Assessment of LES models for a fully developed wind-turbine array boundary layer

D.Folch, F.X.Trias, A.Oliva

Heat and Mass Transfer Technological Center, Technical University of Catalonia
david.folch@upc.edu

Centre Tecnològic de Transferència de Calor
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Introduction

To test and compare the performance of S3PQR Large Eddy Simulation models on boundary layer and wind farm cases.

Spatially filtered incompressible Navier-Stokes equations

$$\begin{aligned} \partial_t \overline{\boldsymbol{u}} + C(\overline{\boldsymbol{u}}, \overline{\boldsymbol{u}}) &= D(\overline{\boldsymbol{u}}) - \nabla p - \nabla \cdot \tau(\overline{\boldsymbol{u}}); \\ \nabla \cdot \overline{\boldsymbol{u}} &= 0 \end{aligned}$$

$\tau(\overline{\boldsymbol{u}}) \approx -2\nu_e S(\overline{\boldsymbol{u}})$ is the LES closure
$S(\overline{\boldsymbol{u}}) = 1/2(\nabla \overline{\boldsymbol{u}} + \nabla \overline{\boldsymbol{u}}^T)$ is the rate-of-strain tensor
ν_e is the eddy viscosity for each model

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S3PQR theory review

Besides the trace, several mathematical invariants can be calculated from the gradient tensor $\mathsf{G} = \nabla\overline{\boldsymbol{u}}$, namely:

$$
\begin{aligned}
Q_{\mathsf{G}} &= (1/2)(tr^2(\mathsf{G}) - tr(\mathsf{G}^2)) \\
R_{\mathsf{G}} &= det(\mathsf{G}) \\
Q_{\mathsf{S}} &= (1/2)(tr^2(\mathsf{S}) - tr(\mathsf{S}^2)) \\
R_{\mathsf{S}} &= det(\mathsf{S}) \\
V_{\mathsf{G}}^2 &= 4(tr(S^2\Omega^2) - 2Q_S Q_\Omega)
\end{aligned}
$$

$\mathsf{S} = 1/2(\mathsf{G} + \mathsf{G}^T)$ and $\Omega = 1/2(\mathsf{G} - \mathsf{G}^T)$ are the symmetric and the skew-symmetric parts of the gradient tensor

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Smagorinsky, Verstappen's, WALE, Vreman's, and all the S3PQR models as a function of the invariants.

The symmetric tensor GG^T formally based on the lowest-order approximation of the subgrid stress tensor is

$$\tau(\overline{\boldsymbol{u}}) = \frac{\Delta^2}{12} GG^T + \mathcal{O}(\Delta^4)$$

Three invariants of this tensor can be defined and are directly related to the previous ones

$$\begin{aligned} P_{GG^T} &= tr(GG^T) = 2(Q_\Omega - Q_S) \\ Q_{GG^T} &= 2(Q_\Omega - Q_S)^2 - Q_G^2 + 4tr(S^2\Omega^2) \\ R_{GG^T} &= det(GG^T) = det(G)det(G^T) = R_G^2 \end{aligned}$$

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S3PQR: a combination of two invariants of GG^T (Trias et al. 2015)

$$\nu_e^{S3PQ} = (C_{s3pq}\Delta)^2 P_{GG^T}^{-5/2} Q_{GG^T}^{3/2}$$
$$\nu_e^{S3PR} = (C_{s3pr}\Delta)^2 P_{GG^T}^{-1} R_{GG^T}^{1/2}$$
$$\nu_e^{S3QR} = (C_{s3qr}\Delta)^2 Q_{GG^T}^{-1} R_{GG^T}^{5/6}$$

where Δ is the subgrid characteristic length.

Two ways to determine the model constant C_{s3pq}:

1. Less or equal dissipation than Vreman's model.

$C_{s3pq} = C_{s3pr} = C_{s3qr} = \sqrt{3}C_{Vr} \approx 0.458$

2. The averaged dissipation of the models is equal to that of the Smagorinsky model.

$C_{s3pq} = 0.572, \; C_{s3pr} = 0.709, \; C_{s3qr} = 0.762$

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Algorithm details

- Six possible combinations to test (PQ1,PQ2,...)
- For all the current computations, the grid size of the domain is $Nx = 32$, $Ny = 64$, and $Nz = 32$ points
- $Re_{\delta*} = 1000$, where δ^* is the displacement thickness.
- Pseudo-spectral algorithm: strong formulation with Poisson - pressure correction term; Chebyshev polynomials
- Algebraic scaling $y_\infty = L\frac{1+y}{1-y}$, for the semi-infinite domain
- Fully explicit AB time-integration method.
- We will test the zero mean pressure gradient case

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Boundary layer

Spalart and Leonard (1987):
- Normal coordinate similarity transformations
- Growing terms $GT(\overline{\boldsymbol{u}}, \overline{U})$

