

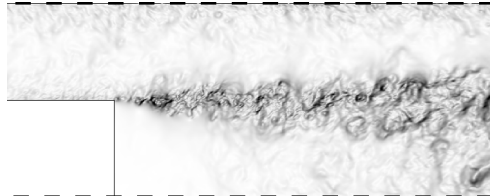
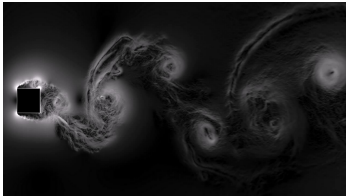


Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Interactions between numerical discretization, subgrid modeling and filtering in LES

F.Xavier Trias

Heat and Mass Transfer Technological Center, Technical University of Catalonia (UPC)





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- 1 Background
- 2 Motivation
- 3 Revisiting FVM
- 4 A rational length scale for LES
- 5 Conclusions

Contents

1 Background

2 Motivation

3 Revisiting FVM

4 A rational length scale for LES

5 Conclusions

About myself...

Professional...

- Current position (since 2024): **Full Professor** at UPC
- Previous positions: PostDoc at University of Groningen (2007-2009) and UPC (2010-2013), *Ramón y Cajal* Senior Researcher at UPC (2013-2018) and Associate Professor (2018-2023).
- My **research** focus is on fluid mechanics, turbulence modeling, physics and numerics of complex flows, applied mathematics and numerical methods.
- Some numbers: 58 papers, 190 conferences, 12 PhD's+5 (on-going)
- Stays and collaborations: Groningen (Netherlands), UCLA, KIAM (Russian Academy of Sciences), Stanford, Tsinghua (China), TokioTech (Japan), Napoli (Italy), CWI (Netherlands),...
- More info: www.fxtrias.com

About myself...

... and more personal stuff

- My complete name: Francesc Xavier Trias Miquel
- Born in Barcelona
- My mother tongue is Catalan. I also speak Spanish at native level.
- Hobbies? I like my work but also sports. Most practiced ones are running and football:



Groningen (2009?)



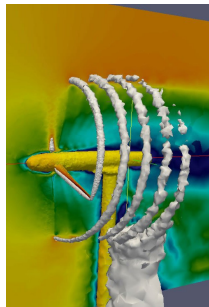
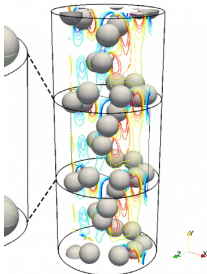
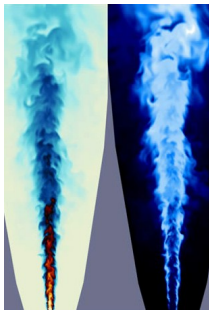
Barcelona (2019)



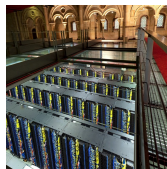
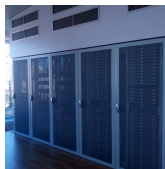
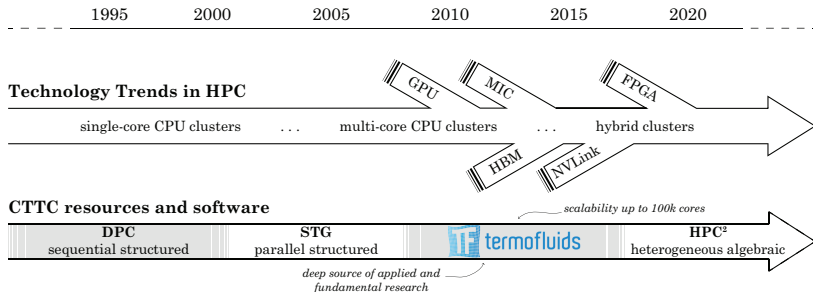
The CTTC research group

Heat and Mass Transfer Technological Center (Catalan: *Centre Tecnològic de Transferència de Calor*) has more than 30 years experience on CFD:

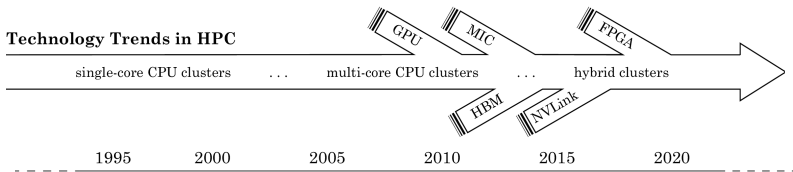
- **Fundamental research** on numerical methods, fluid dynamics and heat and mass transfer phenomena.
- **Applied research** on thermal and fluid dynamic optimization of thermal system and equipment.



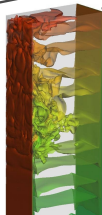
CTTC's historical background in HPC



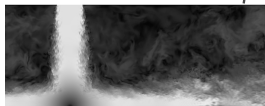
Technology Trends in HPC



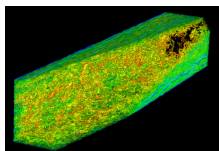
DHC
 $Ra=10^9$
(3.2M)



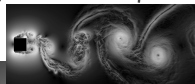
DHC
 $Ra=10^{11}$
(111M)



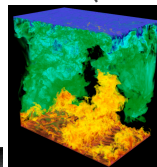
Impinging jet
 $Re=20000$
(102M)



DUCT $Re_t=1200$ (172M)

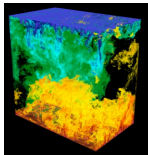


SqCyl $Re=22000$ (324M)



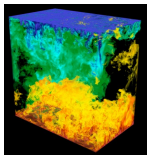
RB
 $Ra=10^{10}$ (607M)
 $Ra=10^{11}$ (5600M)

General motivation: (very) large-scale DNS/LES



DNS {

General motivation: (very) large-scale DNS/LES



MareNostrum 5-ACC
(Barcelona)



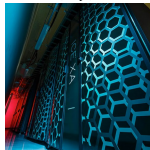
rank #8
1120 nodes with:
2x Intel Shippore Rapids 8460
1x NVIDIA Hopper 64 GB HBM
1x Infiniband NDR200

Snellius
(Amsterdam)



rank #165
714 nodes with:
2x AMD EPYC 9654 96C
1x Infiniband NDR200

TSUBAME4.0
(Tokyo)



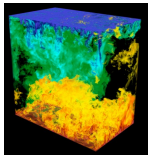
rank #31
240 nodes with:
2x AMD EPYC 9654 96-Core
4x NVIDIA H100 SXM5
1x Infiniband NDR200



