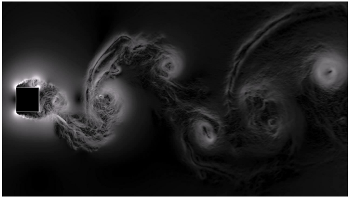
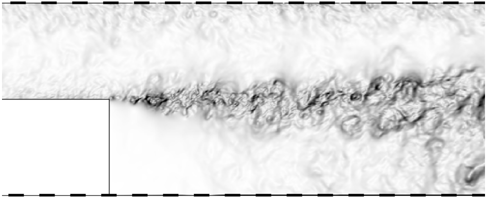
Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Interactions between numerical discretization, subgrid modeling and filtering in LES

F.Xavier Trias

Heat and Mass Transfer Technological Center, Technical University of Catalonia (UPC)





CTTC UPC
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Contents

Contents

About myself...

Professional...

- Current position (since 2024): **Full Professor** at UPC
- Previous positions: PostDoc at University of Groningen (2007-2009) and UPC (2010-2013), *Ramón y Cajal* Senior Researcher at UPC (2013-2018) and Associate Professor (2018-2023).
- My **research** focus is on fluid mechanics, turbulence modeling, physics and numerics of complex flows, applied mathematics and numerical methods.
- Some numbers: 58 papers, 190 conferences, 12 PhD's+5 (on-going)
- Stays and collaborations: Groningen (Netherlands), UCLA, KIAM (Russian Academy of Sciences), Stanford, Tsinghua (China), TokioTech (Japan), Napoli (Italy), CWI (Netherlands),...
- More info: www.fxtrias.com

About myself...

... and more personal stuff

- My complete name: Francesc Xavier Trias Miquel
- Born in Barcelona
- My mother tongue is Catalan. I also speak Spanish at native level.
- Hobbies? I like my work but also sports. Most practiced ones are running and football:

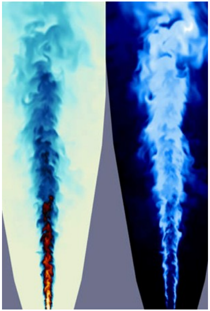
Groningen (2009?)

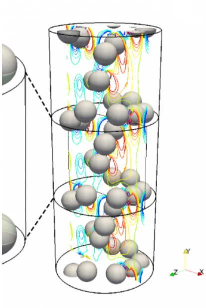
Barcelona (2019)

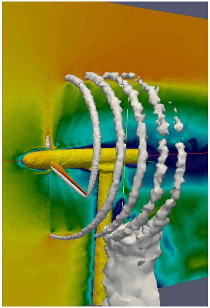
The CTTC research group

Heat and Mass Transfer Technological Center (Catalan: *Centre Tecnològic de Transferència de Calor*) has more than 30 years experience on CFD:

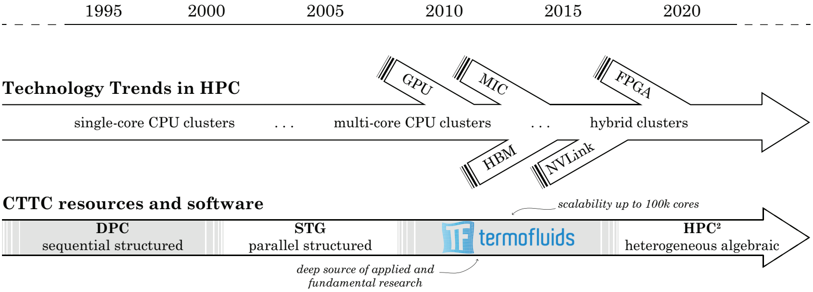
- **Fundamental research** on numerical methods, fluid dynamics and heat and mass transfer phenomena.
- **Applied research** on thermal and fluid dynamic optimization of thermal system and equipment.

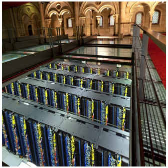


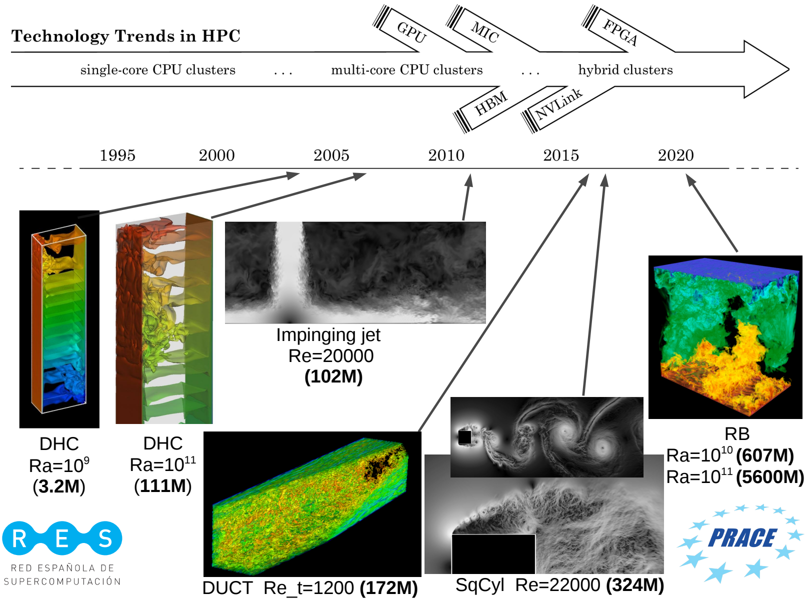




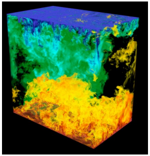
CTTC's historical background in HPC





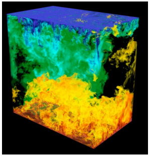

Technology Trends in HPC
GPU
MIC
FPGA
single-core CPU clusters
. . .
multi-core CPU clusters
. . .
hybrid clusters
HBM
NVLink
1995
2000
2005
2010
2015
2020
Impinging jet
Re=20000
(102M)
DHC
Ra=10^9
(3.2M)
DHC
Ra=10^11
(111M)
DUCT Re_t=1200 (172M)
SqCyl Re=22000 (324M)
RB
Ra=10^10 (607M)
Ra=10^11 (5600M)
RES
RED ESPAÑOLA DE SUPERCOMPUTACIÓN
PRACE

