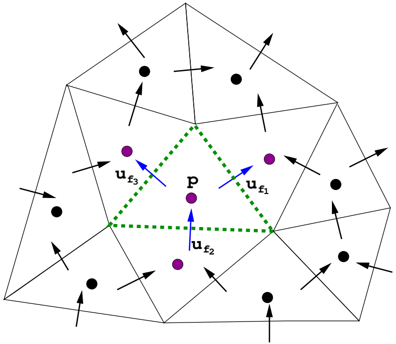
$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c$$

Let's consider we have $\boldsymbol{u}_s$ such as

$$\mathsf{M}\boldsymbol{u}_s \neq \mathbf{0}_c$$

then, we can easily project $\boldsymbol{u}_s$

$$\mathsf{M}\boldsymbol{u}_s = \mathsf{M}(\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c) = \mathbf{0}_c$$



Why staggered?

$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}(\boldsymbol{u}_s)\,\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c$$

Let's consider we have $\boldsymbol{u}_s$ such as

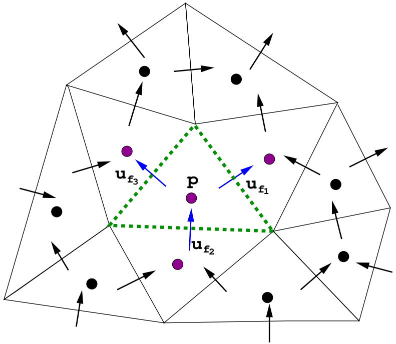
$$\mathsf{M}\boldsymbol{u}_s \neq \mathbf{0}_c$$

then, we can easily project $\boldsymbol{u}_s$

$$\mathsf{M}\boldsymbol{u}_s = \mathsf{M}(\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c) = \mathbf{0}_c$$

Finally, this leads to a Poisson eq.

$$\mathsf{M}\mathsf{G}\boldsymbol{p}_c = \mathsf{M}\boldsymbol{u}_s$$



Why staggered?

$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}(\boldsymbol{u}_s)\,\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \boldsymbol{0}_c$$

Let's consider we have $\boldsymbol{u}_s$ such as

$$\mathsf{M}\boldsymbol{u}_s \neq \boldsymbol{0}_c$$

then, we can easily project $\boldsymbol{u}_s$

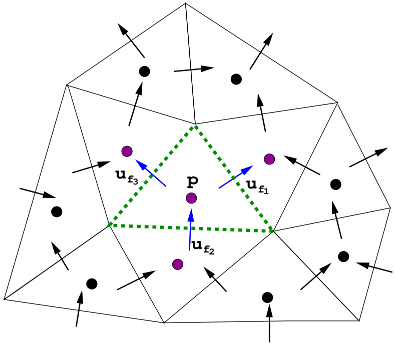
$$\mathsf{M}\boldsymbol{u}_s = \mathsf{M}(\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c) = \boldsymbol{0}_c$$

Finally, this leads to a Poisson eq.

$$\mathsf{M}\mathsf{G}\boldsymbol{p}_c = \mathsf{M}\boldsymbol{u}_s$$

If $\Omega_s \mathsf{G} = -\mathsf{M}^T$

$$\langle \nabla\cdot\boldsymbol{a}, \varphi\rangle = -\langle \boldsymbol{a}, \nabla\varphi\rangle$$



Why staggered? Everything seems to be in the right place!

$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c$$

Let's consider we have $\boldsymbol{u}_s$ such as

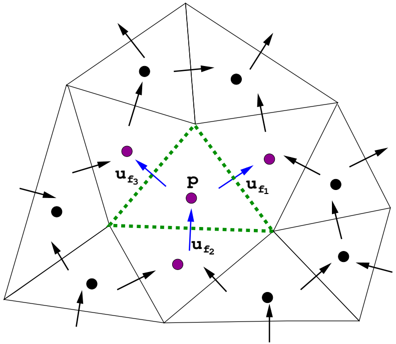
$$\mathsf{M}\boldsymbol{u}_s \neq \mathbf{0}_c$$

then, we can easily project $\boldsymbol{u}_s$

$$\mathsf{M}\boldsymbol{u}_s = \mathsf{M}(\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c) = \mathbf{0}_c$$

Finally, this leads to a Poisson eq.

$$\mathsf{M}\mathsf{G}\boldsymbol{p}_c = \mathsf{M}\boldsymbol{u}_s$$

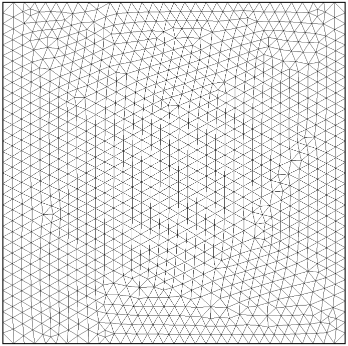


$$\text{If } \Omega_s \mathsf{G} = -\mathsf{M}^T \implies \langle \boldsymbol{u}_s, \mathsf{G}\boldsymbol{p}_c \rangle_h = \boldsymbol{u}_s^T \Omega_s \mathsf{G}\boldsymbol{p}_c = -(\mathsf{M}\boldsymbol{u}_s)^T \boldsymbol{p}_c = 0$$

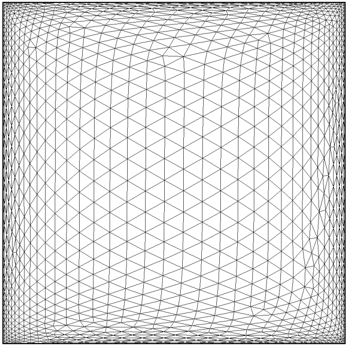
$$\langle \nabla\cdot\boldsymbol{a}, \varphi \rangle = -\langle \boldsymbol{a}, \nabla\varphi \rangle \implies \langle \boldsymbol{u}, \nabla p \rangle = -\langle \nabla\cdot\boldsymbol{u}, p \rangle = 0$$

But is this discrete Laplacian accurate?

Without stretching



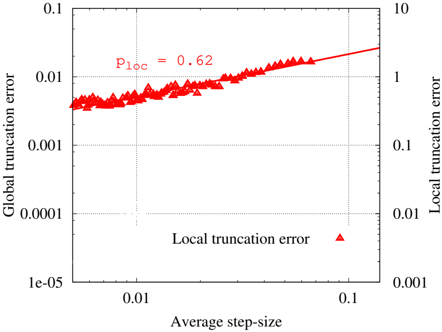
With stretching



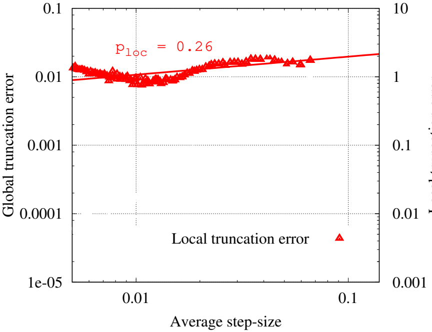
$$\nabla^2\varphi = f(x,y) \quad \text{with } f(x,y) = \nabla^2(k^{-2}\sin(kx)\sin(ky)) \text{ and } k = 25\pi$$

But is this discrete Laplacian accurate?

Without stretching



With stretching

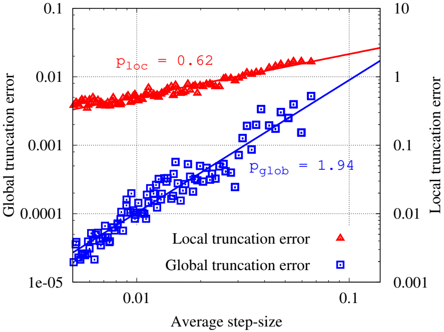


$$\nabla^2\varphi = f(x,y) \quad \text{with } f(x,y) = \nabla^2(k^{-2}\sin(kx)\sin(ky)) \text{ and } k = 25\pi$$

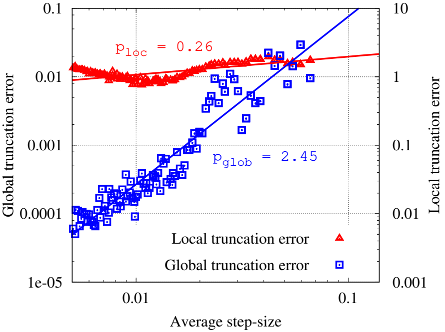
But is this discrete Laplacian accurate?

Yes, even for distorted unstructured meshes! And symmetries are preserved!

