On a checkerboard-free, conservative method for turbulent flows

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Abstract

Central-difference discretisation applied to collocated grids result in a wide-stencil Laplacian that can lead to oscillatory pressure modes, known as the checkerboard problem. Rhie-Chow like corrections and a compact-stencil Laplacian solve this problem at the cost of numerical dissipation. To find a conservative solution, a quantification method was derived that sheds more light onto the origins of the problem. The quantification method returns a global non-dimensional normalised coefficient, independent of time-step, C_{cb}, also able to detect local oscillations which lie outside of the kernel of the discrete wide-stencil Laplacian. C_{cb} was used to self-regulate the pressure predictor through negative feedback, since it is known to be a cause of checkerboarding. The quantification method predicted levels of checkerboarding consistent with qualitative results of a turbulent channel flow at $Re_\tau = 180$. Moreover, the levels of checkerboarding were successfully self-regulated to diminish oscillations at similar accuracy. Through this quantification method, other new methods could be developed that mitigate oscillations only when necessary, leading to more conservative solutions to the checkerboard problem.

Equations

The fractional step method that leads to checkerboarding and two ways to avoid it are given in table 1. The pressure error is caused by the divergence in the velocity fields found in the oscillation-free methods result, which are non-zero and proportional to $\Delta t(L\mathbf{p}_c - L_c\mathbf{p}_c)$.

Table 1: Occurrence of and solutions to checkerboarding in the fractional step method. Notation from [1].

<table>
<tr><th>Wide-stencil</th><th>Wide & Rhie-Chow</th><th>Compact-stencil</th></tr>
<tr><td colspan="3">$\mathbf{u}_c^p = R(\mathbf{u}_c^n, \mathbf{u}_s^n) - G_c\tilde{\mathbf{p}}_c^p$</td></tr>
<tr><td colspan="2">$L_c\tilde{\mathbf{p}}_c' = M\Gamma_{cs}\mathbf{u}_c^p$</td><td>$L\tilde{\mathbf{p}}_c' = M\Gamma_{cs}\mathbf{u}_c^p$</td></tr>
<tr><td colspan="3">$\mathbf{u}_c^{n+1} = \mathbf{u}_c^p - G_c\tilde{\mathbf{p}}_c'$</td></tr>
<tr><td>$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^{n+1}$</td><td colspan="2">$\mathbf{u}_s^{n+1} = \Gamma_{cs}\mathbf{u}_c^p - G\tilde{\mathbf{p}}_c'$</td></tr>
<tr><td>Checkerboarding</td><td>$M\mathbf{u}_s^{n+1} \neq \mathbf{0}_c$</td><td>$M\Gamma_{cs}\mathbf{u}_c^{n+1} \neq \mathbf{0}_c$</td></tr>
</table>

Checkerboard coefficient

Oscillations in the pressure field are not felt by the wide-stencil gradient, suggesting the kernel of L_c would be a good definition to quantify checkerboarding. However, local oscillations lie outside of the kernel, and should be accounted for [3]. A less strict definition is found by looking at the global pressure diffusion budget term:

$$-\mathbf{u}_c^T\Omega G_c\mathbf{p}_c = \mathbf{p}_c^T M_c\mathbf{u}_c = \Delta t\mathbf{p}_c^T(L - L_c)\mathbf{p}_c \in [\Delta t\mathbf{p}_c^T L\mathbf{p}_c, 0] \tag{1}$$

which is strictly dissipative. If we divide out $\Delta t\mathbf{p}_c^T L\mathbf{p}_c$, we find:

$$C_{cb} = 1 - \frac{\mathbf{p}_c^T L_c\mathbf{p}_c}{\mathbf{p}_c^T L\mathbf{p}_c} = 1 - \frac{\mathbf{p}_c^T G_c\Omega G_c\mathbf{p}_c}{\mathbf{p}_c^T G\Omega_s G\mathbf{p}_c} = 1 - \frac{||G_c\mathbf{p}_c||}{||G\mathbf{p}_c||} \in [0, 1] \tag{2}$$