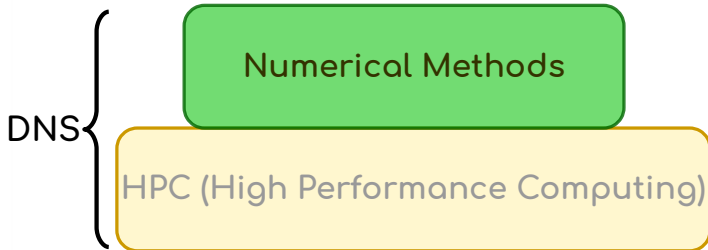
DNS

HPC (High Performance Computing)

General motivation: (very) large-scale DNS/LES

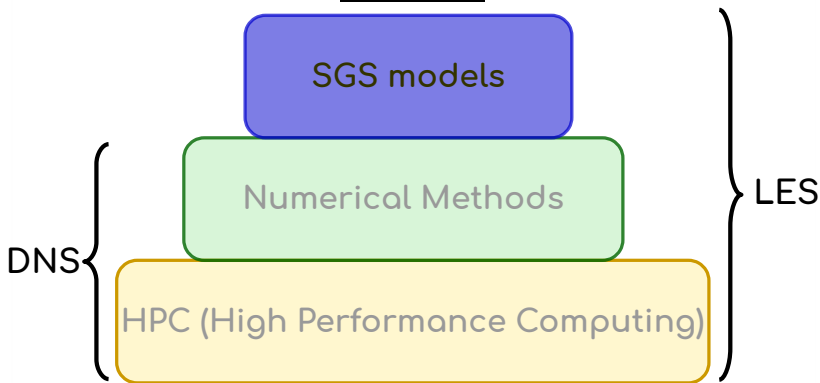
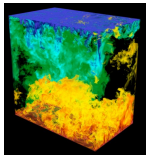


How to properly discretize NS?

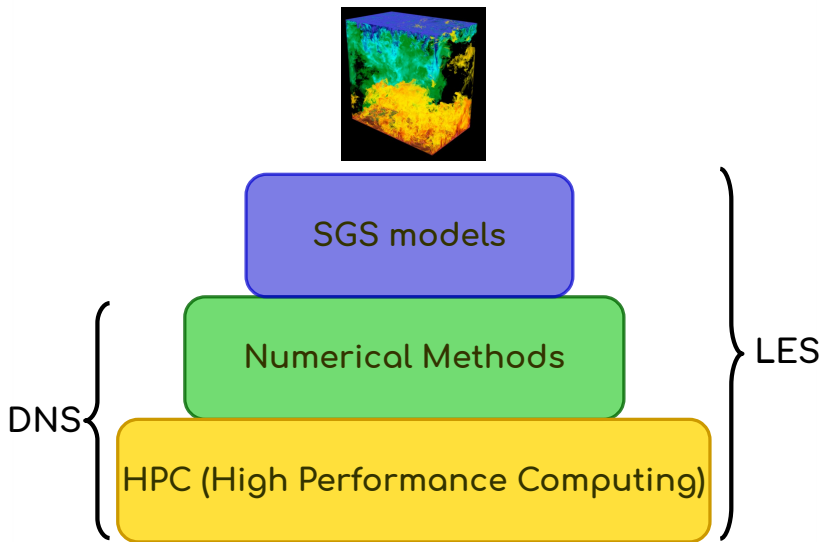


General motivation: (very) large-scale DNS/LES

How to
properly
model SGS?



General motivation: (very) large-scale DNS/LES



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2 Motivation

3 Revisiting FVM

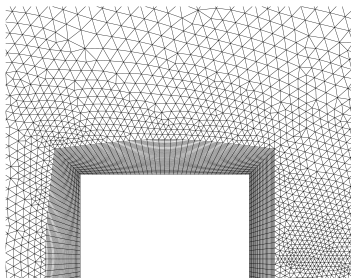
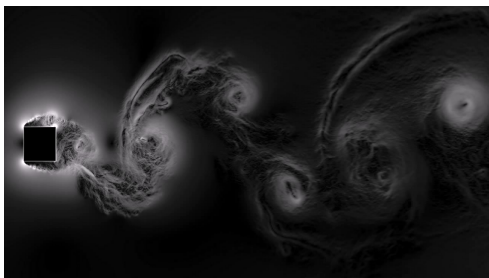
4 A rational length scale for LES

5 Conclusions

Motivation

Research question #1:

- What are we indeed solving with **finite volume method**?



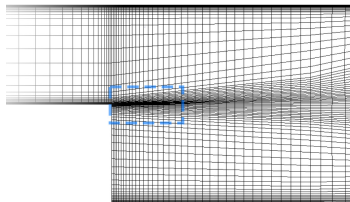
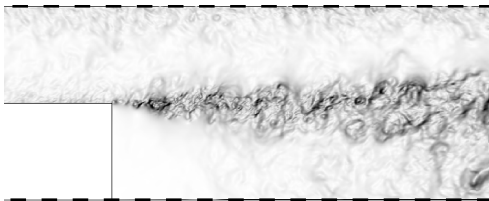
DNS¹ of the turbulent flow around a square cylinder at $Re = 22000$

¹F.X.Trias, A.Gorobets, A.Oliva. *Turbulent flow around a square cylinder at Reynolds number 22000: a DNS study*, **Computers&Fluids**, 123:87-98, 2015.

Motivation

Research question #1:

- What are we indeed solving with **finite volume method**?



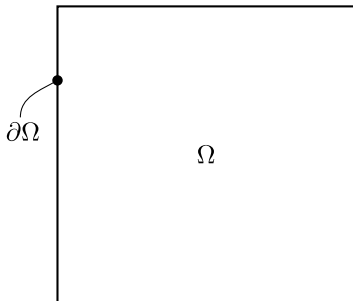
DNS² of backward-facing step at $Re_\tau = 395$ and expansion ratio 2

²A.Pont-Vílchez, F.X.Trias, A.Gorobets, A.Oliva. *DNS of Backward-Facing Step flow at $Re_\tau = 395$ and expansion ratio 2*. **Journal of Fluid Mechanics**, 863:341-363, 2019.

Motivation

Research question #2:

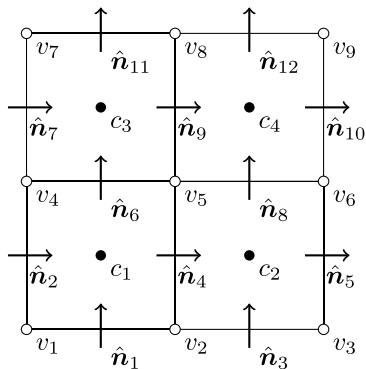
- What are we interpolating? What is the correct interpretation?