Without stretching



With stretching



$$\nabla^2 \varphi = f(x,y) \quad \text{with } f(x,y) = \nabla^2 (k^{-2} \sin(kx) \sin(ky)) \text{ and } k = 25\pi$$

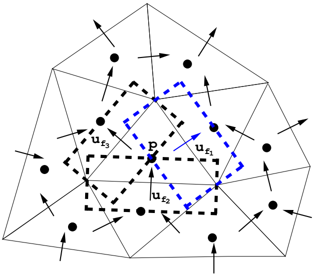
Then, why collocated arrangements are so popular?

- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM

$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_s = \mathsf{D}\boldsymbol{u}_s - \mathsf{G}\boldsymbol{p}_c; \quad \mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c$$

In staggered meshes

- p-$\boldsymbol{u}_s$ coupling is naturally solved ✓
- $\mathsf{C}\left(\boldsymbol{u}_s\right)$ and D difficult to discretize ✗



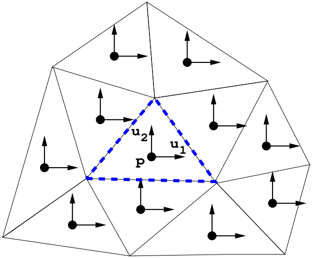
Then, why collocated arrangements are so popular?

- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM

$$\Omega_c \frac{d\boldsymbol{u}_c}{dt} + \mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_c = \mathsf{D}\boldsymbol{u}_c - \mathsf{G}_c\boldsymbol{p}_c; \quad \mathsf{M}_c\boldsymbol{u}_c = \boldsymbol{0}_c$$

In <u>collocated meshes</u>

- p-$\boldsymbol{u}_c$ coupling is cumbersome ✗
- $\mathsf{C}\left(\boldsymbol{u}_s\right)$ and D easy to discretize ✓
- Cheaper, less memory,... ✓



Then, why collocated arrangements are so popular?

Everything is easy except the pressure-velocity coupling...

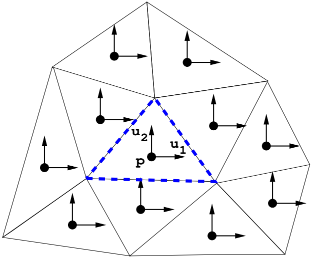
- STAR-CCM+ — CD-adapco, SIEMENS
- ANSYS-FLUENT — ANSYS FLUENT

- Code-Saturne — CODE SATURNE, eDF, GPL
- OpenFOAM — Open∇FOAM®, GPL v3 Free Software

$$\Omega_c \frac{d\boldsymbol{u}_c}{dt} + \mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_c = \mathsf{D}\boldsymbol{u}_c - \mathsf{G}_c \boldsymbol{p}_c ; \quad \mathsf{M}_c \boldsymbol{u}_c = \boldsymbol{0}_c$$

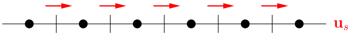
In collocated meshes

- p-$\boldsymbol{u}_c$ coupling is cumbersome ✗
- $\mathsf{C}\left(\boldsymbol{u}_s\right)$ and D easy to discretize ✓
- Cheaper, less memory,... ✓



Pressure-velocity coupling on staggered grids

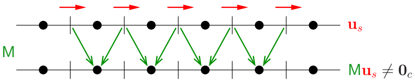
Works perfectly!



$\boldsymbol{u_s}$

Pressure-velocity coupling on staggered grids

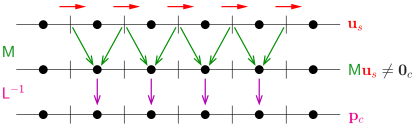
Works perfectly!



$\mathsf{M}\boldsymbol{u_s}$

Pressure-velocity coupling on staggered grids

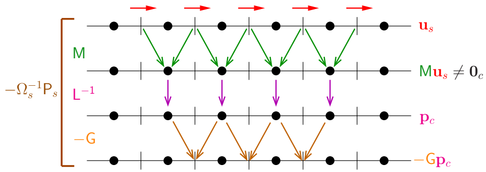
Works perfectly!



$$\underbrace{\mathsf{L}^{-1}\mathsf{M}\boldsymbol{u}_s}_{\boldsymbol{p}_c}$$

Pressure-velocity coupling on staggered grids

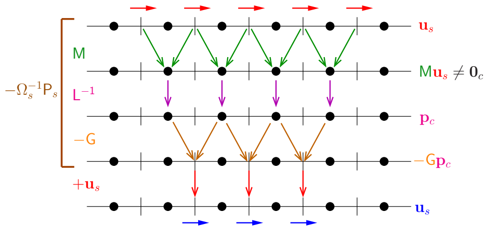
Works perfectly!



$$-\mathsf{G}\,\underbrace{\mathsf{L}^{-1}\mathsf{M}\boldsymbol{u_s}}_{\boldsymbol{p_c}}$$

Pressure-velocity coupling on staggered grids

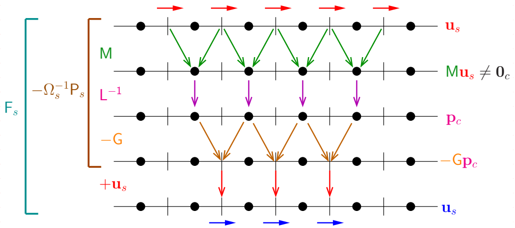
Works perfectly!



$$\boldsymbol{u}_s = \boldsymbol{u}_s - \mathsf{G}\underbrace{\mathsf{L}^{-1}\mathsf{M}\boldsymbol{u}_s}_{\boldsymbol{p}_c}$$

Pressure-velocity coupling on staggered grids

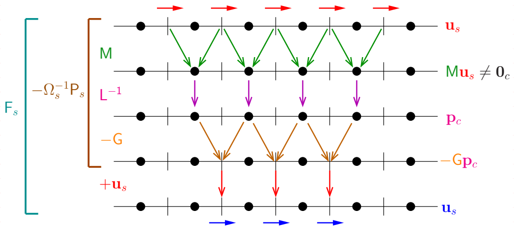
Works perfectly!



$$\boldsymbol{u}_s = \boldsymbol{u}_s - \mathsf{G}\underbrace{\mathsf{L}^{-1}\mathsf{M}\boldsymbol{u}_s}_{\boldsymbol{p}_c} = (I - \Omega_s^{-1}\mathsf{P}_s)\boldsymbol{u}_s = \mathsf{F}_s\boldsymbol{u}_s$$

Pressure-velocity coupling on staggered grids

Works perfectly!

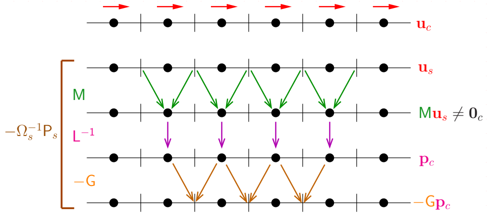


$$\boldsymbol{u}_s = \boldsymbol{u}_s - \mathsf{G}\underbrace{\mathsf{L}^{-1}\mathsf{M}\boldsymbol{u}_s}_{\boldsymbol{p}_c} = (I - \Omega_s^{-1}\mathsf{P}_s)\boldsymbol{u}_s = \mathsf{F}_s\boldsymbol{u}_s$$

$$\mathsf{M}\boldsymbol{u}_s = \mathsf{M}\boldsymbol{u}_s - \underbrace{\mathsf{M}\mathsf{G}}_{\mathsf{L}}\mathsf{L}^{-1}\mathsf{M}\boldsymbol{u}_s = \mathbf{0}_c$$

Pressure-velocity coupling on collocated grids

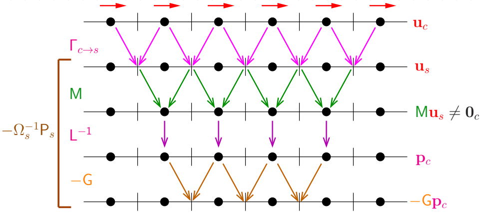
A vicious circle that cannot be broken...



$\boldsymbol{u}_c$

Pressure-velocity coupling on collocated grids

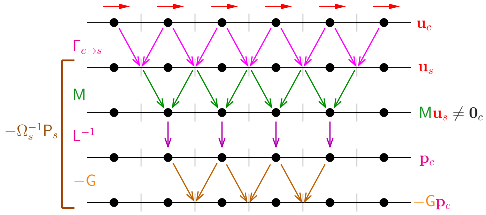
A vicious circle that cannot be broken...



