Turbulence, Heat and Mass Transfer 10

Can we hit the ultimate regime of thermal turbulence using LES simulations at low Prandtl numbers?

F.X.Trias[1], A.Gorobets[2], A.Oliva[1]
[1] *Heat and Mass Transfer Technological Center, Technical University of Catalonia C/Colom 11, 08222 Terrassa (Barcelona)* *E-mail: francesc.xavier.trias@upc.edu*
[2] *Keldysh Institute of Applied Mathematics, 4A, Miusskaya Sq., Moscow 125047, Russia*

Abstract – In this work, we plan to shed light on the following research question: *can we hit the ultimate regime of thermal turbulence using large-eddy simulations (LES) at low Prandtl numbers?* This is motivated by our recent findings showing the reliability of LES techniques at low-Pr where no subgrid heat flux activity is expected. Hence, we are carrying out a set of LES simulations of a Rayleigh–Bénard configuration at $Pr = 0.005$ (liquid sodium) using two levels of refinement. According to our estimations the two highest Rayleigh numbers (2.25×10^{10} and 7.14×10^{10}) are located in the region of the $\{Ra, Pr\}$-phase space where the $Nu\ Ra^{1/2}$ power-law scaling of the ultimate regime should be observed. This asymptotic regime was theoretically predicted by Kraichnan in the early 60s; however, despite the great efforts devoted, it still remains elusive.

1. Introduction

Buoyancy-driven flows have always been an important subject of scientific studies with numerous applications in environment and technology. The most famous example thereof is the thermally driven flow developed in a fluid layer heated from below and cooled from above, *i.e.* the Rayleigh-Bénard convection (RBC). It constitutes a canonical flow configuration that resembles many natural and industrial processes, such as solar thermal power plants, indoor space heating and cooling, flows in nuclear reactors, electronic devices, and convection in the atmosphere, oceans and the deep mantle.

In the last decades significant efforts, both numerically and experimentally, have been directed at investigating the mechanisms and the detailed scaling behavior of the Nusselt number as a function of Rayleigh and Prandtl numbers in the general form $Nu \propto Ra^{\gamma} Pr^{\beta}$ [1]. In this regard, Figure 1 shows the predictions of the Nu-number based on the classical Grossmann-Lohse (GL) theory [2] and its subsequent corrections [3, 4] where different scaling regimes, characterized by their corresponding exponents γ and β, are identified. Assuming this power-law scalings and following the same reasoning as in Ref. [5] leads to the estimations for the number of grid points shown in Figure 2 (left). This corresponds to mesh resolution requirements for DNS and it clearly explains why nowadays DNS of RBC is still limited to relatively low Ra-numbers [1]. However, many of the above-mentioned applications are governed by much higher Ra numbers, located in the region of the $\{Ra, Pr\}$ phase space where the thermal boundary layer becomes turbulent (*i.e.* below the black dash-dotted line in Figure 2). This region corresponds to the so-called asymptotic Kraichnan or ultimate regime of turbulence [6], with $\gamma = 1/2$. On the other hand, reaching such Ra-numbers experimentally while keeping the basic assumptions (Boussinesq approximation, adiabaticity of the closing walls, isothermal horizontal walls, perfectly smooth surfaces...) is a very hard task; therefore, the observation of the Kraichnan regime also remains elusive [3, 4].

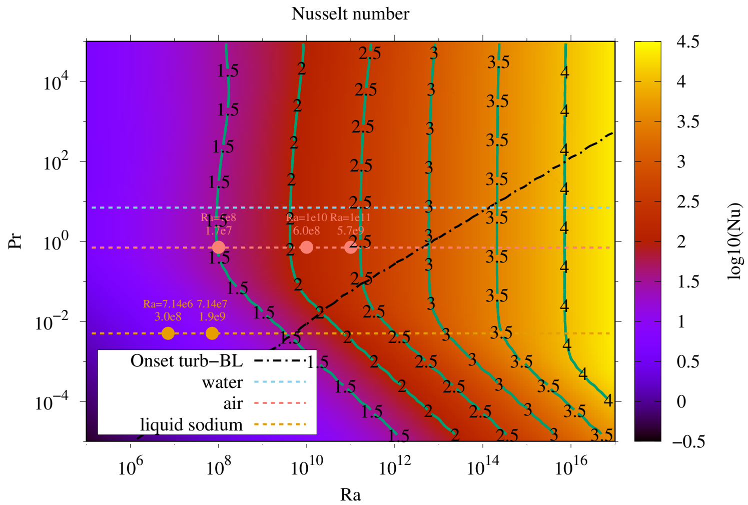

Figure 1: Estimation of the Nusselt number of a RBC in the $\{Ra, Pr\}$ phase space given by the classical GL theory [2] and its subsequent corrections [3]. Green solid isolines represent the $log10$ of the Nusselt. Three dashed horizontal lines correspond to three different working fluids: water ($Pr = 7$), air ($Pr = 0.7$) and liquid sodium ($Pr = 0.005$). Dots displayed correspond to the DNS simulations carried out in previous studies [5, 7, 8]. Black dash-dotted line is an estimation for the onset of turbulence in the thermal boundary layer.