Which nicely gives a global non-dimensional normalised coefficient, independent of time-step, with $C_{cb} = 1$ for pure oscillatory fields fully inside $Ker(L_c)$. One common cause of checkerboarding is the inclusion of a pressure predictor, which can be weighted by a non-dimensional coefficient as:

$$\mathbf{p}_c^p = \theta_p\mathbf{p}_c^n \tag{3}$$

By setting $\theta_p = 1 - C_{cb} = \frac{||G_c\mathbf{p}_c||}{||G\mathbf{p}_c||}$, a self-regulating solver with variable inclusion of pressure predictor is derived. This solver was tested and compared to other values of θ_p:

Solver	θ_0	θ_1	θ_{cb}
θ_p	0	1	$1 - C_{cb}$

Results

A turbulent channel flow at $Re_\tau = 180$ was run on a $4\pi \times 2 \times \frac{4}{3}\pi$ domain with $38 \times 108 \times 38$ cells in x, y, z respectively. The mesh was stretched in the y-direction from $\Delta y_w^+ = 0.66$ to $\Delta y_B^+ = 9.4$. The symmetry-preserving Runge-Kutta OpenFOAM solver *RKSymFoam* of [2] was used, which can be found in the GitHub repository given in the author's affiliations. The Runge-Kutta 3 scheme was used with $\Delta t = 0.001$.

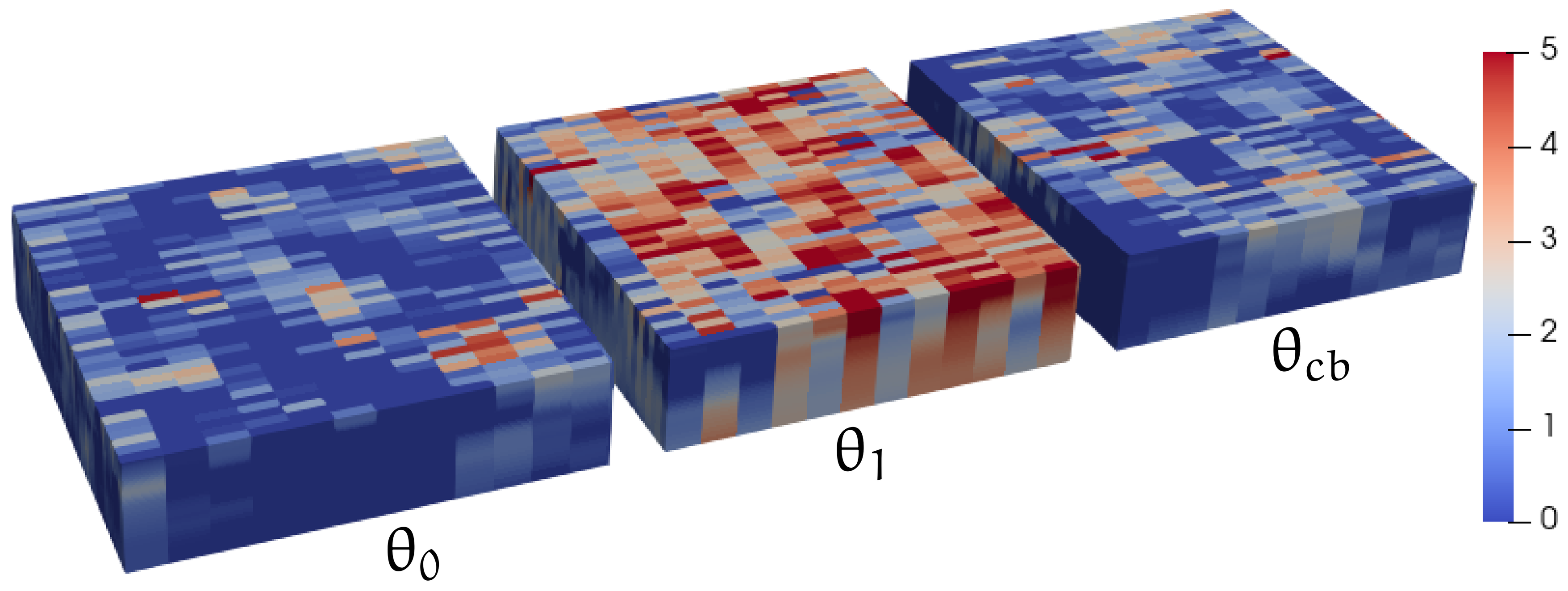

Figure 1: Checkerboarding in turbulent channel flow at $Re_\tau = 180$, xz−plane at $y = 1$