$\Gamma_{c\to s}\boldsymbol{u}_c$

Pressure-velocity coupling on collocated grids

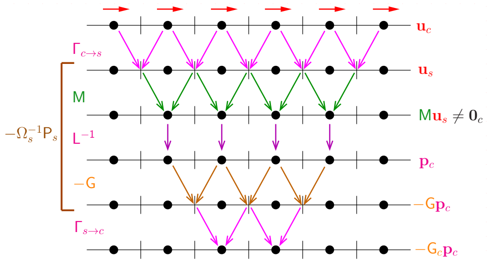
A vicious circle that cannot be broken...



$$\mathsf{G}\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c$$

Pressure-velocity coupling on collocated grids

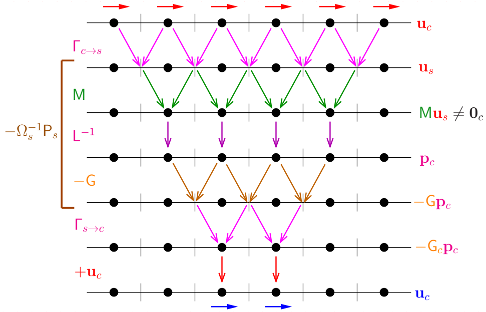
A vicious circle that cannot be broken...



$$-\Gamma_{s\to c}\mathsf{G}\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c$$

Pressure-velocity coupling on collocated grids

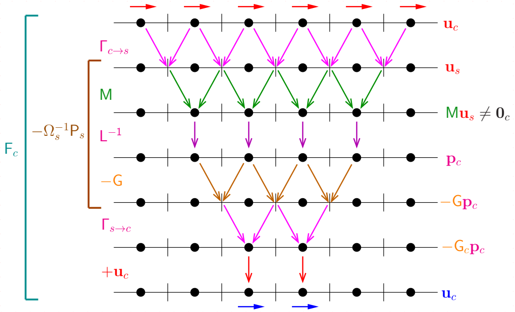
A vicious circle that cannot be broken...



$$\boldsymbol{u}_c = \boldsymbol{u}_c - \Gamma_{s\to c}\mathsf{G}\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c$$

Pressure-velocity coupling on collocated grids

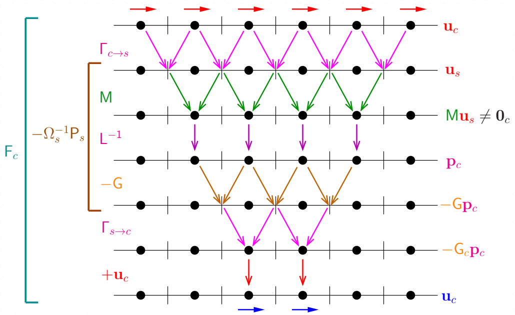
A vicious circle that cannot be broken...



$$\boldsymbol{u}_c = \boldsymbol{u}_c - \Gamma_{s\to c}\mathsf{G}\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c = \mathsf{F}_c\boldsymbol{u}_c$$

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...



$$\boldsymbol{u}_c = \boldsymbol{u}_c - \Gamma_{s\to c}\mathsf{G}\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c = \mathsf{F}_c\boldsymbol{u}_c$$

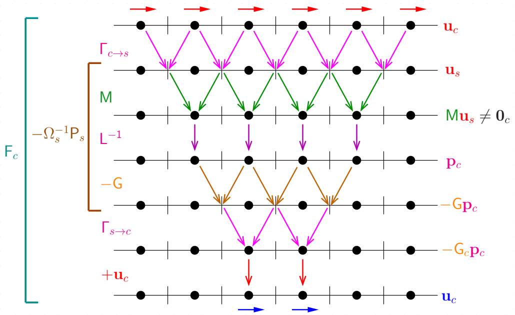
To preserve symmetry we impose $\Gamma_{s\to c} = \Omega_c^{-1}\Gamma_{c\to s}^{T}\Omega_s$. This leads to

$$\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c = \mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c - \mathsf{L}_c\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c \approx \mathbf{0}_c \text{ ✗}$$

where $\mathsf{L}_c = -\mathsf{M}\Gamma_{c\to s}\Omega_c^{-1}\Gamma_{c\to s}^{T}\mathsf{M}$ (wide-stencil discrete Laplacian).

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...



$$\boldsymbol{u}_c = \boldsymbol{u}_c - \Gamma_{s\to c}\mathsf{G}\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c = \mathsf{F}_c\boldsymbol{u}_c$$

To preserve symmetry we impose $\Gamma_{s\to c} = \Omega_c^{-1}\Gamma_{c\to s}^T\Omega_s$. This leads to

$$\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c = \mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c - \mathsf{L}_c\mathsf{L}^{-1}\mathsf{M}\Gamma_{c\to s}\boldsymbol{u}_c \approx \mathbf{0}_c \text{ ✗}$$

where $\mathsf{L}_c = -\mathsf{M}\Gamma_{c\to s}\Omega_c^{-1}\Gamma_{c\to s}^T\mathsf{M}$ (wide-stencil discrete Laplacian).

Moreover, **contribution to kinetic enegy**: $\boldsymbol{p}_c(\mathsf{L} - \mathsf{L}_c)\boldsymbol{p}_c \neq 0$ ✗

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $M\Gamma_{c\rightarrow s}\boldsymbol{u}_c = M\Gamma_{c\rightarrow s}\boldsymbol{u}_c - \mathsf{L}_c\mathsf{L}^{-1} M\Gamma_{c\rightarrow s}\boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c \left(\mathsf{L} - \mathsf{L}_c\right) \boldsymbol{p}_c \neq 0$ ✗

[8] F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen. *Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured grids*, **Journal of Computational Physics**, 258 (1): 246-267, 2014.

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $\mathsf{M}\Gamma_{c \to s} \boldsymbol{u}_c = \mathsf{M}\Gamma_{c \to s} \boldsymbol{u}_c - \boxed{\mathsf{L}_c \mathsf{L}^{-1}} \mathsf{M}\Gamma_{c \to s} \boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c \boxed{(\mathsf{L} - \mathsf{L}_c)} \boldsymbol{p}_c \neq 0$ ✗

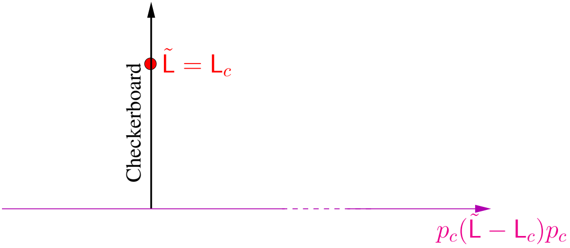
[8] F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen. *Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured grids*, **Journal of Computational Physics**, 258 (1): 246-267, 2014.

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $\mathsf{M}\Gamma_{c\rightarrow s}\boldsymbol{u}_c = \mathsf{M}\Gamma_{c\rightarrow s}\boldsymbol{u}_c - \boxed{\mathsf{L}_c\mathsf{L}^{-1}}\mathsf{M}\Gamma_{c\rightarrow s}\boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c\boxed{(\mathsf{L}-\mathsf{L}_c)}\boldsymbol{p}_c \neq 0$ ✗



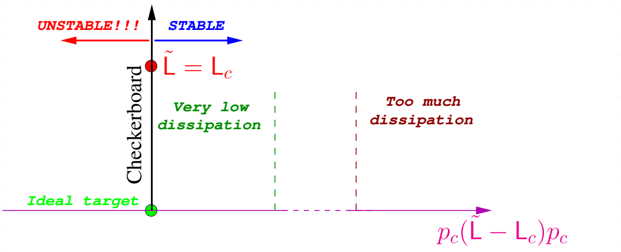
[8]F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen. *Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured grids*, **Journal of Computational Physics**, 258 (1): 246-267, 2014.

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $M\Gamma_{c\to s}\boldsymbol{u}_c = M\Gamma_{c\to s}\boldsymbol{u}_c - \boxed{\mathsf{L}_c\mathsf{L}^{-1}} M\Gamma_{c\to s}\boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c \boxed{(\mathsf{L}-\mathsf{L}_c)} \boldsymbol{p}_c \neq 0$ ✗



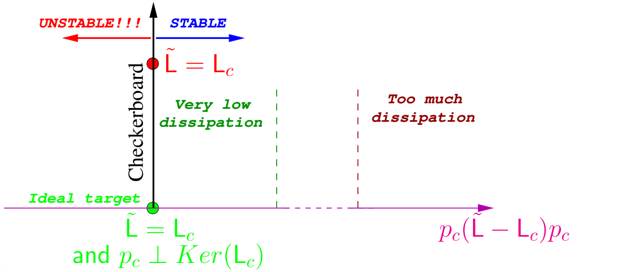
[8] F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen. *Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured grids*, **Journal of Computational Physics**, 258 (1): 246-267, 2014.