2. LES of buoyancy-driven turbulence

In this context, we may turn to LES to predict the large-scale behavior of incompressible turbulent flows driven by buoyancy at very high Ra-numbers. In LES, the large-scale motions are explicitly computed, whereas the effects of small-scale motions are modeled. Since the advent of CFD, many subgrid-scale (SGS) models have been proposed and successfully applied to a wide range of flows. However, there still exist inherent difficulties in the proper modelization of the SGS heat flux. This was analyzed in detail in the PRACE project entitled ”Exploring new frontiers in Rayleigh–Bénard convection” (33.1 millions of CPU hours on MareNostrum 4 in 2018-2019), where DNS simulations of air-filled ($Pr = 0.7$) RBC up to $Ra = 10^{11}$ were carried out using meshes up to 5600M grid points (see dots displayed in Figures 1 and 2, left). These results shed light into the flow topology and the small-scale dynamics [7]. Moreover, it also provided new insights into the preferential alignments of the SGS and its dependence with the Ra-numbers [8], highlighting that the modelization of the SGS heat flux is the main difficulty that (still) precludes reliable LES of buoyancy-driven flows at (very) high Ra-numbers. This inherent difficulty can be by-passed by carrying out simulations at low-Prandtl numbers. In this case, the ratio between the Kolmogorov length scale and the Obukhov–Corrsin length scale (the smallest scale for the temperature field) is given by $Pr^{3/4}$; therefore, for instance, at $Pr = 0.005$ (liquid sodium) we have a separation of more than one decade. Hence, it is possible to combine an LES simulation for the velocity field (momentum equation) with the numerical resolution of all the thermal scales. Results obtained in Ref. [8] suggest that accurate predictions of the overall Nu can be obtained with meshes significantly coarser than those needed for a DNS (*e.g.* in practice for $Pr = 0.005$ we can expect mesh reductions in the range 10^2-10^3 for the total number of grid points leading to computational cost reductions in the range 10^3-10^4). This can be clearly observed in Figure 2 (right), where estimations of the mesh size for LES are given with

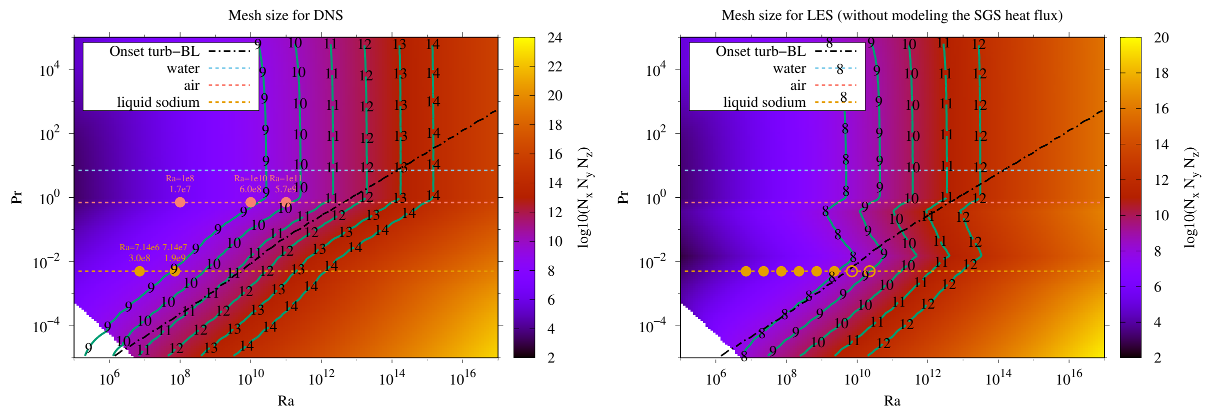

Figure 2: Estimation of the mesh sizes for DNS (left) and LES (right) simulations of RBC in the $\{Ra, Pr\}$ phase space. LES estimations assume that thermal scales are fully resolved, *i.e.* no SGS heat flux model is needed. Green solid isolines represent the $log10$ of the total number of grid points. Three dashed horizontal lines correspond to three different working fluids: water ($Pr = 7$), air ($Pr = 0.7$) and liquid sodium ($Pr = 0.005$). Dots displayed in the left figure correspond to the DNS simulations carried out in previous studies [5, 7, 8] whereas the dots shown in the right figure are the set of LES simulations (being) carried out in the present work. Black dash-dotted line is an estimation for the onset of turbulence in the thermal boundary layer.

the assumption that thermal scales are fully resolved. This opens the possibility to reach the ultimate regime carrying out LES at low-Pr using meshes.