Motivation

Research question #2:

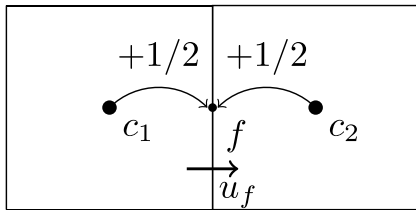
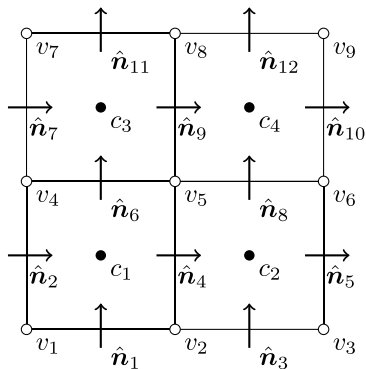
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Motivation

Research question #2:

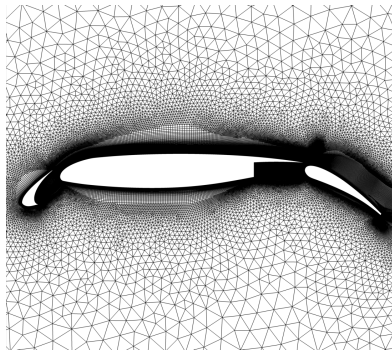
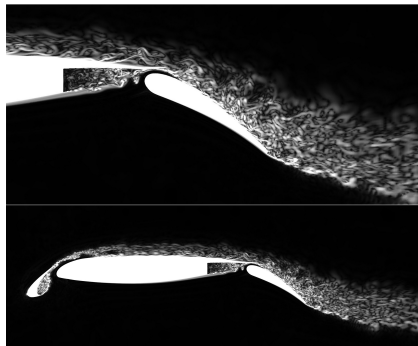
- What are we interpolating? What is the correct interpretation?



Motivation

Research question #3:

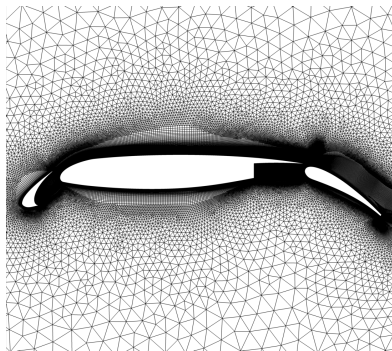
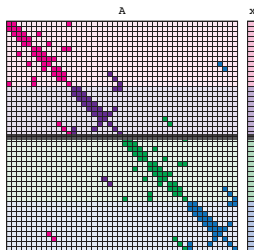
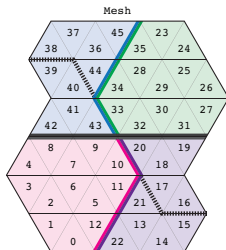
- Can we developed **high-order** symmetry-preserving schemes for **unstructured grids**?



Motivation

Research question #3:

- Can we developed **high-order** symmetry-preserving schemes for **unstructured grids**? And, therefore, get better performance



Reminder: symmetry-preserving discretization

Continuous

$$\frac{\partial \mathbf{u}}{\partial t} + \mathcal{C}(\mathbf{u}, \mathbf{u}) = \nu \nabla^2 \mathbf{u} - \nabla p$$

$$\nabla \cdot \mathbf{u} = 0$$

Reminder: symmetry-preserving discretization

Continuous

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{C}(\mathbf{u}, \mathbf{u}) = \nu \nabla^2 \mathbf{u} - \nabla p$$

$$\nabla \cdot \mathbf{u} = 0$$

Discrete

$$\Omega \frac{d\mathbf{u}_h}{dt} + \mathbf{C}(\mathbf{u}_h) \mathbf{u}_h = \mathbf{D} \mathbf{u}_h - \mathbf{G}_h \mathbf{p}_h$$

$$\mathbf{M} \mathbf{u}_h = \mathbf{0}_h$$

Reminder: symmetry-preserving discretization

Continuous

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{C}(\mathbf{u}, \mathbf{u}) = \nu \nabla^2 \mathbf{u} - \nabla p$$

$$\nabla \cdot \mathbf{u} = 0$$

$$(\mathbf{a}, \mathbf{b}) = \int_{\Omega} \mathbf{a} \mathbf{b} d\Omega$$

Discrete

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$$\mathbf{M} \mathbf{u}_h = \mathbf{0}_h$$

$$(\mathbf{a}_h, \mathbf{b}_h)_h = \mathbf{a}_h^T \Omega \mathbf{b}_h$$

Reminder: symmetry-preserving discretization

Continuous

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$$\nabla \cdot \mathbf{u} = 0$$

$$(\mathbf{a}, \mathbf{b}) = \int_{\Omega} \mathbf{a} \mathbf{b} d\Omega$$

$$(\mathbf{C}(\mathbf{u}, \phi_1), \phi_2) = -(\mathbf{C}(\mathbf{u}, \phi_2), \phi_1)$$

Discrete

$$\Omega \frac{d\mathbf{u}_h}{dt} + \mathbf{C}(\mathbf{u}_h) \mathbf{u}_h = \mathbf{D} \mathbf{u}_h - \mathbf{G}_h \mathbf{p}_h$$

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Reminder: symmetry-preserving discretization

Continuous

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$$(\mathbf{a}, \mathbf{b}) = \int_{\Omega} \mathbf{a} \mathbf{b} d\Omega$$

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$$(\nabla \cdot \mathbf{a}, \phi) = -(\mathbf{a}, \nabla \phi)$$

Discrete

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$$\Omega \mathbf{G}_h = -\mathbf{M}^T$$

Reminder: symmetry-preserving discretization

Continuous

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{C}(\mathbf{u}, \mathbf{u}) = \nu \nabla^2 \mathbf{u} - \nabla p$$

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$$(\nabla \cdot \mathbf{a}, \phi) = -(\mathbf{a}, \nabla \phi)$$

$$(\nabla^2 \mathbf{a}, \mathbf{b}) = (\mathbf{a}, \nabla^2 \mathbf{b})$$

Discrete

$$\Omega \frac{d\mathbf{u}_h}{dt} + \mathbf{C}(\mathbf{u}_h) \mathbf{u}_h = \mathbf{D} \mathbf{u}_h - \mathbf{G}_h \mathbf{p}_h$$

$$\mathbf{M} \mathbf{u}_h = \mathbf{0}_h$$

$$(\mathbf{a}_h, \mathbf{b}_h)_h = \mathbf{a}_h^T \Omega \mathbf{b}_h$$

$$\mathbf{C}(\mathbf{u}_h) = -\mathbf{C}^T(\mathbf{u}_h)$$

$$\Omega \mathbf{G}_h = -\mathbf{M}^T$$

$$\mathbf{D} = \mathbf{D}^T \quad \text{def --}$$

Contents

1 Background

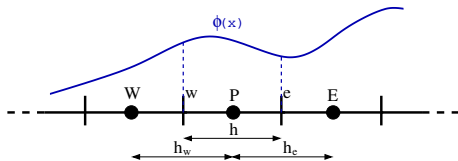
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3 Revisiting FVM