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $M\Gamma_{c\rightarrow s}\boldsymbol{u}_c = M\Gamma_{c\rightarrow s}\boldsymbol{u}_c - \boxed{L_c L^{-1}} M\Gamma_{c\rightarrow s}\boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c \boxed{(L - L_c)} \boldsymbol{p}_c \neq 0$ ✗



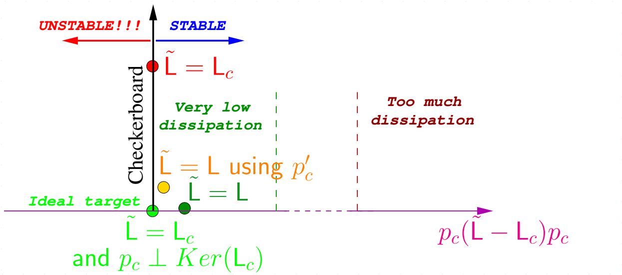
[8] Shashank, J.Larsson, G.Iaccarino. *A co-located incompressible Navier-Stokes solver with exact mass, momentum and kinetic energy conservation in the inviscid limit*, **Journal of Computational Physics**, 229: 4425-4430,2010.

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

- Mass: $M\Gamma_{c\to s}\boldsymbol{u}_c = M\Gamma_{c\to s}\boldsymbol{u}_c - \boxed{L_c L^{-1}} M\Gamma_{c\to s}\boldsymbol{u}_c \approx \mathbf{0}_c$ ✗
- Energy: $\boldsymbol{p}_c \boxed{(L - L_c)} \boldsymbol{p}_c \neq 0$ ✗



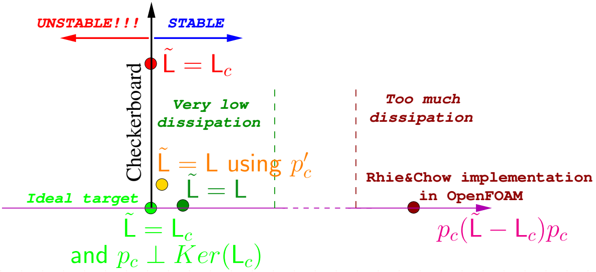
[8] F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen. *Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured grids*, **Journal of Computational Physics**, 258 (1): 246-267, 2014.

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary[8]:

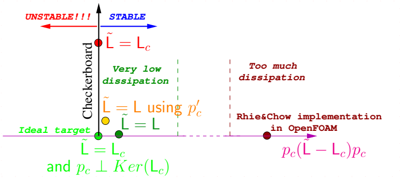
- Mass: $M\Gamma_{c\rightarrow s}\boldsymbol{u}_c = M\Gamma_{c\rightarrow s}\boldsymbol{u}_c - \boxed{L_c L^{-1}} M\Gamma_{c\rightarrow s}\boldsymbol{u}_c \approx \boldsymbol{0}_c$ ✗
- Energy: $\boldsymbol{p}_c \boxed{(L - L_c)} \boldsymbol{p}_c \neq 0$ ✗

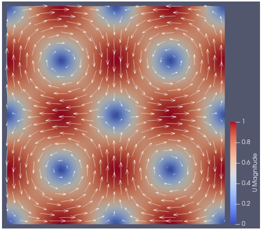


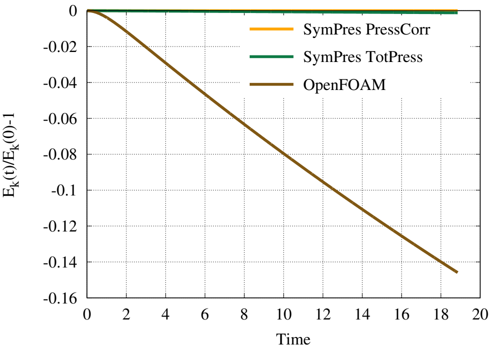
[8] E.Komen, J.A.Hopman, E.M.A.Frederix, F.X.Trias, R.W.C.P.Verstappen. "A symmetry-preserving second-order time-accurate PISO-based method". **Computers & Fluids**, (accepted).

Pressure-velocity coupling on collocated grids

A vicious circle that ~~cannot be broken~~ can almost be broken...





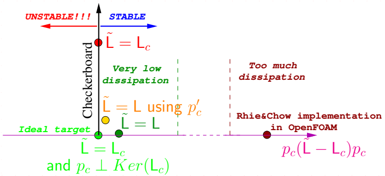


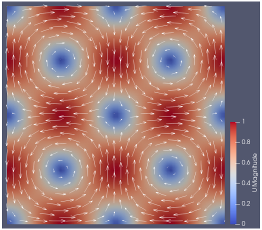
Results for an inviscid Taylor-Green vortex[9]

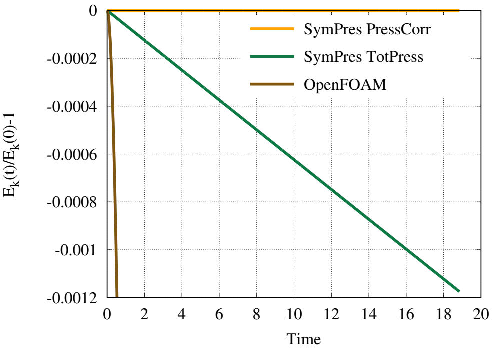
[9] E.Komen, J.A.Hopman, E.M.A.Frederix, F.X.Trias, R.W.C.P.Verstappen. "A symmetry-preserving second-order time-accurate PISO-based method". **Computers & Fluids**, (accepted).

Pressure-velocity coupling on collocated grids

A vicious circle that ~~cannot be broken~~ can almost be broken...







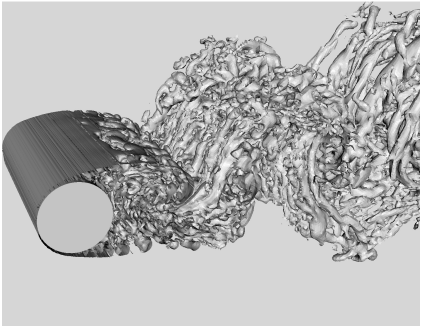
Results for an inviscid Taylor-Green vortex[9]

[9] E.Komen, J.A.Hopman, E.M.A.Frederix, F.X.Trias, R.W.C.P.Verstappen. "A symmetry-preserving second-order time-accurate PISO-based method". **Computers & Fluids**, (accepted).

Pressure-velocity coupling on collocated grids

Examples of simulations

Despite these inherent limitations, symmetry-preserving collocated formulation has been successfully used for DNS/LES simulations[10]:

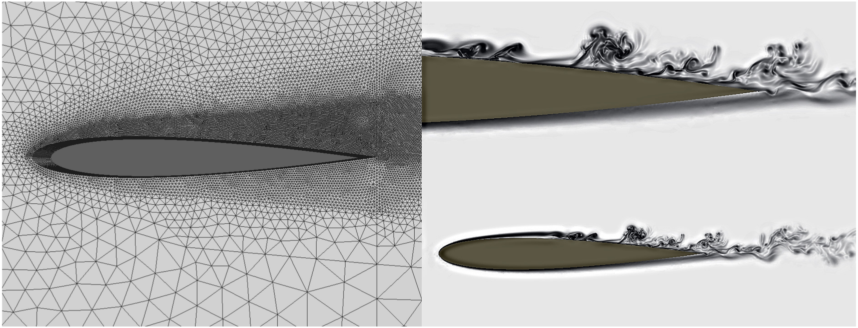


[10] R.Borrell, O.Lehmkuhl, F.X.Trias, A.Oliva. *Parallel Direct Poisson solver for discretizations with one Fourier diagonalizable direction*. **Journal of Computational Physics**, 230:4723-4741, 2011.

Pressure-velocity coupling on collocated grids

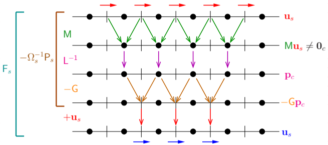
Examples of simulations

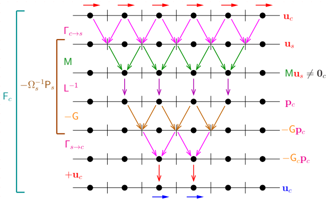
Despite these inherent limitations, symmetry-preserving collocated formulation has been successfully used for DNS/LES simulations[10]:



[10] F.X.Trias and O.Lehmkuhl. *A self-adaptive strategy for the time-integration of Navier-Stokes equations*. **Numerical Heat Transfer, part B**, 60(2):116-134, 2011.