General motivation: (very) large-scale DNS/LES

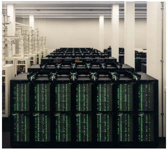


DNS

General motivation: (very) large-scale DNS/LES

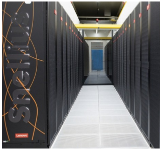


MareNostrum 5-ACC
(Barcelona)



rank #8
1120 nodes with:
2× Intel Shappire Rapids 8460
1× NVIDIA Hopper 64 GB HBM
1× Infiniband NDR200

Snellius
(Amsterdam)



rank #165
714 nodes with:
2× AMD EPYC 9654 96C
1× Infiniband NDR200

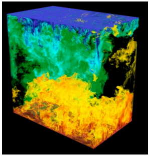
TSUBAME4.0
(Tokyo)

rank #31
240 nodes with:
2× AMD EPYC 9654 96-Core
4× NVIDIA H100 SXM5
1× Infiniband NDR200

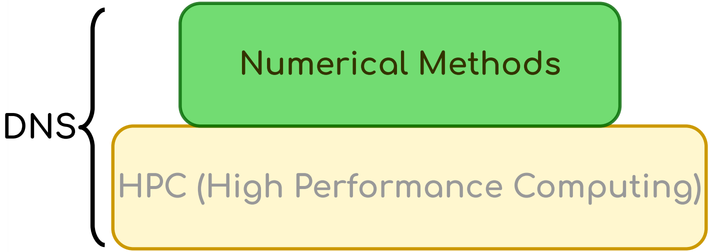
DNS

HPC (High Performance Computing)

General motivation: (very) large-scale DNS/LES

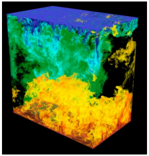


How to properly discretize NS?



General motivation: (very) large-scale DNS/LES

How to properly model SGS?



SGS models

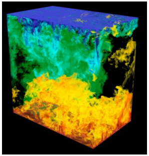
Numerical Methods

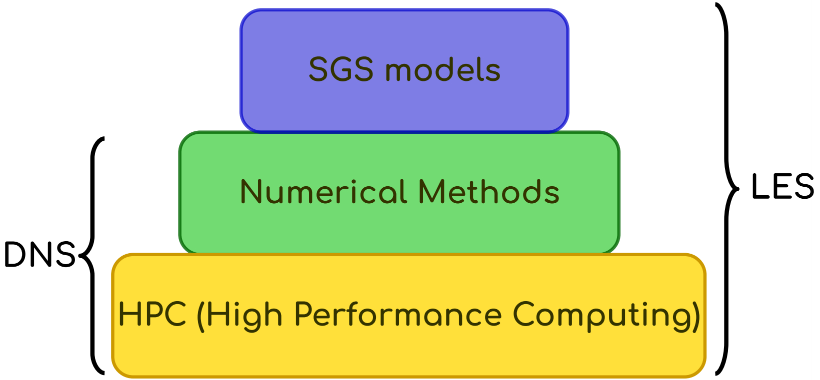
HPC (High Performance Computing)

DNS

LES

General motivation: (very) large-scale DNS/LES



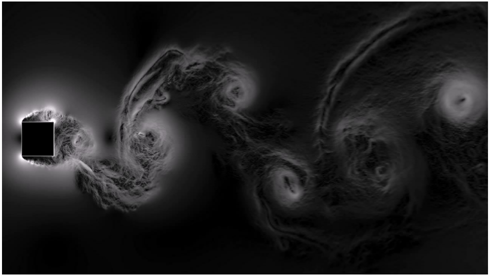


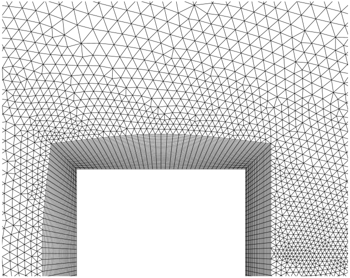
Contents

Motivation

Research question #1:

- What are we indeed solving with **finite volume method**?





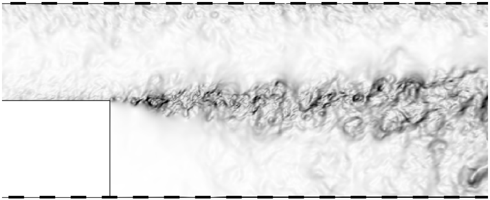
DNS[1] of the turbulent flow around a square cylinder at $Re = 22000$

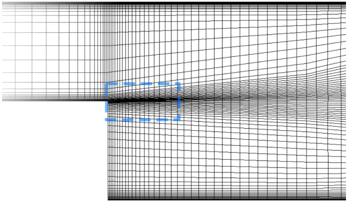
[1] F.X.Trias, A.Gorobets, A.Oliva. *Turbulent flow around a square cylinder at Reynolds number 22000: a DNS study*, **Computers&Fluids**, 123:87-98, 2015.

Motivation

Research question #1:

- What are we indeed solving with **finite volume method**?





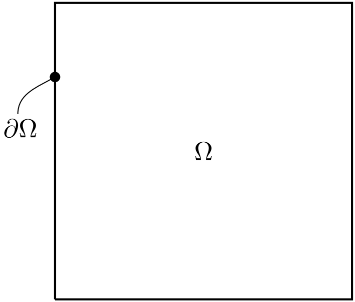
DNS[2] of backward-facing step at $Re_\tau = 395$ and expansion ratio 2

[2] A.Pont-Vílchez, F.X.Trias, A.Gorobets, A.Oliva. *DNS of Backward-Facing Step flow at* $Re_\tau = 395$ *and expansion ratio 2*. **Journal of Fluid Mechanics**, 863:341-363, 2019.

Motivation

Research question #2:

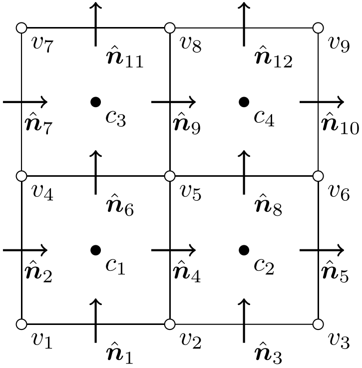
- What are we interpolating? What is the correct interpretation?



Motivation

Research question #2:

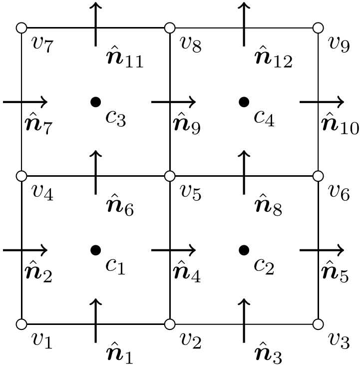
- What are we interpolating? What is the correct interpretation?