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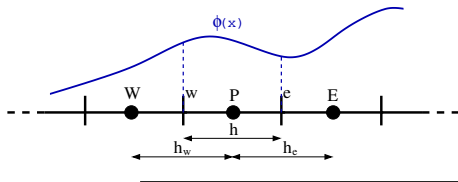
Revisiting FVM



Box filter:
$$\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$$

$$\partial_x \bar{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

Revisiting FVM

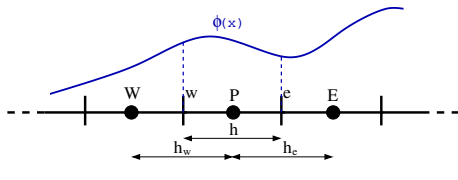


Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \bar{\phi}|_e = \bar{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial(u\phi)}{\partial x} = \nu \frac{\partial^2 \phi}{\partial x^2}$$

Revisiting FVM



Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \bar{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

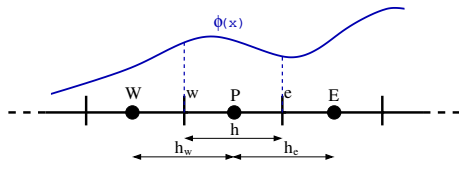
$$\frac{\partial \phi}{\partial t} + \frac{\partial(u\phi)}{\partial x} = \nu \frac{\partial^2 \phi}{\partial x^2}$$

→

$$\boxed{\frac{\partial \bar{\phi}}{\partial t} + \frac{\partial(u\bar{\phi})}{\partial x} = \nu \frac{\partial^2 \bar{\phi}}{\partial x^2}}$$

Exact FVM eq!

Revisiting FVM



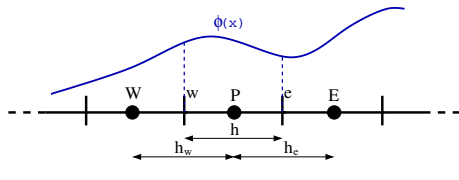
Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \bar{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\mathbf{u}\phi)}{\partial x} = \nu \frac{\partial^2 \phi}{\partial x^2} \longrightarrow \boxed{\frac{\partial \bar{\phi}}{\partial t} + \frac{\partial(\overline{\mathbf{u}\phi})}{\partial x} = \nu \overline{\frac{\partial^2 \phi}{\partial x^2}}} \text{ Exact FVM eq!}$$

$$h \frac{\partial \bar{\phi}_P}{\partial t} + (\mathbf{u}\phi)_e - (\mathbf{u}\phi)_w = \nu \left(\left. \frac{\partial \phi}{\partial x} \right|_e - \left. \frac{\partial \phi}{\partial x} \right|_w \right)$$

Revisiting FVM



Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \bar{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

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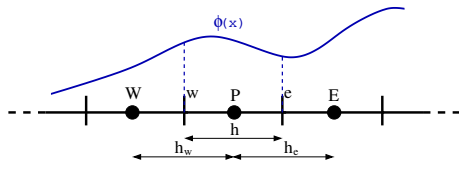
$$h \frac{\partial \bar{\phi}_P}{\partial t} + (\mathbf{u}\phi)_e - (\mathbf{u}\phi)_w = \nu \left(\left. \frac{\partial \phi}{\partial x} \right|_e - \left. \frac{\partial \phi}{\partial x} \right|_w \right)$$

$$h \frac{\partial \bar{\phi}_P}{\partial t} +$$

=

$$\boxed{\frac{\partial \bar{\phi}}{\partial t} + \quad =}$$

Revisiting FVM



Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \bar{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

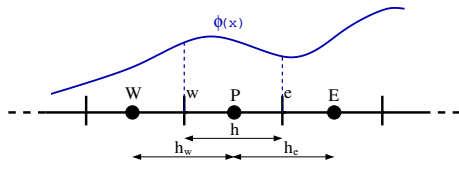
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$$h \frac{\partial \bar{\phi}_P}{\partial t} + (\mathbf{u}\phi)_e - (\mathbf{u}\phi)_w = \nu \left(\left. \frac{\partial \phi}{\partial x} \right|_e - \left. \frac{\partial \phi}{\partial x} \right|_w \right)$$

$$h \frac{\partial \bar{\phi}_P}{\partial t} + = \nu \left(\frac{\bar{\phi}_E - \bar{\phi}_P}{h_e} - \frac{\bar{\phi}_P - \bar{\phi}_W}{h_w} \right)$$

$$\boxed{\frac{\partial \bar{\phi}}{\partial t} + = \nu \overline{\frac{\partial^2 \phi}{\partial x^2}}}$$

Revisiting FVM



Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \bar{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

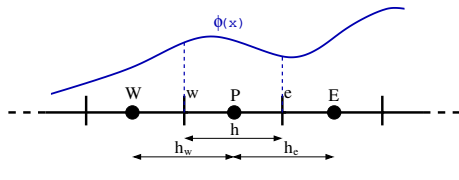
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Revisiting FVM



Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

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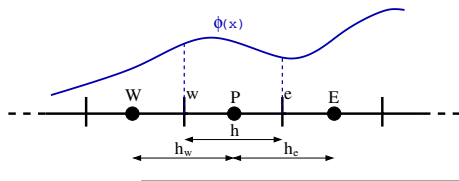
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where $\tilde{\phi}_e = \frac{\bar{\phi}_P + \bar{\phi}_E}{2} = \bar{\phi}_e + \mathcal{O}(h^2)$

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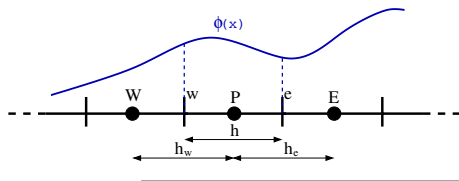
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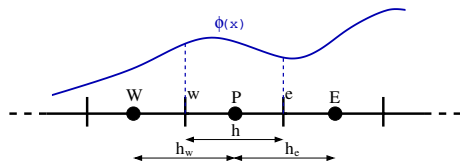
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Towards high-order via deconvolution



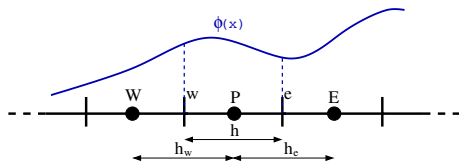
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$$\frac{\partial^2 \varphi}{\partial x^2} \approx \overline{\frac{\partial^2 \bar{\varphi}}{\partial x^2}} + \mathcal{O}(h^2) \quad \text{where} \quad \overline{\frac{\partial^2 \bar{\varphi}}{\partial x^2}} = \frac{\varphi_E - 2\varphi_P + \varphi_W}{h^2}$$