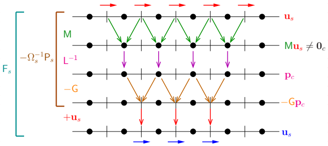
Are staggered and collocated so different at the end?

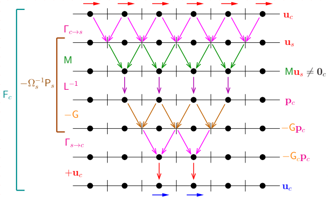




Collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\left(I_c - \Gamma_{s\to c}\Omega_s^{-1}\mathsf{P}_s\Gamma_{c\to s}\right)}_{\mathsf{F}_c} \overbrace{\underbrace{\left[I_c + \partial_t^c\right]}_{\mathsf{T}_c}\boldsymbol{u}_c^n}^{\boldsymbol{u}_c^p} = \underbrace{\mathsf{F}_c\mathsf{T}_c}_{NS_c}\boldsymbol{u}_c^n$$

Are staggered and collocated so different at the end?

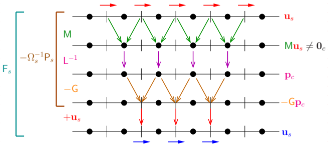


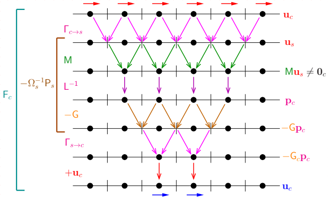


Collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\left(I_c - \Gamma_{s\to c}\Omega_s^{-1}\mathsf{P}_s\Gamma_{c\to s}\right)}_{\mathsf{F}_c} \overbrace{\underbrace{\left[I_c + \partial_t^c\right]}_{\mathsf{T}_c} \boldsymbol{u}_c^n}^{\boldsymbol{u}_c^p} = \underbrace{\mathsf{F}_c\mathsf{T}_c}_{NS_c} \boldsymbol{u}_c^n$$

Naïve collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\Gamma_{s\to c}\mathsf{F}_s\Gamma_{c\to s}\mathsf{T}_c}_{\widetilde{NS}_c} \boldsymbol{u}_c^n$$

Are staggered and collocated so different at the end?



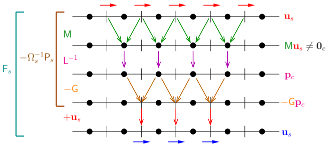


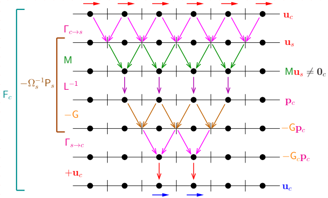
Collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\left(I_c - \Gamma_{s\to c}\Omega_s^{-1}\mathsf{P}_s\Gamma_{c\to s}\right)}_{\mathsf{F}_c} \overbrace{\underbrace{\left[I_c + \partial_t^c\right]}_{\mathsf{T}_c} \boldsymbol{u}_c^n}^{\boldsymbol{u}_c^p} = \underbrace{\mathsf{F}_c\mathsf{T}_c}_{NS_c} \boldsymbol{u}_c^n$$

Naïve collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\Gamma_{s\to c}\mathsf{F}_s\Gamma_{c\to s}\mathsf{T}_c}_{\widetilde{NS}_c} \boldsymbol{u}_c^n$$

Naïve staggered: $$\boldsymbol{u}_s^{n+1} = \underbrace{\mathsf{F}_s\Gamma_{c\to s}\mathsf{T}_c\Gamma_{s\to c}}_{\widetilde{NS}_s} \boldsymbol{u}_s^n$$

Are staggered and collocated so different at the end?





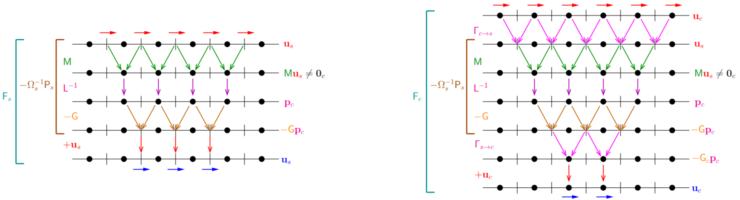
Collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\left(I_c - \Gamma_{s\to c}\Omega_s^{-1}\mathsf{P}_s\Gamma_{c\to s}\right)}_{\mathsf{F}_c}\underbrace{\overbrace{\left[I_c + \partial_t^c\right]\boldsymbol{u}_c^n}^{\boldsymbol{u}_c^p}}_{\mathsf{T}_c} = \underbrace{\mathsf{F}_c\mathsf{T}_c}_{NS_c}\boldsymbol{u}_c^n$$

Naïve collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\Gamma_{s\to c}\mathsf{F}_s\Gamma_{c\to s}\mathsf{T}_c}_{\widetilde{NS}_c}\boldsymbol{u}_c^n$$

Naïve staggered: $$\boldsymbol{u}_s^{n+1} = \underbrace{\mathsf{F}_s\Gamma_{c\to s}\mathsf{T}_c\Gamma_{s\to c}}_{\widetilde{NS}_s}\boldsymbol{u}_s^n$$

$$\boxed{\Gamma_{s\to c}(\widetilde{NS}_s)^n = (\widetilde{NS}_c)^n\Gamma_{c\to s}}$$

Are staggered and collocated so different at the end?



Collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\left(I_c - \Gamma_{s\to c}\Omega_s^{-1}\mathsf{P}_s\Gamma_{c\to s}\right)}_{\mathsf{F}_c} \overbrace{\underbrace{\left[I_c + \partial_t^c\right]}_{\mathsf{T}_c} \boldsymbol{u}_c^n}^{\boldsymbol{u}_c^p} = \underbrace{\mathsf{F}_c\mathsf{T}_c}_{NS_c} \boldsymbol{u}_c^n$$

Staggered: $$\boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c\Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$

Can we have a staggered formulation based only on collocated operators?

Then, it could be easily implemented in existing collocated codes such as OpenFOAM

$$\text{Staggered:} \quad \mathbf{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \mathbf{u}_s}^{\mathbf{u}_s^p}$$

Similar approaches have been proposed in the literature before[11,12,13,14,15].

[11] B.Perot. *Conservative properties of unstructured staggered meshs chemes*. **Journal of Comp. Physics**, 159: 58-89, 2000

[12] X.Zhang, D.Schmidt, B.Perot. *Accuracy and conservation properties of a three-dimensional unstructured staggered mesh scheme for fluid dynamics*. **Journal of Computational Physics**, 175: 764-791, 2002.

[13] K.Mahesh, G.Constantinescu, P.Moin. *A numerical method for large-eddy simulation in complex geometries*. **Journal of Computational Physics**,197: 215-240, 2004.

[14] J.E.Hicken, F.E.Ham, J.Militzer, M.Koksal. *A shift transformation for fully conservative methods: turbulence simulation on complex, unstructured grids*. **Journal of Computational Physics**, 208:704-734, 2005.

[15] L.Jofre, O.Lehmkuhl, J.Ventosa, F.X.Trias, A.Oliva. *Conservation properties of unstructured finite-volume mesh schemes for the Navier-Stokes equations*. **Numerical Heat Transfer, Part B**, 65:1-27, 2014.

Can we have a staggered formulation based only on collocated operators?

Then, it could be easily implemented in existing collocated codes such as OpenFOAM

$$\text{Staggered:} \quad \mathbf{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \mathbf{u}_s}^{\mathbf{u}_s^p}$$

Similar approaches have been proposed in the literature before[11,12,13,14,15].

Research question: then, why at the end collocated approach seems to be the winner?

- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM

[11] B.Perot. *Conservative properties of unstructured staggered meshs chemes*. **Journal of Comp. Physics**, 159: 58-89, 2000

[12] X.Zhang, D.Schmidt, B.Perot. *Accuracy and conservation properties of a three-dimensional unstructured staggered mesh scheme for fluid dynamics*. **Journal of Computational Physics**, 175: 764-791, 2002.