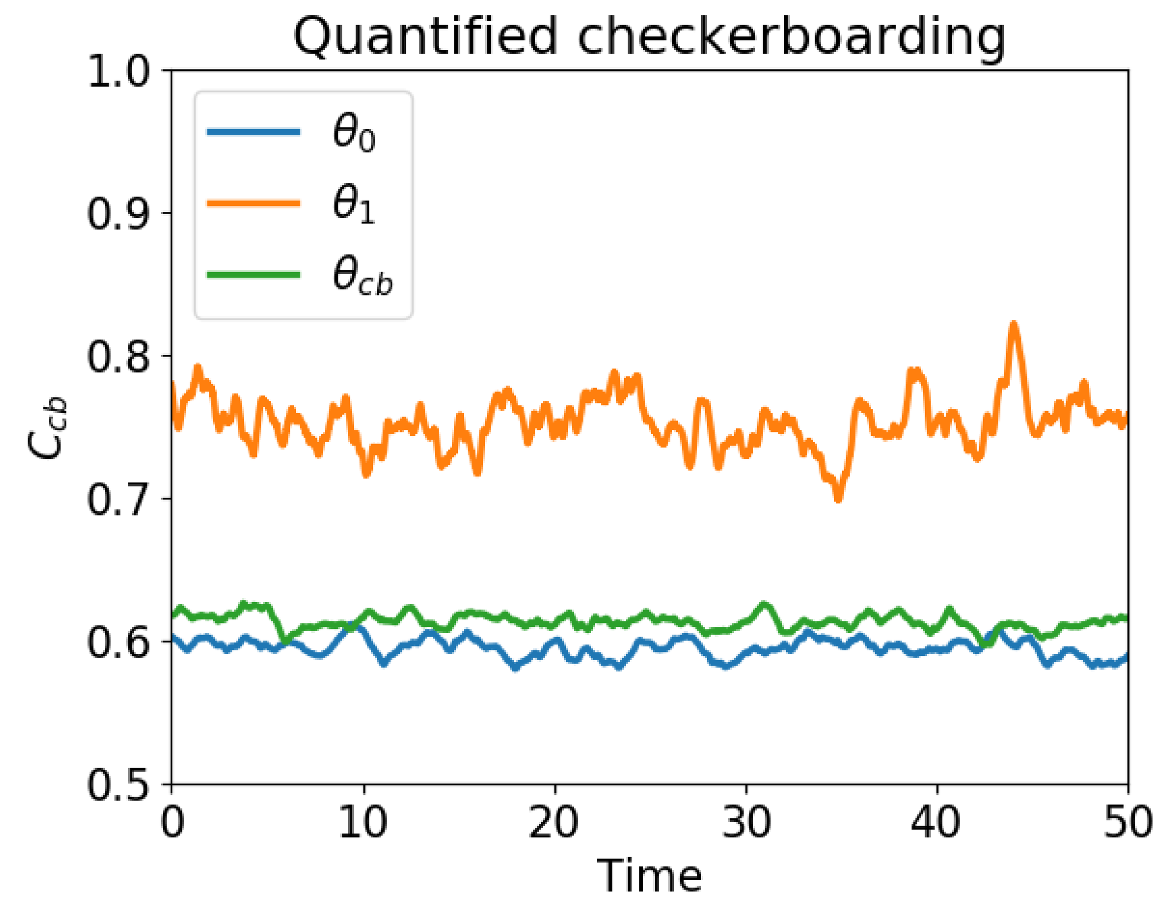

Figure 2: The heavy oscillations present in θ_1 are clearly depicted by C_{cb}

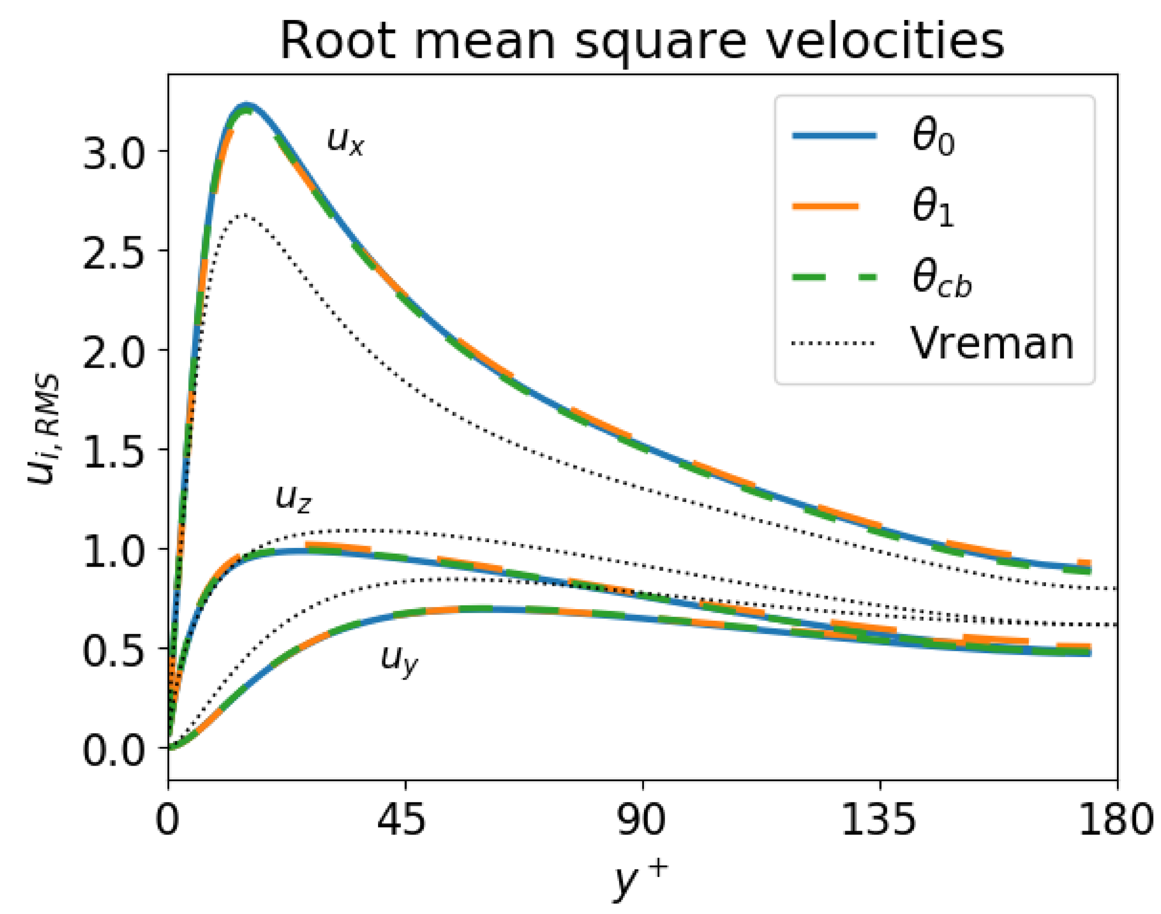

Figure 3: Solvers show comparable accuracy through RMS velocities

From figures 1 and 2 it can be seen that the oscillations occur most heavily for θ_1, which is correctly portrayed by C_{cb}. Since the case is prone to checkerboarding, C_{cb} adjusts θ_p to diminish the problem, resulting in a relatively low C_{cb}. Moreover, figure 3 shows that there is no significant difference in accuracy between the methods.

Conclusions

- C_{cb} quantifies checkerboarding with a global non-dimensional normalised coefficient, independent of time-step.
- Using negative feedback of C_{cb} to determine the pressure predictor, levels of checkerboarding could be self-regulated.
- C_{cb} could be used to develop other methods that only diminish oscillations (locally) at the cost of numerical dissipation, whenever necessary.

References

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THMT'23 - 10th international conference on Turbulence, Heat and Mass Transfer September 11-15, 2023, Rome, Italy