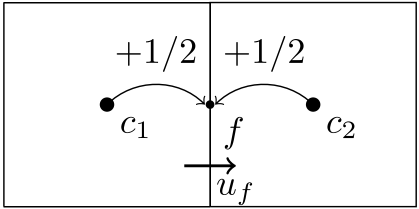


Motivation

Research question #2:

- What are we interpolating? What is the correct interpretation?

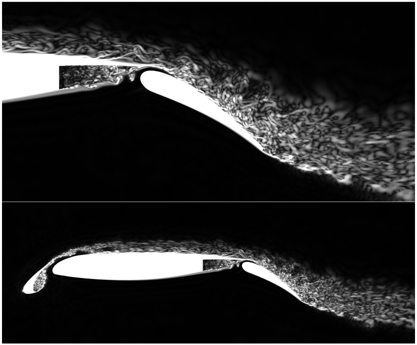


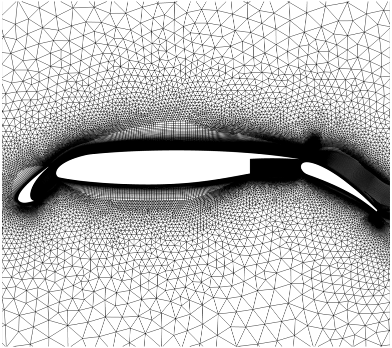


Motivation

Research question #3:

- Can we developed **high-order** symmetry-preserving schemes for **unstructured grids**?

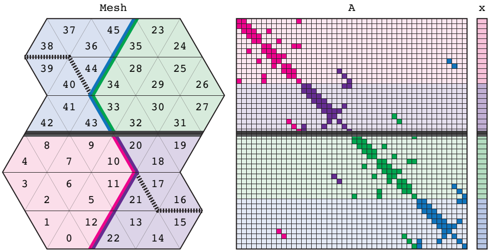


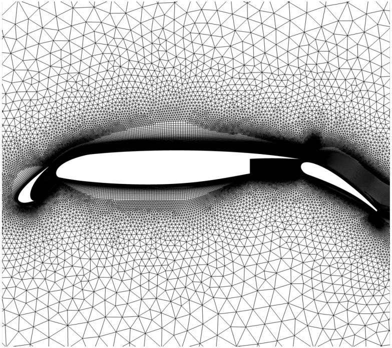


Motivation

Research question #3:

- Can we developed **high-order** symmetry-preserving schemes for **unstructured grids**? And, therefore, get better performance





Reminder: symmetry-preserving discretization

Continuous

$$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$$

$$\nabla \cdot \boldsymbol{u} = 0$$

Reminder: symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\, \boldsymbol{u}_h = \mathsf{D} \boldsymbol{u}_h - \mathsf{G}_h \boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M} \boldsymbol{u}_h = \mathbf{0}_h$

Reminder: symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d \boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\, \boldsymbol{u}_h = \mathsf{D} \boldsymbol{u}_h - \mathsf{G}_h \boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M} \boldsymbol{u}_h = \boldsymbol{0}_h$
$(\boldsymbol{a}, \boldsymbol{b}) = \int_\Omega \boldsymbol{a} \boldsymbol{b} \, d\Omega$	$(\boldsymbol{a}_h, \boldsymbol{b}_h)_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$

Reminder: symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\, \boldsymbol{u}_h = \mathsf{D}\boldsymbol{u}_h - \mathsf{G}_h \boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M}\boldsymbol{u}_h = \boldsymbol{0}_h$
$(\boldsymbol{a}, \boldsymbol{b}) = \int_\Omega \boldsymbol{a}\boldsymbol{b}\, d\Omega$	$(\boldsymbol{a}_h, \boldsymbol{b}_h)_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$
$(C(\boldsymbol{u}, \phi_1), \phi_2) = -(C(\boldsymbol{u}, \phi_2), \phi_1)$	$\mathsf{C}(\boldsymbol{u}_h) = -\mathsf{C}^T(\boldsymbol{u}_h)$

Reminder: symmetry-preserving discretization

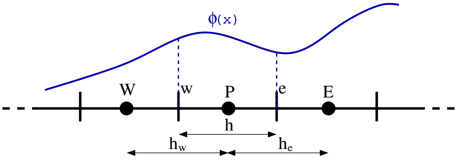
Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d\boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\, \boldsymbol{u}_h = \mathsf{D}\boldsymbol{u}_h - \mathsf{G}_h \boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M}\boldsymbol{u}_h = \boldsymbol{0}_h$
$(\boldsymbol{a}, \boldsymbol{b}) = \int_\Omega \boldsymbol{a}\boldsymbol{b}\, d\Omega$	$(\boldsymbol{a}_h, \boldsymbol{b}_h)_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$
$(C(\boldsymbol{u}, \phi_1), \phi_2) = -(C(\boldsymbol{u}, \phi_2), \phi_1)$	$\mathsf{C}(\boldsymbol{u}_h) = -\mathsf{C}^T(\boldsymbol{u}_h)$
$(\nabla \cdot \boldsymbol{a}, \phi) = -(\boldsymbol{a}, \nabla \phi)$	$\Omega \mathsf{G}_h = -\mathsf{M}^T$

Reminder: symmetry-preserving discretization

Continuous	Discrete
$\frac{\partial \boldsymbol{u}}{\partial t} + C(\boldsymbol{u}, \boldsymbol{u}) = \nu \nabla^2 \boldsymbol{u} - \nabla p$	$\Omega \frac{d \boldsymbol{u}_h}{dt} + \mathsf{C}(\boldsymbol{u}_h)\, \boldsymbol{u}_h = \mathsf{D} \boldsymbol{u}_h - \mathsf{G}_h \boldsymbol{p}_h$
$\nabla \cdot \boldsymbol{u} = 0$	$\mathsf{M} \boldsymbol{u}_h = \boldsymbol{0}_h$
$(\boldsymbol{a}, \boldsymbol{b}) = \int_\Omega \boldsymbol{a} \boldsymbol{b} d\Omega$	$(\boldsymbol{a}_h, \boldsymbol{b}_h)_h = \boldsymbol{a}_h^T \Omega \boldsymbol{b}_h$
$(C(\boldsymbol{u}, \phi_1), \phi_2) = -(C(\boldsymbol{u}, \phi_2), \phi_1)$	$\mathsf{C}(\boldsymbol{u}_h) = -\mathsf{C}^T(\boldsymbol{u}_h)$
$(\nabla \cdot \boldsymbol{a}, \phi) = -(\boldsymbol{a}, \nabla \phi)$	$\Omega \mathsf{G}_h = -\mathsf{M}^T$
$(\nabla^2 \boldsymbol{a}, \boldsymbol{b}) = (\boldsymbol{a}, \nabla^2 \boldsymbol{b})$	$\mathsf{D} = \mathsf{D}^T \quad def -$