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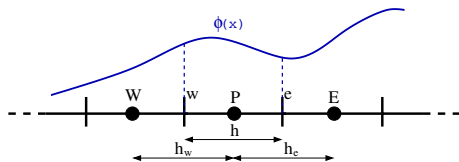
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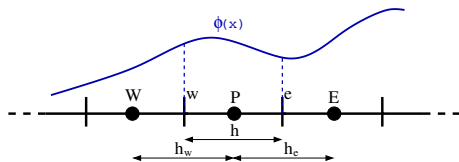
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Recalling that $\phi = \bar{\phi} + \phi'$ and $\phi'_P \approx -\frac{1}{24}(\varphi_E - 2\varphi_P + \varphi_W) + \mathcal{O}(h^2)$

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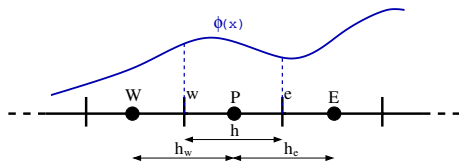
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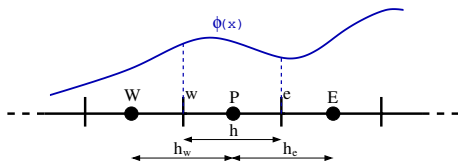
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$$\frac{\partial^2 \varphi}{\partial x^2} \approx \frac{-\varphi_{EE} + 16\varphi_E - 30\varphi_P + 16\varphi_W - \varphi_{WW}}{12h^2} + \mathcal{O}(h^4)$$

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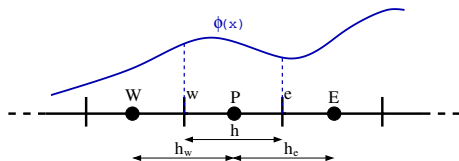
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This can be extended to 2D/3D (notice that $\varphi' = -\frac{h^2}{24} \nabla^2 \varphi + \mathcal{O}(h^4)$)

$$\boxed{\nabla^2 \varphi = \overline{\nabla^2 \varphi} + \overline{\nabla^2 \varphi'} + (\nabla^2 \bar{\varphi})' + \mathcal{O}(h^4)}$$

Contents

- 1 Background
- 2 Motivation
- 3 Revisiting FVM
- 4 A rational length scale for LES
- 5 Conclusions

Subgrid characteristic length for LES: state of the art

$$\partial_t \bar{\mathbf{u}} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{u}} = \nabla^2 \bar{\mathbf{u}} - \nabla \bar{p} - \nabla \cdot \boldsymbol{\tau}(\bar{\mathbf{u}}) ; \quad \nabla \cdot \bar{\mathbf{u}} = 0$$

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Subgrid characteristic length for LES: state of the art

- In the context of **LES**, most popular (by far) is:

$$\boxed{\delta_{\text{vol}} = (\Delta x \Delta y \Delta z)^{1/3}} \Longleftarrow \text{Deardorff (1970)}$$

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- In the context of **DES**:

$$\delta_{\text{max}} = \max(\Delta x, \Delta y, \Delta z) \Leftarrow \text{Sparlart et al. (1997)}$$

Flow-dependant definitions

$$\delta_{\omega} = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y) / |\omega|^2} \Leftarrow \text{Chauvet et al. (2007)}$$

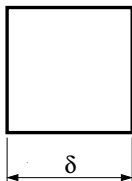
$$\tilde{\delta}_{\omega} = \frac{1}{\sqrt{3}} \max_{n,m=1,\dots,8} |I_n - I_m| \Leftarrow \text{Mockett et al. (2015)}$$

$$\delta_{\text{SLA}} = \tilde{\delta}_{\omega} F_{\text{KH}}(VTM) \Leftarrow \text{Shur et al. (2015)}$$

A rational length scale for LES

Research question #4:

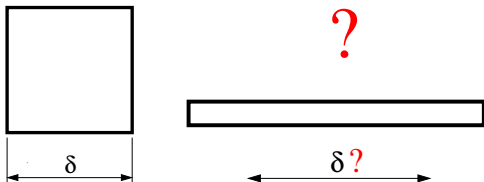
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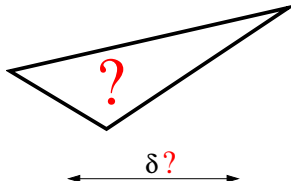
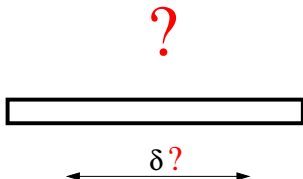
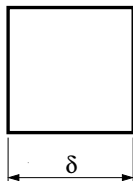
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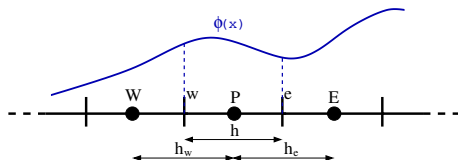
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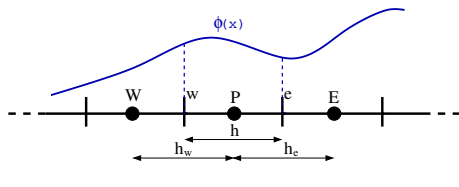


Box filter:
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Remark #1: the actual filter length, δ , when computing the face derivative is h_e , i.e., the distance between the adjacent nodes P and E .

A rational length scale for LES



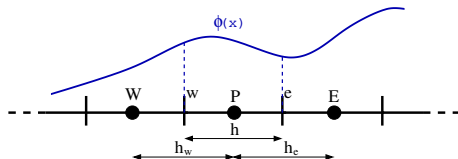
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The diffusive term in a FVM framework is approximated as follows

$$\left. \frac{\partial}{\partial x} \left(\Gamma \frac{\partial \phi}{\partial x} \right) \right|_P \approx \frac{1}{h} \left(\left. \Gamma \frac{\partial \phi}{\partial x} \right|_e - \left. \Gamma \frac{\partial \phi}{\partial x} \right|_w \right) = \overline{\left. \frac{\partial}{\partial x} \left(\Gamma \frac{\partial \phi}{\partial x} \right) \right|_P}$$

A rational length scale for LES



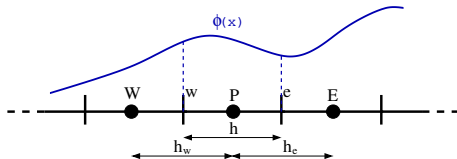
Box filter: $\bar{\phi}(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \bar{\phi} = \overline{\partial_x \phi} = (\phi_e - \phi_w)/h$$