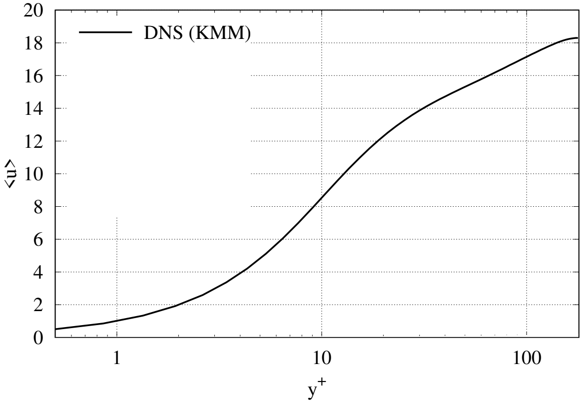
[13] K.Mahesh, G.Constantinescu, P.Moin. *A numerical method for large-eddy simulation in complex geometries*. **Journal of Computational Physics**,197: 215-240, 2004.

[14] J.E.Hicken, F.E.Ham, J.Militzer, M.Koksal. *A shift transformation for fully conservative methods: turbulence simulation on complex, unstructured grids*. **Journal of Computational Physics**, 208:704-734, 2005.

[15] L.Jofre, O.Lehmkuhl, J.Ventosa, F.X.Trias, A.Oliva. *Conservation properties of unstructured finite-volume mesh schemes for the Navier-Stokes equations*. **Numerical Heat Transfer, Part B**, 65:1-27, 2014.

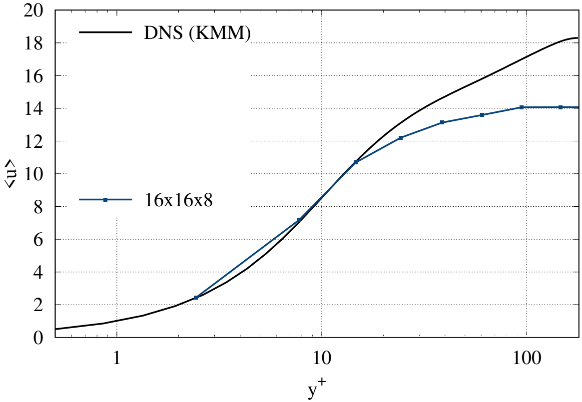
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^0: $\quad \boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1} \mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$



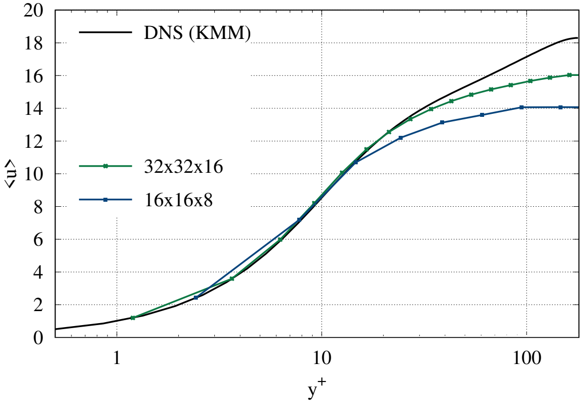
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^0: $$\boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1} \mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s} \partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



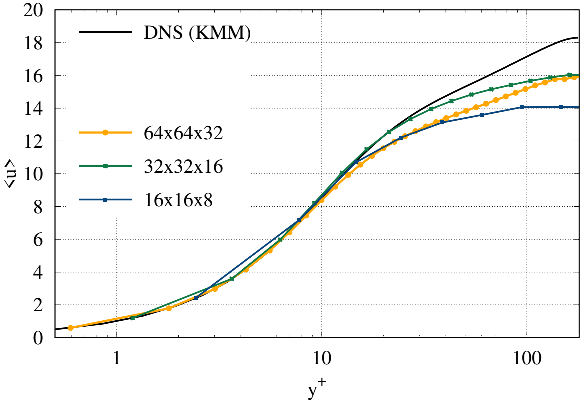
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^0: $\quad \boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1} \mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s} \partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$



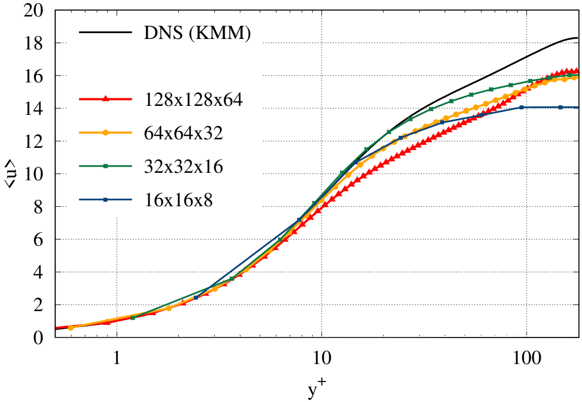
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^0: $$\boldsymbol{u}_s^{n+1} = \underbrace{(I_s - \Omega_s^{-1} \mathsf{P}_s)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



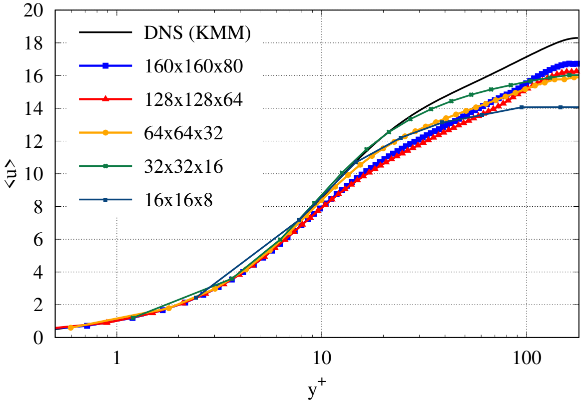
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^0: $$\boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1} \mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s} \partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



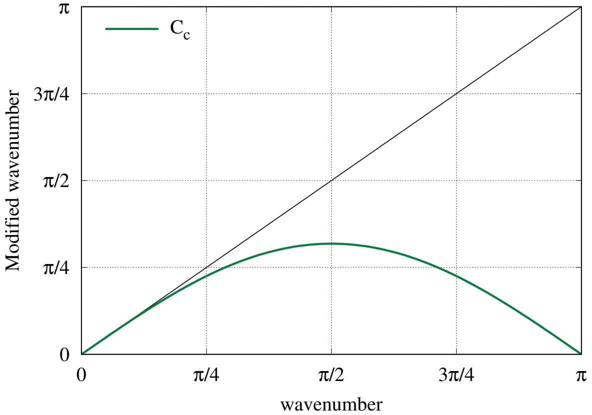
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C^0_s: $$\mathbf{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1} \mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \mathbf{u}_s}^{\mathbf{u}_s^p}$$



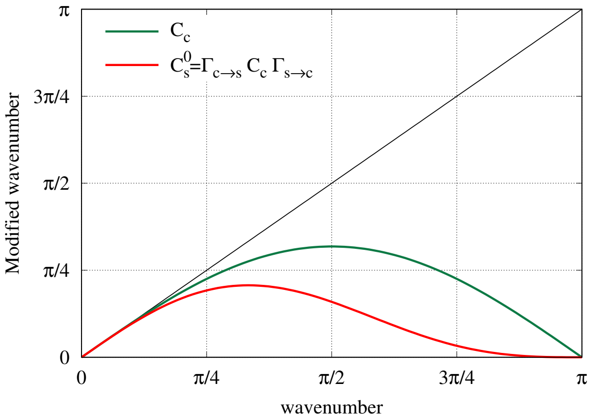
Dispersion errors analysis

Staggered C_s^0: $$\boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1} \mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



Dispersion errors analysis

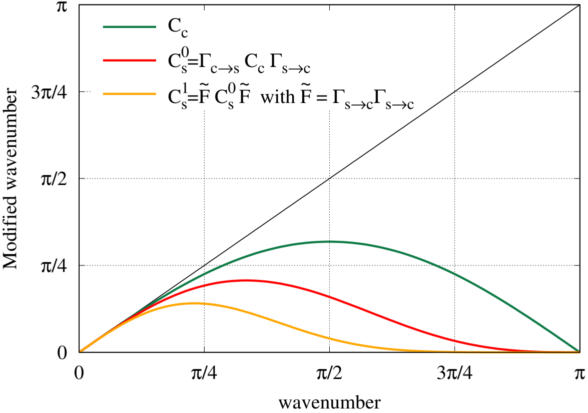
$$\text{Staggered } C_s^0: \quad \boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \quad \Gamma_{c\to s}\partial_t^c \Gamma_{s\to c} \quad\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