Contents

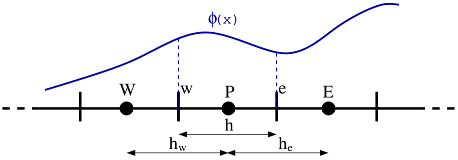
Revisiting FVM



Box filter: $$\overline{\phi}(x) = \frac{1}{h}\int_{x-h/2}^{x+h/2} \phi dx$$

$$\partial_x \overline{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

Revisiting FVM



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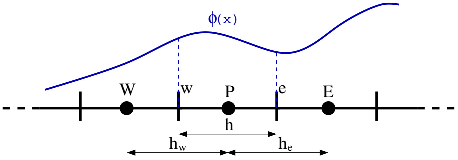
$$\partial_x \overline{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\boldsymbol{u}\phi)}{\partial x} = \nu \frac{\partial^2 \phi}{\partial x^2}$$

Revisiting FVM



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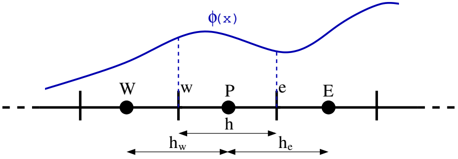
$$\partial_x \overline{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial (\boldsymbol{u}\phi)}{\partial x} = \nu \frac{\partial^2 \phi}{\partial x^2} \longrightarrow \boxed{\frac{\partial \overline{\phi}}{\partial t} + \frac{\overline{\partial (\boldsymbol{u}\phi)}}{\partial x} = \nu \frac{\overline{\partial^2 \phi}}{\partial x^2}}$$ Exact FVM eq!

Revisiting FVM



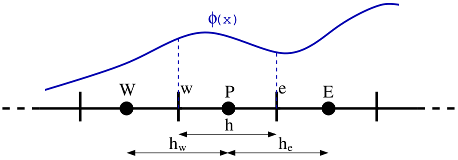
Box filter: $\overline{\phi}(x) = \dfrac{1}{h}\displaystyle\int_{x-h/2}^{x+h/2} \phi dx$

$$\partial_x \overline{\phi}|_e = \overline{\partial_x \phi}|_e = (\phi_E - \phi_P)/h_e$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial (\boldsymbol{u}\phi)}{\partial x} = \nu \frac{\partial^2 \phi}{\partial x^2} \longrightarrow \boxed{\frac{\partial \overline{\phi}}{\partial t} + \overline{\frac{\partial (\boldsymbol{u}\phi)}{\partial x}} = \nu \overline{\frac{\partial^2 \phi}{\partial x^2}}}$$ Exact FVM eq!

$$h\frac{\partial \overline{\phi}_P}{\partial t} + (\boldsymbol{u}\phi)_e - (\boldsymbol{u}\phi)_w = \nu \left(\left.\frac{\partial \phi}{\partial x}\right|_e - \left.\frac{\partial \phi}{\partial x}\right|_w \right)$$

Revisiting FVM



Box filter: $$\overline{\phi}(x) = \frac{1}{h}\int_{x-h/2}^{x+h/2} \phi dx$$

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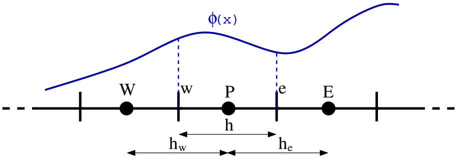
$$h\frac{\partial \overline{\phi}_P}{\partial t} + (\boldsymbol{u}\phi)_e - (\boldsymbol{u}\phi)_w = \nu \left(\left.\frac{\partial \phi}{\partial x}\right|_e - \left.\frac{\partial \phi}{\partial x}\right|_w \right)$$

$$h\frac{\partial \overline{\phi}_P}{\partial t} + \qquad =$$

$$\boxed{\frac{\partial \overline{\phi}}{\partial t} + \qquad = \qquad}$$

Revisiting FVM



Box filter: $\overline{\phi}(x) = \dfrac{1}{h}\displaystyle\int_{x-h/2}^{x+h/2} \phi\, dx$

$$\partial_x \overline{\phi}\big|_e = \overline{\partial_x \phi}\big|_e = (\phi_E - \phi_P)/h_e$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\boldsymbol{u}\phi)}{\partial x} = \nu \frac{\partial^2 \phi}{\partial x^2} \quad \longrightarrow \quad \boxed{\frac{\partial \overline{\phi}}{\partial t} + \overline{\frac{\partial(\boldsymbol{u}\phi)}{\partial x}} = \nu\, \overline{\frac{\partial^2 \phi}{\partial x^2}}}$$

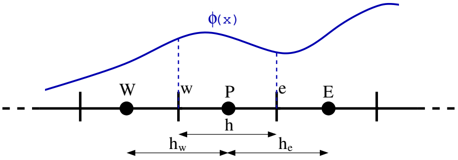
Exact FVM eq!

$$h\frac{\partial \overline{\phi}_P}{\partial t} + (\boldsymbol{u}\phi)_e - (\boldsymbol{u}\phi)_w = \nu \left(\left.\frac{\partial \phi}{\partial x}\right|_e - \left.\frac{\partial \phi}{\partial x}\right|_w \right)$$

$$h\frac{\partial \overline{\phi}_P}{\partial t} + \qquad = \nu \left(\frac{\overline{\phi}_E - \overline{\phi}_P}{h_e} - \frac{\overline{\phi}_P - \overline{\phi}_W}{h_w} \right)$$

$$\boxed{\frac{\partial \overline{\phi}}{\partial t} + \qquad = \nu\, \overline{\frac{\partial^2 \overline{\overline{\phi}}}{\partial x^2}}}$$

Revisiting FVM



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Exact FVM eq!