The diffusive term in a FVM framework is approximated as follows

$$\begin{aligned} \frac{\partial}{\partial x} \left(\Gamma \frac{\partial \phi}{\partial x} \right) \Big|_P &\approx \frac{1}{h} \left(\Gamma \frac{\partial \phi}{\partial x} \Big|_e - \Gamma \frac{\partial \phi}{\partial x} \Big|_w \right) = \overline{\frac{\partial}{\partial x} \left(\Gamma \frac{\partial \phi}{\partial x} \right)} \Big|_P \\ &\approx \frac{1}{h} \left(\Gamma_e \frac{\phi_E - \phi_P}{h_e} - \Gamma_w \frac{\phi_P - \phi_W}{h_w} \right) = \overline{\frac{\partial}{\partial x} \left(\Gamma \frac{\partial \phi}{\partial x} \right)} \Big|_P \end{aligned}$$

A rational length scale for LES



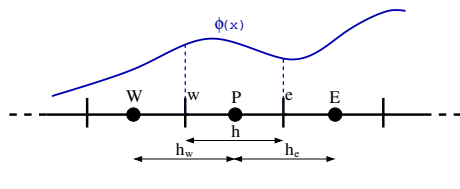
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A rational length scale for LES



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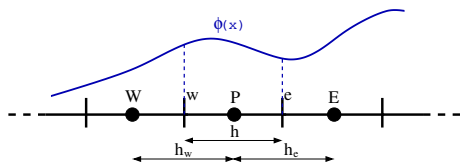
Remark #1: the actual filter length, δ , when computing the face derivative is h_e , i.e., the distance between the adjacent nodes P and E .

Remark #2: two filtering operations are performed when computing the diffusive term:

- the calculation of the **face derivative**
- the cell-to-face **interpolation of Γ**

$$\left. \frac{\partial}{\partial x} \left(\overline{\Gamma} \frac{\partial \phi}{\partial x} \right) \right|_P$$

A rational length scale for LES



Box filter:
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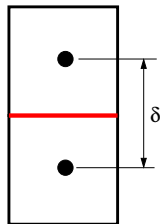
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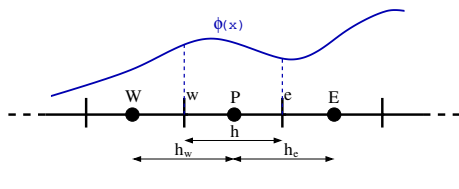
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Both filtering operators share the same filter length, δ ; namely, the distance between the nodes adjacent to the corresponding **face**.



A rational length scale for LES



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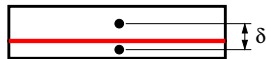
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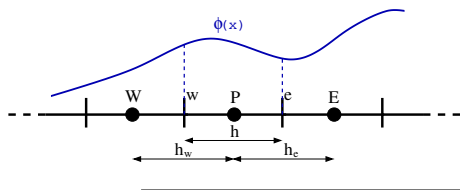
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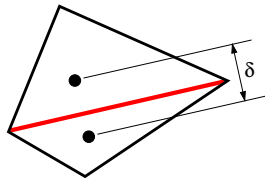
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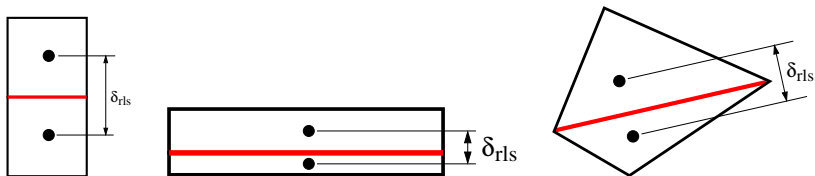
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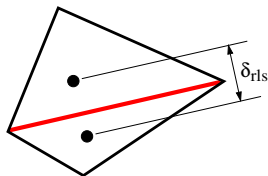
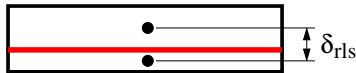
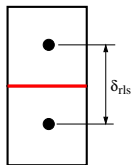
A rational length scale

Properties of new definition, δ_{rls}



A rational length scale

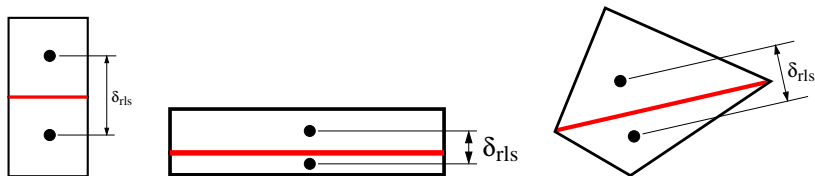
Properties of new definition, δ_{rls}



- Locally defined

A rational length scale

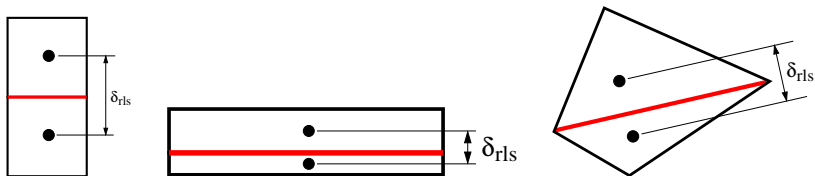
Properties of new definition, δ_{rls}



- Locally defined
- Well-bounded: $\Delta x \leq \delta_{rls} \leq \Delta z$ (assuming $\Delta x \leq \Delta y \leq \Delta z$)

A rational length scale

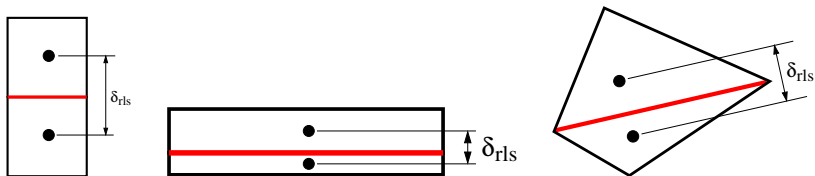
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A rational length scale

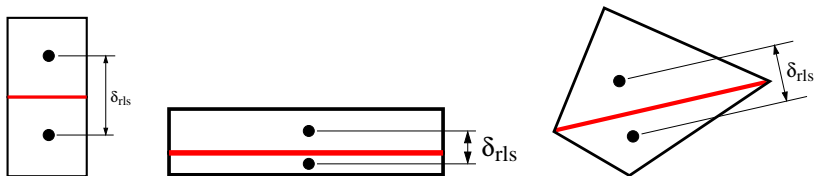
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- Applicable to unstructured grids

A rational length scale

Properties of new definition, δ_{rls}



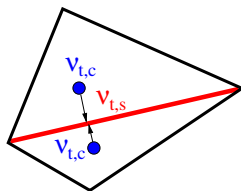
- Locally defined
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- Sensitive to flow orientation, e.g. shear layers
- Applicable to unstructured grids
- Easy and cheap