Dispersion errors analysis

Staggered C_s^1: $$\boldsymbol{u}_s^{n+1} = \underbrace{(I_s - \Omega_s^{-1}\mathsf{P}_s)}_{\mathsf{F}_s} \underbrace{\overbrace{[I_s + \widetilde{\mathsf{F}}\Gamma_{c\rightarrow s}\partial_t^c\Gamma_{s\rightarrow c}\widetilde{\mathsf{F}}\ ]}^{\boldsymbol{u}_s^p}}_{\mathsf{T}_s} \boldsymbol{u}_s$$

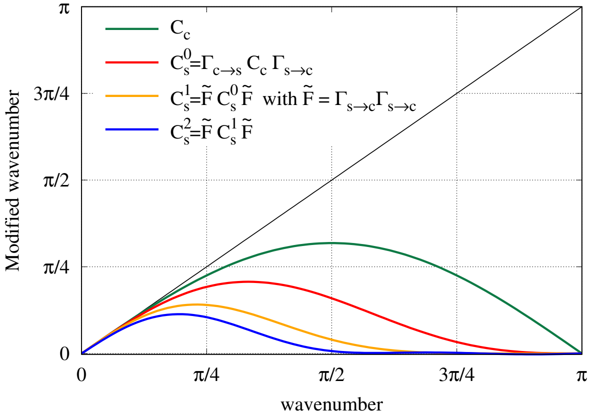
Filter: $\widetilde{\mathsf{F}} = \Gamma_{s\rightarrow c}\Gamma_{c\rightarrow s}$ ($\widetilde{\mathsf{F}} = \widetilde{\mathsf{F}}^T$ and positive semi-definite)



Dispersion errors analysis

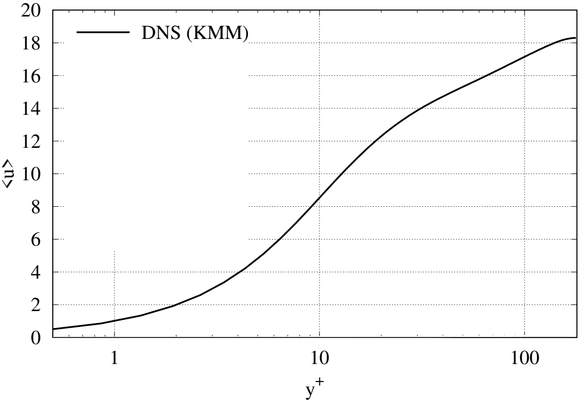
Staggered C_s^2: $$\boldsymbol{u}_s^{n+1} = \underbrace{(I_s - \Omega_s^{-1}\mathsf{P}_s)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\Gamma_{c\rightarrow s}\partial_t^c\Gamma_{s\rightarrow c}\widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$

Filter: $\widetilde{\mathsf{F}} = \Gamma_{s\rightarrow c}\Gamma_{c\rightarrow s}$ ($\widetilde{\mathsf{F}} = \widetilde{\mathsf{F}}^T$ and positive semi-definite)



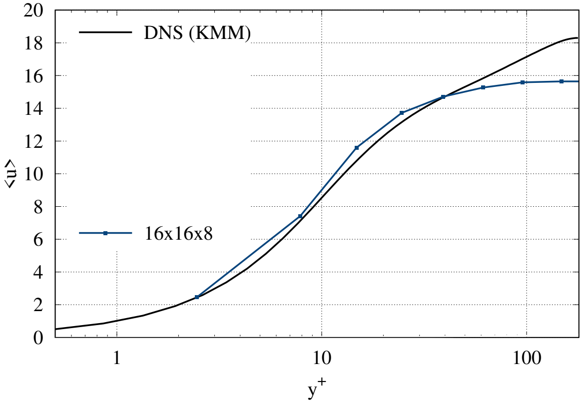
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^2: $$\mathbf{u}_s^{n+1} = \underbrace{(I_s - \Omega_s^{-1} \mathsf{P}_s)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \tilde{\mathsf{F}}\tilde{\mathsf{F}} \Gamma_{c \to s} \partial_t^c \Gamma_{s \to c} \tilde{\mathsf{F}}\tilde{\mathsf{F}} \right]}_{\mathsf{T}_s} \mathbf{u}_s}^{\mathbf{u}_s^p}$$



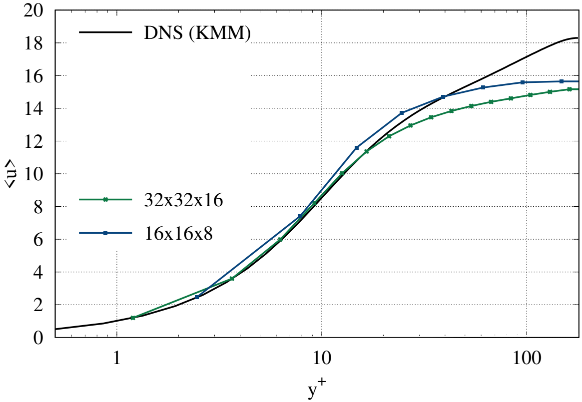
Results for a turbulent channel flow at $Re_\tau = 180$

$$\text{Staggered } C_s^2: \quad \boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\Gamma_{c\rightarrow s}\partial_t^c\Gamma_{s\rightarrow c}\widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



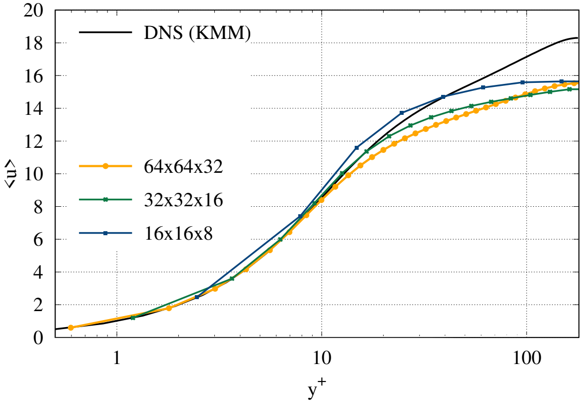
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^2: $$\boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\Gamma_{c\to s}\partial_t^c\Gamma_{s\to c}\widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



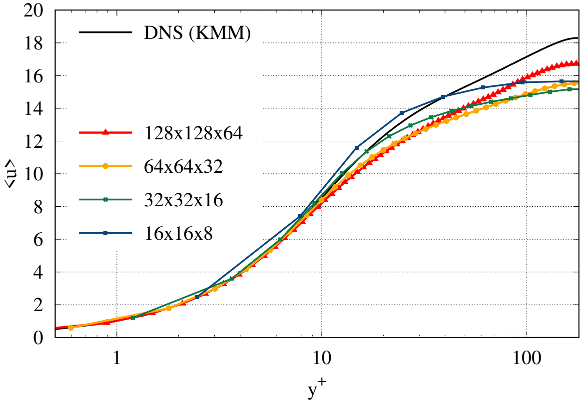
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^2: $\quad \mathbf{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\Gamma_{c\to s}\partial_t^c\Gamma_{s\to c}\widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\right]}_{\mathsf{T}_s} \mathbf{u}_s}^{\mathbf{u}_s^p}$



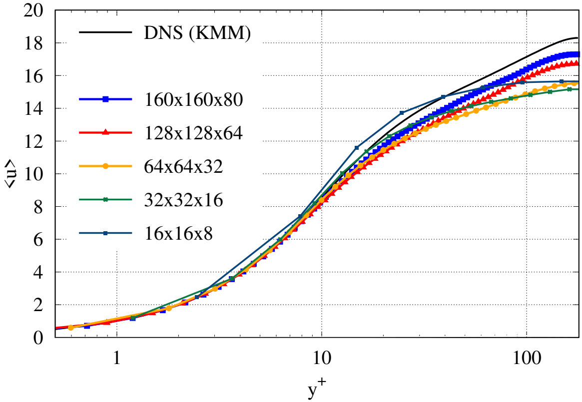
Results for a turbulent channel flow at $Re_\tau = 180$

$$\text{Staggered } C_s^2: \quad \boldsymbol{u}_s^{n+1} = \underbrace{(I_s - \Omega_s^{-1}\mathsf{P}_s)}_{\mathsf{F}_s} \overbrace{\underbrace{[I_s + \widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\widetilde{\mathsf{F}}\widetilde{\mathsf{F}}]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