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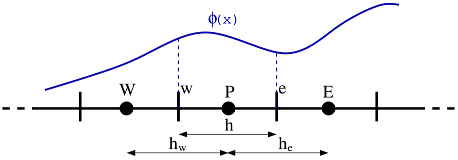
Revisiting FVM



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where $\widetilde{\phi}_e = \frac{\overline{\phi}_P + \overline{\phi}_E}{2} = \overline{\phi}_e + \mathcal{O}(h^2)$

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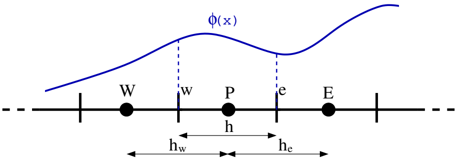

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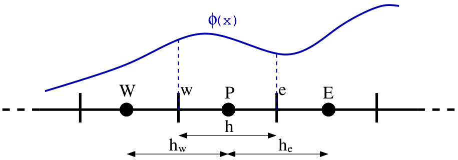

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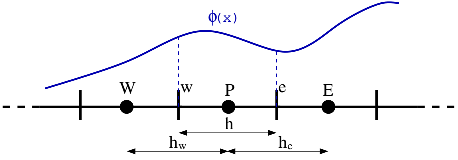
Approx FVM eq!

Towards high-order via deconvolution



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Hence, the standard 2^{nd}-order symmetry-preserving FVM can be viewed as

$$\frac{\partial^2 \varphi}{\partial x^2} \approx \overline{\frac{\partial^2 \overline{\varphi}}{\partial x^2}} + \mathcal{O}(h^2) \quad \text{where} \quad \overline{\frac{\partial^2 \overline{\varphi}}{\partial x^2}} = \frac{\varphi_E - 2\varphi_P + \varphi_W}{h^2}$$

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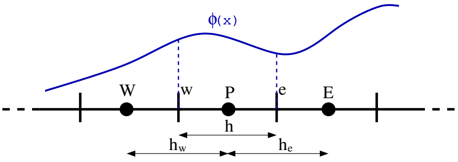

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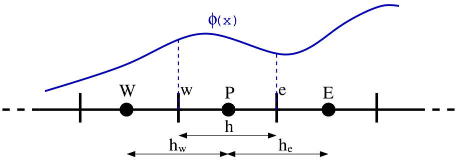

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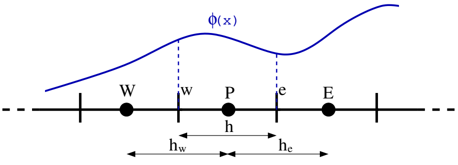

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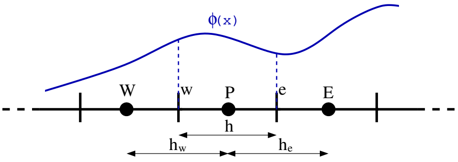

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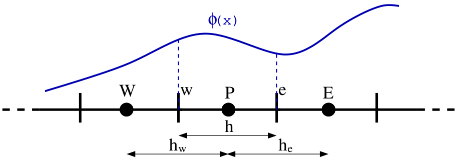

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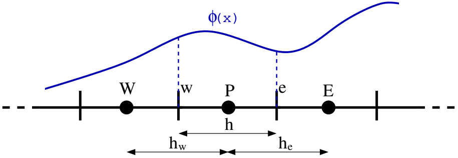

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This can be extended to 2D/3D (notice that $\varphi' = -\frac{h^2}{24}\nabla^2\varphi + \mathcal{O}(h^4)$)

$$\boxed{\nabla^2\varphi = \overline{\nabla^2\overline{\varphi}} + \overline{\nabla^2\varphi'} + (\nabla^2\overline{\varphi})' + \mathcal{O}(h^4)}$$

Contents

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$$\delta_{\mathrm{Sco}} = f(a_1, a_2)\delta_{\mathrm{vol}}, \qquad \delta_{L^2} = \sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)/3}$$

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- In the context of **DES**:

$$\boxed{\delta_{\max} = \max(\Delta x, \Delta y, \Delta z)} \Longleftarrow \textbf{Sparlart et al. (1997)}$$

Flow-dependant definitions

$$\delta_{\boldsymbol{\omega}} = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\boldsymbol{\omega}|^2} \Longleftarrow \textbf{Chauvet et al. (2007)}$$

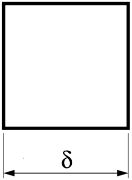
$$\tilde{\delta}_{\boldsymbol{\omega}} = \frac{1}{\sqrt{3}} \max_{n,m=1,\dots,8} |\boldsymbol{l}_n - \boldsymbol{l}_m| \qquad \Longleftarrow \textbf{Mockett et al. (2015)}$$

$$\delta_{\text{SLA}} = \tilde{\delta}_{\boldsymbol{\omega}} F_{\text{KH}}(VTM) \qquad \Longleftarrow \textbf{Shur et al. (2015)}$$

A rational length scale for LES

Research question #4:

- Can we establish a **simple**, **robust**, and **easily implementable** definition of δ for any type of grid that minimizes the impact of mesh anisotropies on the performance of subgrid-scale models?



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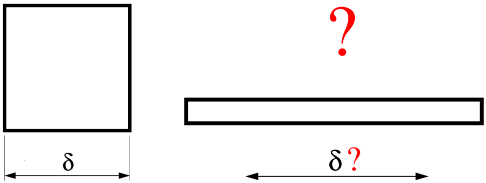

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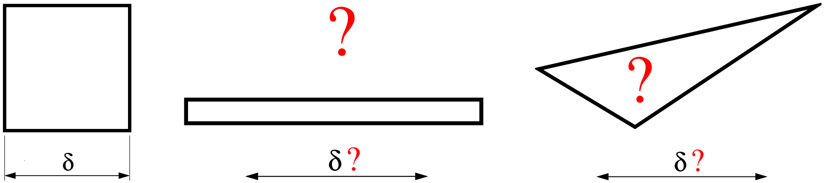

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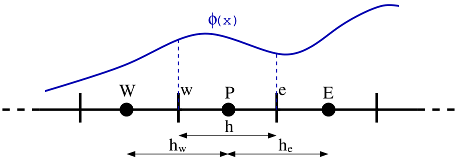

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Remark #1: the actual filter length, δ, when computing the face derivative is h_e, *i.e.*, the distance between the adjacent nodes P and E.