Three main parameters:

1. u_τ as the friction velocity

2. H as the ratio of the displacement thickness to the momentum thickness

3. κ as the Von Kármán constant

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Results

Case:	u_τ	H	κ
Sp-Le DNS	**0.049**	**1.52**	**0.39**
No model	0.049	1.61	0.35
Vreman	0.050	1.51	0.47
WALE	0.046	1.54	0.47
PQ1	0.048	1.58	0.35
PR1	0.050	1.54	0.44
QR1	0.049	1.57	0.35
PQ2	0.046	1.57	0.42
PR2	0.049	1.53	0.39
QR2	0.048	1.57	0.32

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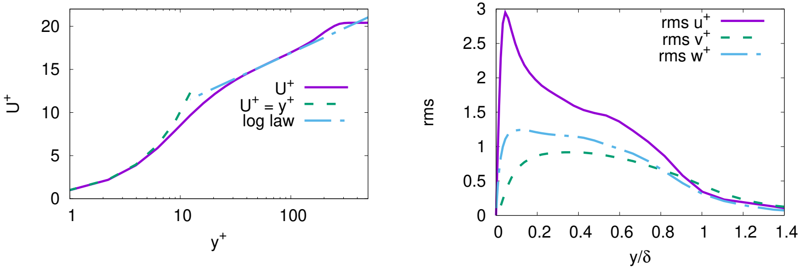
Vertical profiles



Case PR2. Left: normalized average streamwise velocity profile, U^+; log law; $U^+ = y^+$. Right: rms u^+; rms v^+; rms w^+; δ is the boundary layer thickness

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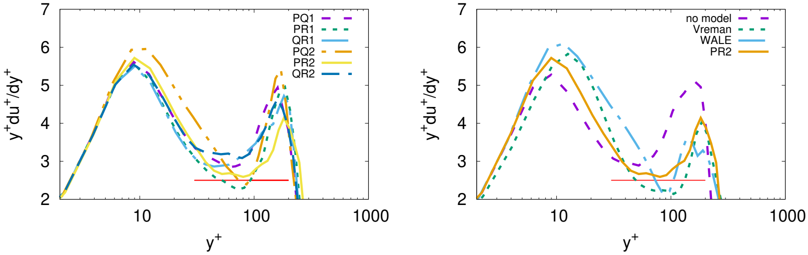
Velocity derivative



Left: S3PQR models. Right: comparison with other LES models. The horizontal line marks the point(s) where the log law would be with $\kappa = 0.4$

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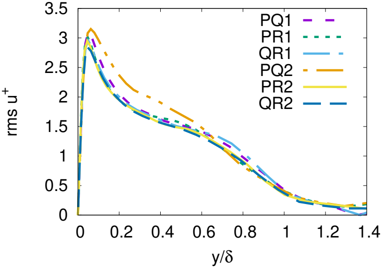
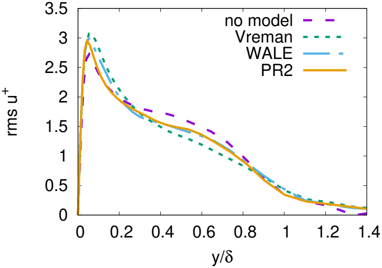
r.m.s. u^+ profiles





Left: S3PQR models. Right: comparison with other LES models

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Wind farm turbines

Calaf, Meneveau, and Meyers (2010):

1- Fully developed boundary layer

2- Disk actuator for every wind turbine. 24 disk actuators evenly distributed in four rows and six columns. The force of the turbine (per unit mass):

$$F(i,j,k) = -\frac{1}{2} C'_T \langle \overline{\boldsymbol{u}}^T \rangle_d^2 \frac{\gamma_{j,k}}{\Delta x}$$

where C'_T is a thrust coefficient, $\langle \overline{\boldsymbol{u}}^T \rangle_d^2$ is the disk averaged local velocity, $\gamma_{j,k}$ is the fraction area overlap of the disk and Δx is the distance between turbines.

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Values of interest

- $z0_{Hi}/zH$, the ratio of the effective roughness above the turbine hub and the height of the turbines' center
- u_{τ}, the usual friction velocity at the wall
- u_{*}, the computed friction velocity above the hub
- P, the time and horizontally averaged power extracted for every turbine
- W_t, the time, horizontally, and vertically (along the hub) averaged power
- $\delta\Phi$, the vertical flux of kinetic energy
- EB, for energy budget