A rational length scale

Implementation and an alternative definition

$$\nu_{t,c} \xrightarrow{\text{interpolation}} \nu_{t,s}$$

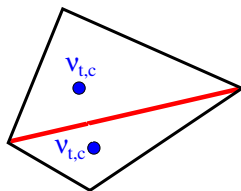
$$\nu_{t,c} \dashrightarrow \hat{\nu}_{t,c} \longrightarrow \hat{\nu}_{t,s} \dashrightarrow \nu_{t,s}$$



A rational length scale

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$$\nu_{t,c} \xrightarrow{\text{interpolation}} \nu_{t,s}$$



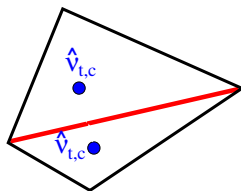
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A rational length scale

Implementation and an alternative definition

$$\nu_{t,c} \xrightarrow{\text{interpolation}} \nu_{t,s}$$

$$\nu_{t,c} \xrightarrow{1/\delta_{vol}^2} \hat{\nu}_{t,c} \longrightarrow \hat{\nu}_{t,s} \dashrightarrow \nu_{t,s}$$

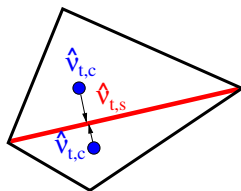


A rational length scale

Implementation and an alternative definition

$$\boldsymbol{\nu}_{t,c} \xrightarrow{\text{interpolation}} \boldsymbol{\nu}_{t,s}$$

$$\boldsymbol{\nu}_{t,c} \xrightarrow{1/\delta_{vol}^2} \hat{\boldsymbol{\nu}}_{t,c} \xrightarrow{\text{interpolation}} \hat{\boldsymbol{\nu}}_{t,s} \xrightarrow{\quad} \boldsymbol{\nu}_{t,s}$$

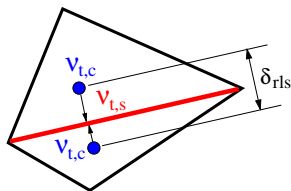


A rational length scale

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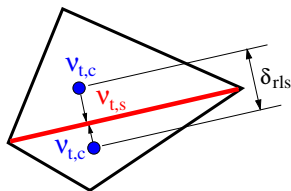


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We can also compute an equivalent filter length, $\tilde{\delta}_{rls}$, that leads to the same local dissipation

$$\tilde{\delta}_{rls}^2 \hat{\nu}_t \mathbf{G} : \mathbf{G} = \hat{\nu}_t \hat{\mathbf{G}} : \hat{\mathbf{G}}$$

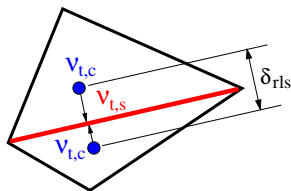
where $\hat{\mathbf{G}} \equiv \mathbf{G} \Delta$ and $\Delta \equiv \text{diag}(\Delta x, \Delta y, \Delta x)$.

A rational length scale

Implementation and an alternative definition

$$\nu_{t,c} \xrightarrow{\text{interpolation}} \nu_{t,s}$$

$$\nu_{t,c} \xrightarrow{1/\delta_{vol}^2} \hat{\nu}_{t,c} \xrightarrow{\text{interpolation}} \hat{\nu}_{t,s} \xrightarrow{\delta_{rls}^2} \nu_{t,s}$$



We can also compute an equivalent filter length, $\tilde{\delta}_{rls}$, that leads to the same local dissipation

$$\tilde{\delta}_{rls}^2 \hat{\nu}_t \mathbf{G} : \mathbf{G} = \hat{\nu}_t \hat{\mathbf{G}} : \hat{\mathbf{G}} \quad \Longrightarrow \quad \boxed{\tilde{\delta}_{rls} = \sqrt{\frac{\hat{\mathbf{G}} : \hat{\mathbf{G}}}{\mathbf{G} : \mathbf{G}}} = \sqrt{\frac{\text{tr}(\hat{\mathbf{G}}\hat{\mathbf{G}}^T)}{\text{tr}(\mathbf{G}\mathbf{G}^T)}}$$

where $\hat{\mathbf{G}} \equiv \mathbf{G}\Delta$ and $\Delta \equiv \text{diag}(\Delta x, \Delta y, \Delta x)$.

A rational length scale

Properties of new definition $\tilde{\delta}_{rls}$

$$\Delta = \begin{pmatrix} \Delta x & 0 \\ 0 & \Delta y \end{pmatrix}$$

$$\mathbf{G} = \begin{pmatrix} \partial_x u & \partial_y u \\ \partial_y u & \partial_y v \end{pmatrix}$$

A rational length scale

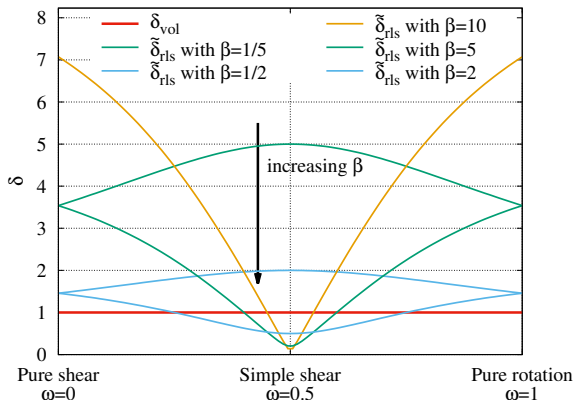
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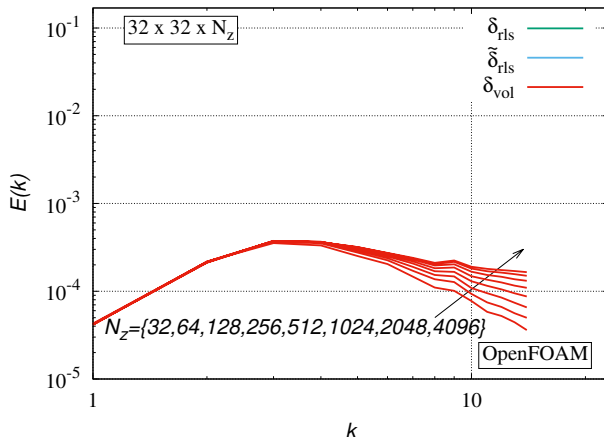
$$\Delta = \begin{pmatrix} \Delta x & 0 \\ 0 & \Delta y \end{pmatrix} = \begin{pmatrix} \beta & 0 \\ 0 & \beta^{-1} \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} \partial_x u & \partial_y u \\ \partial_y u & \partial_y v \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 - 2\omega & 0 \end{pmatrix}$$