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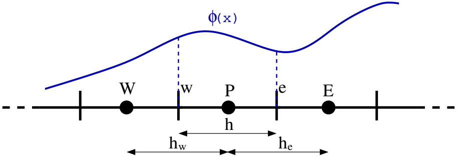

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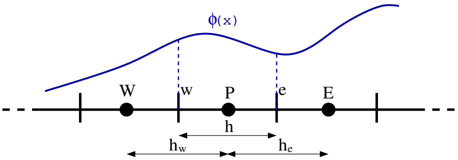
The diffusive term in a FVM framework is approximated as follows

$$\frac{\partial}{\partial x}\left(\Gamma \frac{\partial \phi}{\partial x}\right)\bigg|_P \approx \frac{1}{h}\left(\Gamma \frac{\partial \phi}{\partial x}\bigg|_e - \Gamma \frac{\partial \phi}{\partial x}\bigg|_w\right) = \overline{\frac{\partial}{\partial x}\left(\Gamma \frac{\partial \phi}{\partial x}\right)}\bigg|_P$$

A rational length scale for LES



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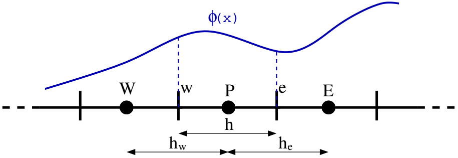

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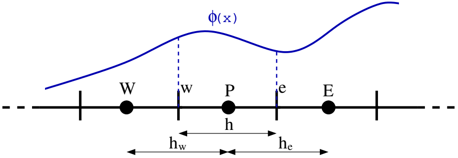
A rational length scale for LES



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Remark #1: the actual filter length, δ, when computing the face derivative is h_e, *i.e.*, the distance between the adjacent nodes P and E.

Remark #2: two filtering operations are performed when computing the diffusive term:

- the calculation of the face derivative
- the cell-to-face interpolation of Γ

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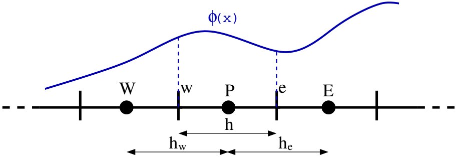
A rational length scale for LES



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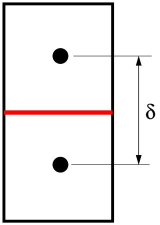
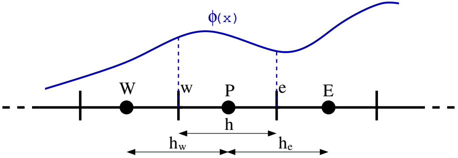


Both filtering operators share the same filter length, δ; namely, the distance between the nodes adjacent to the corresponding **face**.

A rational length scale for LES



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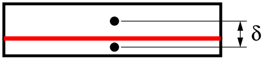

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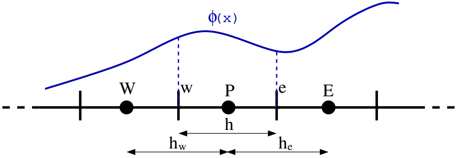

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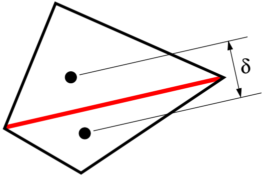

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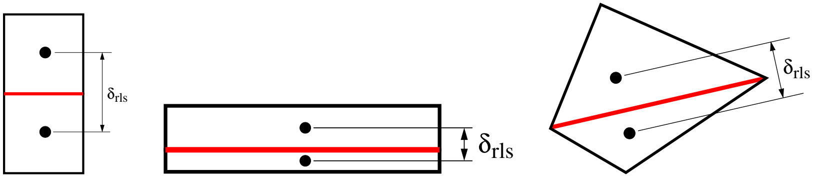
A rational length scale

Properties of new definition, δ_{rls}



A rational length scale

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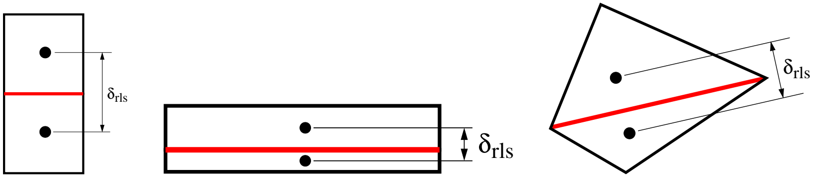
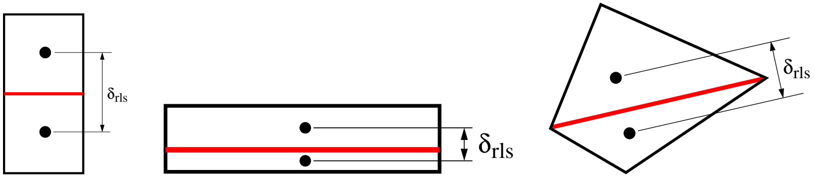


- Locally defined

A rational length scale

Properties of new definition, δ_{rls}



- Locally defined
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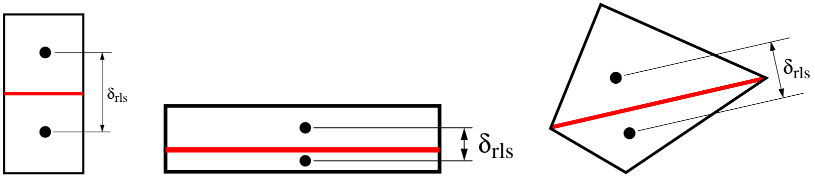
A rational length scale

Properties of new definition, δ_{rls}



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- Sensitive to flow orientation, *e.g.* shear layers

A rational length scale

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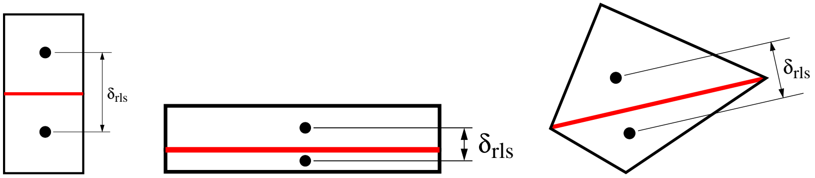

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A rational length scale

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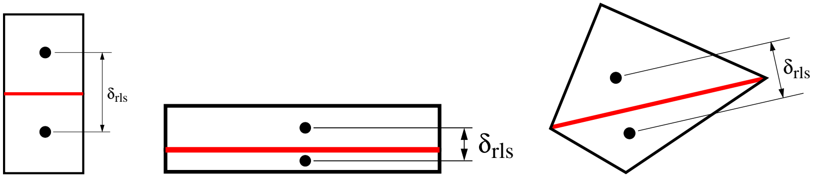