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Computed values

MODEL	$z0_{Hi}/zH$	u_τ	u_*	u_τ/u_*	$P/\delta\Phi$	$W_t/\delta\Phi$	EB
no model	0.160	0.051	0.109	0.47	0.68	0.81	94%
Vreman	0.072	0.056	0.085	0.66	0.67	0.78	94%
WALE	0.082	0.050	0.089	0.56	0.79	0.90	94%
PQ1	0.096	0.052	0.092	0.57	0.75	0.86	96%
PR1	0.105	0.052	0.094	0.55	0.74	0.85	95%
QR1	0.123	0.052	0.100	0.52	0.73	0.84	95%
PQ2	0.074	0.052	0.085	0.61	0.75	0.86	95%
PR2	0.065	0.052	0.083	0.63	0.77	0.88	97%
QR2	0.098	0.052	0.093	0.56	0.74	0.86	95%

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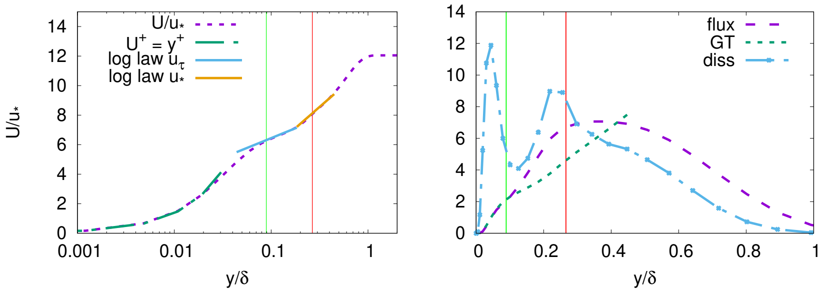
Vertical profiles



PR2. Left: velocity. Right: Normalized mean kinetic energy contributions: **flux**, $\delta\Phi = - < uv > U/u_*^3$; **GT**, normalized growing terms; **diss**, $- < uv > \partial_y U/(u_*^3/\delta)$

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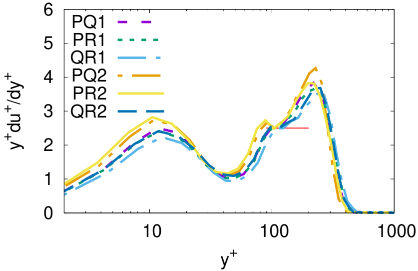
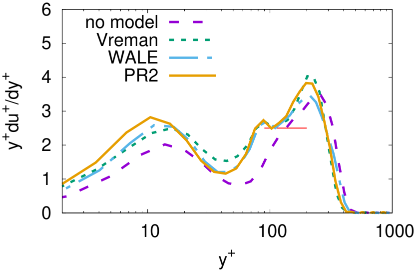
Velocity derivative





Left: S3PQR models. Right: other LES models.

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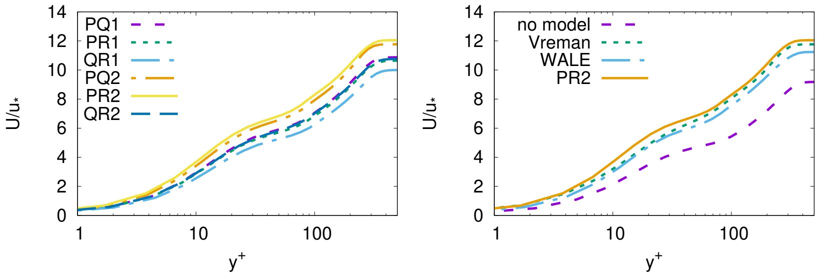
Velocity profile



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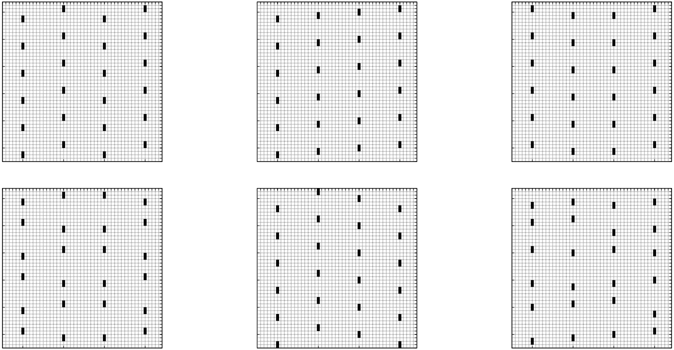
Turbine positions



From the top left, in order: alternated, diagonal, curved#1, curved#2, disordered#1, disordered#2

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Power comparison

PATTERN	$P/\delta\Phi$	$P/{u^*}^3$
ordered	0.77	3.24
alternated	0.80	3.31
diagonal	0.80	3.10
curved#1	0.76	2.87
curved#2	0.77	2.86
disordered#1	0.79	3.05
disordered#2	0.76	3.05

PR2 model. All normalized by u^* of the ordered geometry

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Conclusions

1. S3PQR models yield good performance for the boundary layer with PR standing as the best of all.

2. For the wind farm, most of the S3PQR match the expected behavior with, again, PR as the most reliable.

Thank you for your attention.

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