3. Preliminary results and conclusions

A set of LES simulations of RBC at $Pr = 0.005$ for a wide range of Ra numbers (see dots in Figure 2, right) are being carried out on MareNostrum 4 supercomputer. The configuration is the same as in Ref.[8] where two DNS simulations (solid black dots in Figure 3) were computed using meshes with $488 \times 488 \times 1280 \approx 305M$ ($Ra = 7.14 \times 10^6$) and $996 \times 996 \times 2048 \approx 1911M$ ($Ra = 7.14 \times 10^7$) grid points, respectively. For the LES simulations, two levels of mesh refinement are being used: namely, a fine level that approximately corresponds to estimations shown in Figure 2 (right) and a coarse level which is approximately twice coarser in each spatial direction. For instance, LES meshes at $Ra = 7.14 \times 10^7$ have respectively $44 \times 44 \times 96 \approx 0.19M$ and $90 \times 90 \times 160 \approx 1.3M$ grid points, *i.e.* ≈ 10000 and ≈ 1500 coarser compared with the DNS mesh. Meshes are designed to properly resolve the boundary layer whereas the much coarser bulk region is fine enough to guarantee that thermal scales are fully resolved, *i.e.* no SGS heat flux model is needed. Then, the SGS stress tensor is modeled using the S3PQ model [9] which was already tested for this RBC configuration in Ref. [8].

Results of the overall Nusselt number are displayed in Figure 3. LES simulations up to $Ra = 7.14 \times 10^{10}$ (for the coarse level) and $Ra = 2.26 \times 10^{10}$ (for the fine level) are still being computed on MareNostrum 4 supercomputer. These points are located beyond the transition point for this Pr-number (see Figure 2, right). Nevertheless, these simulations are not statistically converged yet and, therefore, results are not shown here. At first sight, we can observe an accurate agreement with previous DNS results. Furthermore, there is a rather good agreement with the Nu-vs-Ra scaling predicted using the DNS data. In any case, these preliminary

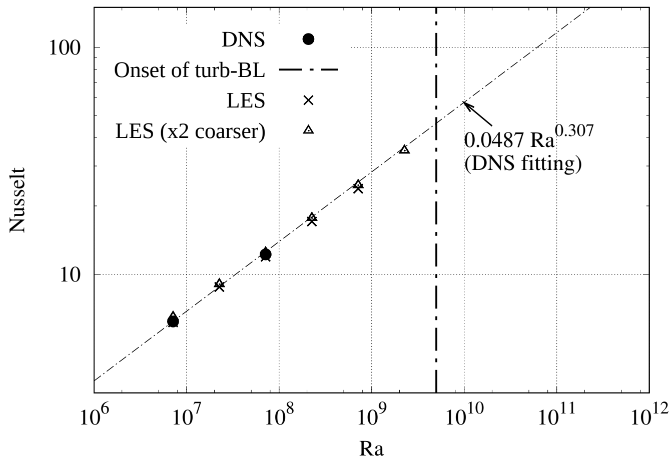

Figure 3: *Nu*-vs-*Ra* results obtained with LES at $Pr = 0.005$ using the same RBC configuration as in Ref.[8] where the two DNS results (solid black dots) were computed. The vertical dash-dotted line corresponds to the estimated *Ra* (for this particular *Pr*) where the thermal boundary layer becomes turbulent.

results show the capability to obtain accurate predictions of the *Nu*-number using LES simulations. Accordingly to the classical GL theory, on-going LES simulations at higher *Ra*-number should possibly show a change in the *Nu*-vs-*Ra* scaling indicating that we are finally hitting the ultimate regime of thermal turbulence.

References

1. R. J. A. M. Stevens, , D. Lohse, and R. Verzicco. Toward DNS of the Ultimate Regime of Rayleigh–Bénard Convection. In *Direct and Large Eddy Simulation XII*, pages 215–224, Madrid, Spain, 2019. Springer International Publishing.
2. S. Grossmann and D. Lohse. Scaling in thermal convection: a unifying theory. *Journal of Fluid Mechanics*, 407:27–56, 2000.
3. R. J. A. M. Stevens, E. P. van der Poel, S. Grossmann, and D. Lohse. The unifying theory of scaling in thermal convection: the updated prefactors. *Journal of Fluid Mechanics*, 730:295–308, 2013.
4. S. Bhattacharya, M. K. Verma, and R. Samtaney. Revisiting Reynolds and Nusselt numbers in turbulent thermal convection. *Physics of Fluids*, 33:015113, 2021.
5. F. Dabbagh, F. X. Trias, A. Gorobets, and A. Oliva. On the evolution of flow topology in turbulent Rayleigh-Bénard convection. *Physics of Fluids*, 28:115105, 2016.
6. R. H. Kraichnan. Turbulent thermal convection at arbitrary Prandtl number. *Physics of Fluids*, 5:1374–1389, 1962.
7. F. Dabbagh, F. X. Trias, A. Gorobets, and A. Oliva. Flow topology dynamics in a three-dimensional phase space for turbulent Rayleigh-Bénard convection. *Physical Review Fluids*, 5:024603, 2020.
8. F.X. Trias, F.Dabbagh, A.Gorobets, and C.Oliet. On a proper tensor-diffusivity model for large-eddy simulation of buoyancy-driven turbulence. *Flow, Turbulence and Combustion*, 105:393–414, 2020.
9. F. X. Trias, D. Folch, A. Gorobets, and A. Oliva. Building proper invariants for eddy-viscosity subgrid-scale models. *Physics of Fluids*, 27(6):065103, 2015.