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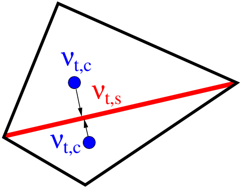
- Sensitive to flow orientation, *e.g.* shear layers
- Applicable to unstructured grids
- Easy and cheap

A rational length scale

Implementation and an alternative definition

$$\nu_{t,c} \xrightarrow{\text{interpolation}} \nu_{t,s}$$



$$\nu_{t,c} \dashrightarrow \hat{\nu}_{t,c} \longrightarrow \hat{\nu}_{t,s} \dashrightarrow \nu_{t,s}$$

A rational length scale

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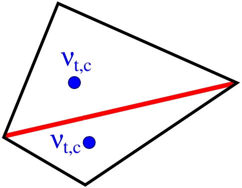

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A rational length scale

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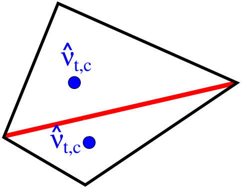

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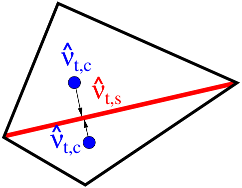

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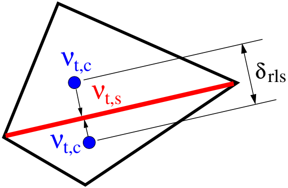

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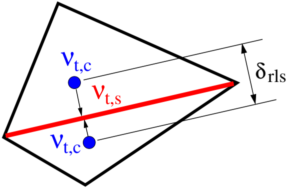

We can also compute an equivalent filter length, $\tilde{\delta}_{\mathrm{rls}}$, that leads to the same local dissipation

$$\tilde{\delta}^2_{\mathrm{rls}} \hat{\nu}_t \mathsf{G} : \mathsf{G} = \hat{\nu}_t \hat{\mathsf{G}} : \hat{\mathsf{G}}$$

where $\hat{\mathsf{G}} \equiv \mathsf{G}\Delta$ and $\Delta \equiv \mathrm{diag}(\Delta x, \Delta y, \Delta x)$.

A rational length scale

Implementation and an alternative definition

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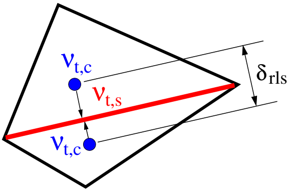

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$$\tilde{\delta}^2_{\mathrm{rls}} \hat{\nu}_t \mathsf{G} : \mathsf{G} = \hat{\nu}_t \hat{\mathsf{G}} : \hat{\mathsf{G}} \quad \Longrightarrow \quad \boxed{\tilde{\delta}_{\mathrm{rls}} = \sqrt{\frac{\hat{\mathsf{G}} : \hat{\mathsf{G}}}{\mathsf{G} : \mathsf{G}}} = \sqrt{\frac{\mathrm{tr}(\hat{\mathsf{G}}\hat{\mathsf{G}}^T)}{\mathrm{tr}(\mathsf{G}\mathsf{G}^T)}}}$$

where $\hat{\mathsf{G}} \equiv \mathsf{G}\Delta$ and $\Delta \equiv \mathrm{diag}(\Delta x, \Delta y, \Delta x)$.

A rational length scale

Properties of new definition $\tilde{\delta}_{\mathrm{rls}}$

$$\Delta = \begin{pmatrix} \Delta x & 0 \\ 0 & \Delta y \end{pmatrix} \qquad G = \begin{pmatrix} \partial_x u & \partial_y u \\ \partial_y u & \partial_y v \end{pmatrix}$$

A rational length scale

Properties of new definition $\tilde{\delta}_{\mathrm{rls}}$

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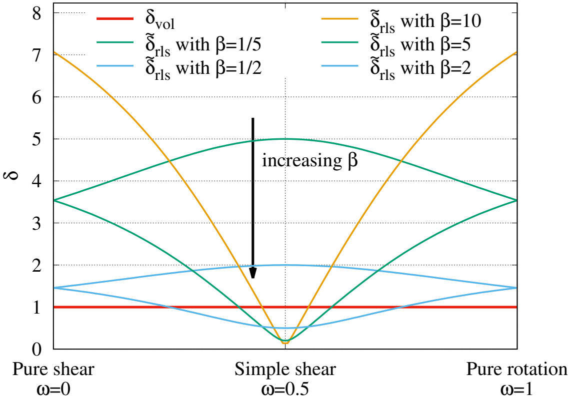
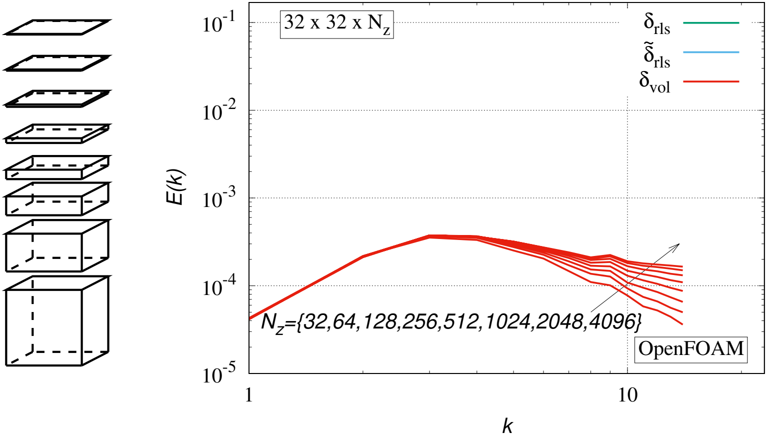