A rational length scale

Isotropic turbulence on anisotropic grids

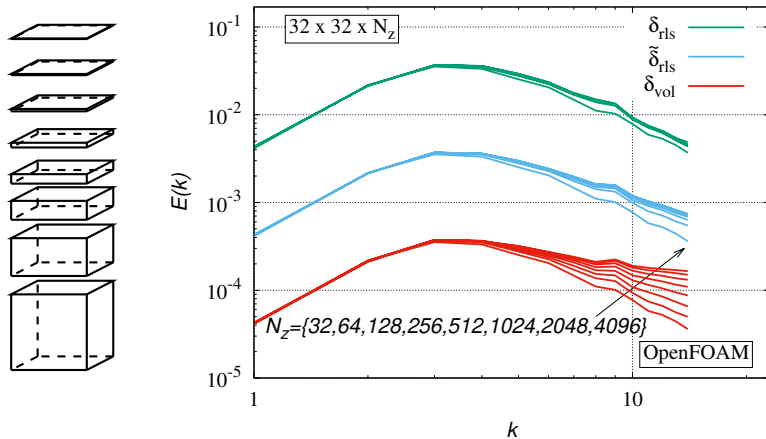
Comparison with classical Comte-Bellot & Corrsin (CBC) experiment



A rational length scale

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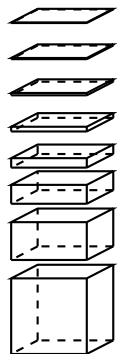
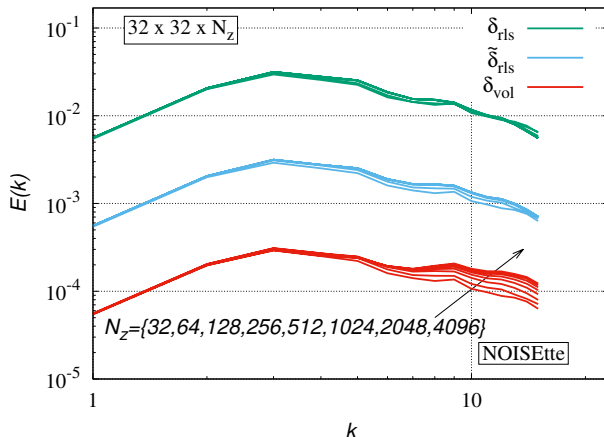
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A rational length scale

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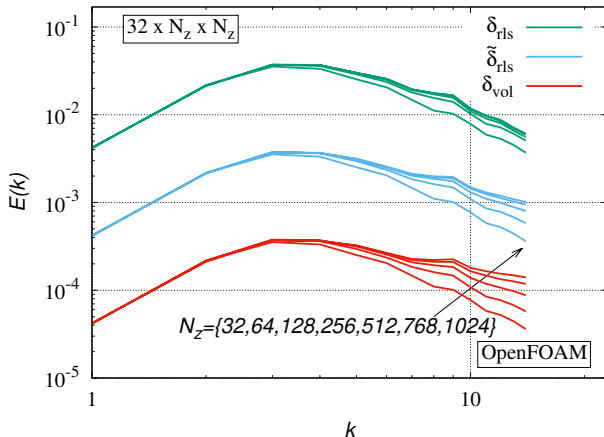
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A rational length scale

Isotropic turbulence on anisotropic grids

Comparison with classical Comte-Bellot & Corrsin (CBC) experiment



Contents

1 Background

2 Motivation

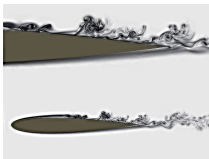
3 Revisiting FVM

4 A rational length scale for LES

5 Conclusions

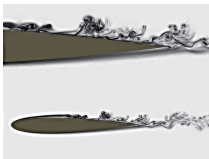
Concluding remarks

- **Preserving operator symmetries** is the key point for **reliable LES/DNS** simulations



Concluding remarks

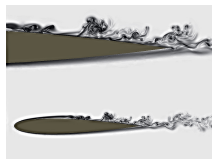
- **Preserving operator symmetries** is the key point for **reliable LES/DNS** simulations
- Numerical **schemes in FVM** can be viewed as **spatial (box) filters**



$$\frac{\partial \varphi}{\partial t} + \frac{\partial(\mathbf{u}\varphi)}{\partial x} = \nu \frac{\partial^2 \varphi}{\partial x^2} \xrightarrow{\text{schemes}} \boxed{\frac{\partial \varphi}{\partial t} + \frac{\partial(\overline{\mathbf{u}\varphi})}{\partial x} = \nu \frac{\partial^2 \overline{\varphi}}{\partial x^2}}$$

Concluding remarks

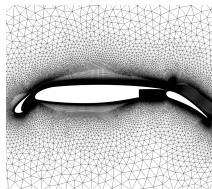
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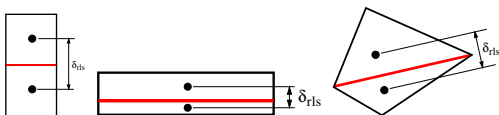
- In principle, the analysis can be extended to **3D problems** and **unstructured grids**

$$\boxed{\nabla^2 \varphi = \overline{\nabla^2 \varphi} + \overline{\nabla^2 \varphi'} + (\nabla^2 \overline{\varphi})' + \mathcal{O}(h^4)}$$



Concluding remarks

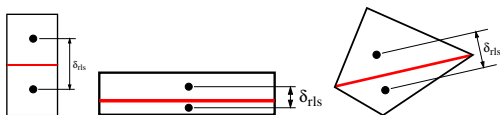
- A new definition for δ has been proposed



$$\tilde{\delta}_{rls} = \sqrt{\frac{\hat{G} : \hat{G}}{G : G}}$$

Concluding remarks

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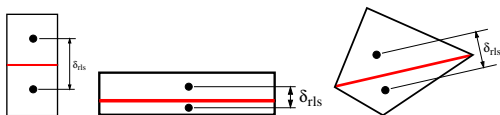


$$\tilde{\delta}_{rls} = \sqrt{\frac{\hat{G} : \hat{G}}{G : G}}$$

- It is locally defined, well-bounded, cheap and easy to implement
- Suitable for unstructured grids

Concluding remarks

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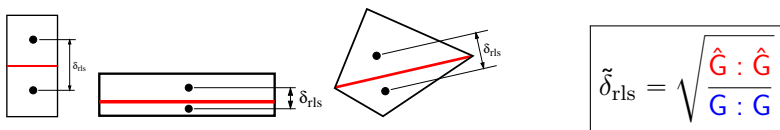


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Concluding remarks

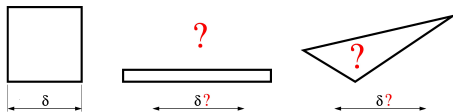
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Takeaway message:

- Definition of δ can have a big effect on simulation results



Thank you for your attendance