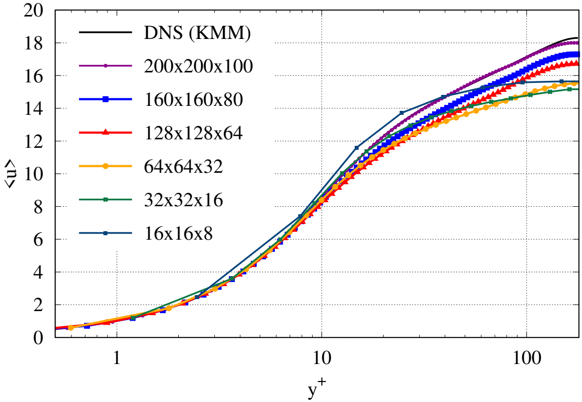
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered C_s^2: $$\boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\Gamma_{c\rightarrow s}\partial_t^c\Gamma_{s\rightarrow c}\widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\right]}_{\mathsf{T}_s}\boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



Results for a turbulent channel flow at $Re_\tau = 180$

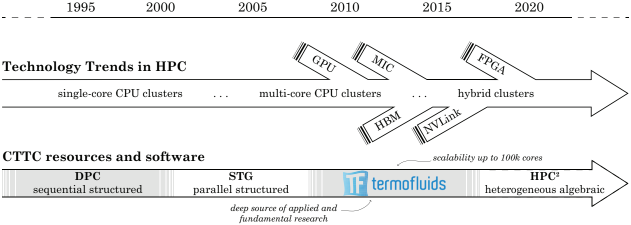
$$\text{Staggered } C_s^2: \quad \boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1} \mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\Gamma_{c\to s}\partial_t^c \Gamma_{s\to c}\widetilde{\mathsf{F}}\widetilde{\mathsf{F}}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$



Algebra-based approach naturally leads to portability

Research question #2:

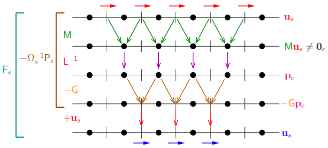
- How can we develop **portable** and **efficient** CFD codes for large-scale simulations on modern supercomputers?

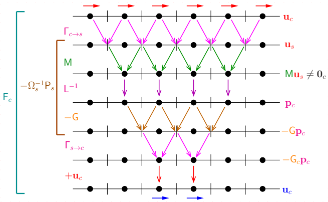


HPC^2: portable, algebra-based framework for heterogeneous computing is being developed. Traditional stencil-based data and sweeps are replaced by algebraic structures (sparse matrices and vectors) and kernels. NUMA-aware execution strategies for CFD are presented in this conference[16].

[16] X.Álvarez, A.Gorobets, F.X.Trias, A.Oliva. *NUMA-aware strategies for the efficient execution of CFD simulations on CPU supercomputers* **ParCFD2021**. Don't miss it!

Algebra-based approach naturally leads to portability, to simple and analyzable formulations

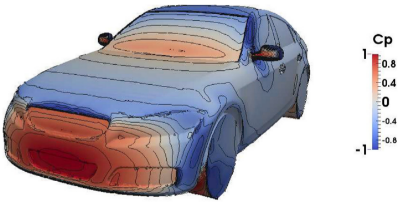


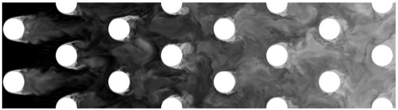


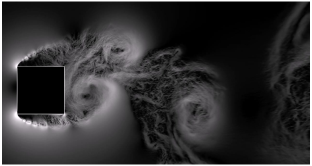
Collocated: $$\boldsymbol{u}_c^{n+1} = \underbrace{\left(I_c - \Gamma_{s\to c}\Omega_s^{-1}\mathsf{P}_s\Gamma_{c\to s}\right)}_{\mathsf{F}_c} \overbrace{\underbrace{\left[I_c + \partial_t^c\right]}_{\mathsf{T}_c} \boldsymbol{u}_c^n}^{\boldsymbol{u}_c^p} = \underbrace{\mathsf{F}_c\mathsf{T}_c}_{NS_c} \boldsymbol{u}_c^n$$

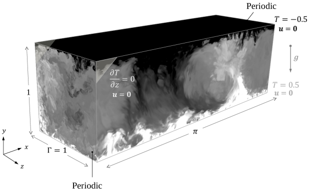
Staggered: $$\boldsymbol{u}_s^{n+1} = \underbrace{\left(I_s - \Omega_s^{-1}\mathsf{P}_s\right)}_{\mathsf{F}_s} \overbrace{\underbrace{\left[I_s + \Gamma_{c\to s}\partial_t^c\Gamma_{s\to c}\right]}_{\mathsf{T}_s} \boldsymbol{u}_s}^{\boldsymbol{u}_s^p}$$

Algebra-based approach naturally leads to portability, to simple and analyzable formulations and opens the door to new strategies[17] to improve its perfomance...



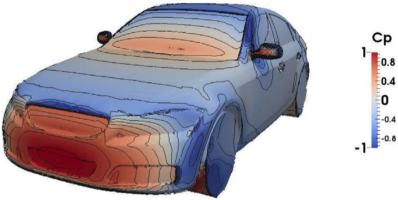


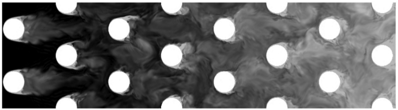




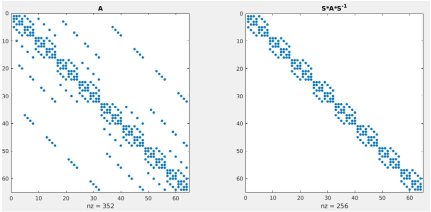
[17] A.Alsalti, X.Álvarez, F.X.Trias, A.Gorobets, A.Oliva. *A highly portable heterogeneous implementation of a Poisson solver for flows with one periodic direction* **ParCFD2021**. Don't miss it!

Algebra-based approach naturally leads to portability, to simple and analyzable formulations and opens the door to new strategies[17] to improve its perfomance...





$$\hat{\mathsf{L}} = \mathsf{S}\mathsf{L}\mathsf{S}^{-1} = \mathsf{I} \otimes \mathsf{L}_{inn} + diag(\boldsymbol{d})$$

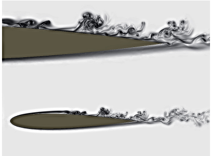


SpMMV can be used $\Longrightarrow$ **higher AI**

[17] A.Alsalti, X.Álvarez, F.X.Trias, A.Gorobets, A.Oliva. *A highly portable heterogeneous implementation of a Poisson solver for flows with one periodic direction* **ParCFD2021**. Don't miss it!

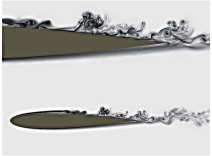
Concluding remarks

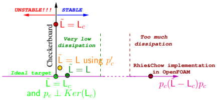
- **Preserving symmetries** either using staggered or collocated formulations is the key point for **reliable LES/DNS** simulations.



Concluding remarks

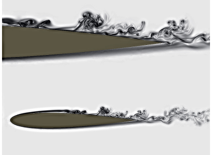
- **Preserving symmetries** either using staggered or collocated formulations is the key point for **reliable LES/DNS** simulations.
- Main drawback of **collocated** formulations: you either have **checkerboard** or some (small) amount of **artificial dissipation** due to pressure term.

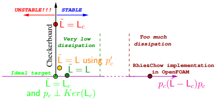




Concluding remarks

- **Preserving symmetries** either using staggered or collocated formulations is the key point for **reliable LES/DNS** simulations.
- Main drawback of **collocated** formulations: you either have **checkerboard** or some (small) amount of **artificial dissipation** due to pressure term.
- Despite this, the CFD community have generally adopted collocated formulations due to the inherent **difficulties** to formulate a simple and robust **staggered discretization of momentum**.
  $\Longrightarrow$ A potential solution has been presented here...

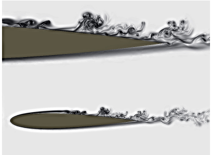


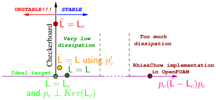


- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM

Concluding remarks

- **Preserving symmetries** either using staggered or collocated formulations is the key point for **reliable LES/DNS** simulations.
- Main drawback of **collocated** formulations: you either have **checkerboard** or some (small) amount of **artificial dissipation** due to pressure term.
- Despite this, the CFD community have generally adopted collocated formulations due to the inherent **difficulties** to formulate a simple and robust **staggered discretization of momentum**.
  $\Longrightarrow$ A potential solution has been presented here...





- STAR-CCM+
- ANSYS-FLUENT
- Code-Saturne
- OpenFOAM

On-going research:

- Complete the analysis for higher Re_τ
- Test for complex geometries using unstructured meshes

Thank you for your virtual attendance