A rational length scale

Isotropic turbulence on anisotropic grids

Comparison with classical Comte-Bellot & Corrsin (CBC) experiment



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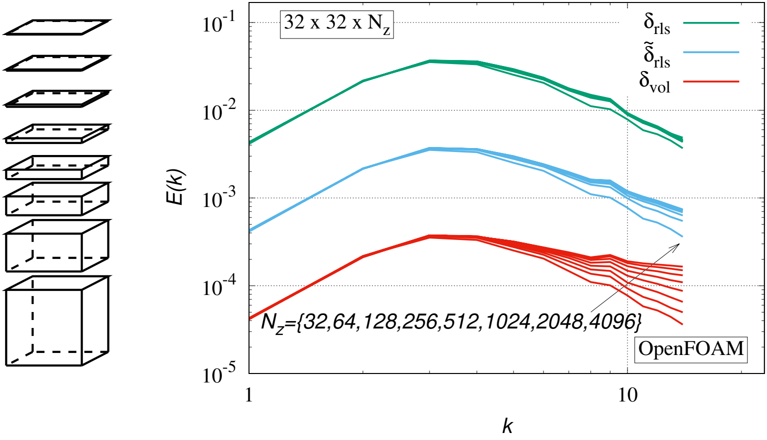

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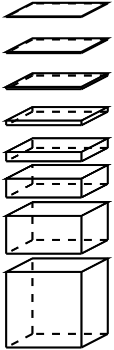

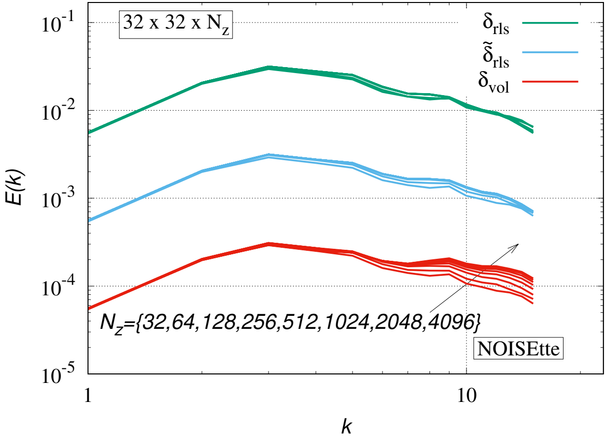

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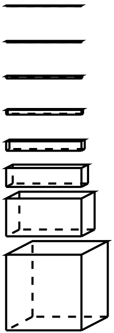
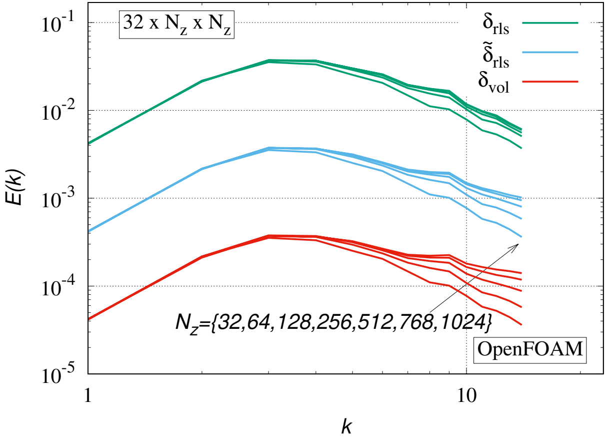
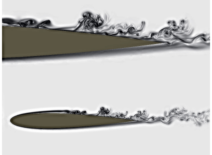




Contents

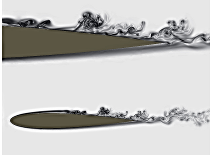
Concluding remarks

- **Preserving operator symmetries** is the key point for **reliable LES/DNS** simulations



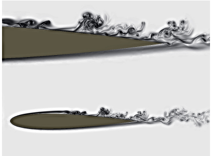
Concluding remarks

- **Preserving operator symmetries** is the key point for **reliable LES/DNS** simulations
- Numerical **schemes in FVM** can be viewed as **spatial (box) filters**



$$\frac{\partial \varphi}{\partial t} + \frac{\partial (\boldsymbol{u}\varphi)}{\partial x} = \nu \frac{\partial^2 \varphi}{\partial x^2} \quad \xrightarrow{\textit{schemes}} \quad \boxed{\frac{\partial \varphi}{\partial t} + \overline{\frac{\partial (\boldsymbol{u}\overline{\varphi})}{\partial x}} = \nu \overline{\frac{\partial^2 \overline{\varphi}}{\partial x^2}}}$$

Concluding remarks



- **Preserving operator symmetries** is the key point for **reliable LES/DNS** simulations
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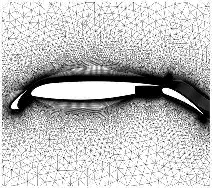
$$\frac{\partial \varphi}{\partial t} + \frac{\partial (\boldsymbol{u}\varphi)}{\partial x} = \nu \frac{\partial^2 \varphi}{\partial x^2} \xrightarrow{\text{schemes}} \boxed{\frac{\partial \varphi}{\partial t} + \overline{\frac{\partial (\boldsymbol{u}\overline{\varphi})}{\partial x}} = \nu \overline{\frac{\partial^2 \overline{\varphi}}{\partial x^2}}}$$

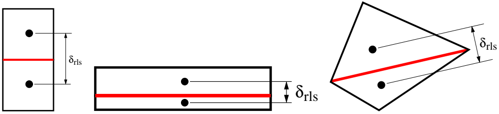
- In principle, the analysis can be extended to **3D problems** and **unstructured grids**

$$\boxed{\nabla^2 \varphi = \overline{\nabla^2 \overline{\varphi}} + \overline{\nabla^2 \varphi'} + (\nabla^2 \overline{\varphi})' + \mathcal{O}(h^4)}$$



Concluding remarks

- A new definition for δ has been proposed



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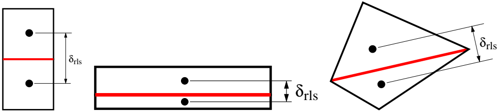

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- It is locally defined, well-bounded, cheap and easy to implement
- Suitable for unstructured grids

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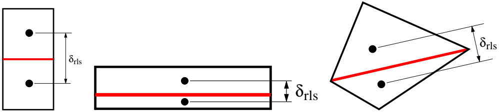

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  - HIT ✓
  - Turbulent channel flow (on-going)
  - Unstructured grids (future work)

Concluding remarks

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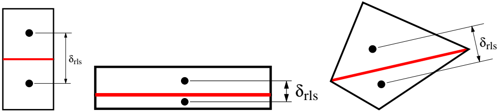

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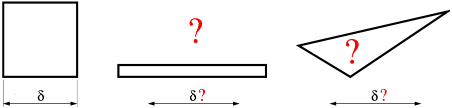
  - Turbulent channel flow (on-going)
  - Unstructured grids (future work)



Takeaway message:

- Definition of δ can have a big effect on simulation results

Thank you for your attendance