A new subgrid length-scale for LES

F.X.Trias[⋆], A.Gorobets[⋆,*], M.H.Silvis[†], R.W.C.P.Verstappen[†], A.Oliva[⋆]

[⋆]Heat and Mass Transfer Technological Center, Technical University of Catalonia

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Symposium for the PhD defense of Maurits Silvis
Groningen (online from Barcelona) October 8^{th}, 2020

A new subgrid length-scale for LES/DES

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rijksuniversiteit groningen

A (not so) new subgrid length-scale for LES/DES

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PHYSICS OF FLUIDS **29**, 115109 (2017)

A new subgrid characteristic length for turbulence simulations on anisotropic grids

F. X. Trias,[1,a)] A. Gorobets,[1,2,b)] M. H. Silvis,[3,c)] R. W. C. P. Verstappen,[3,d)] and A. Oliva[1,e)]
[1]*Heat and Mass Transfer Technological Center, Technical University of Catalonia, C/Colom 11, 08222 Terrassa, Spain*
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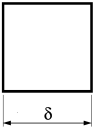

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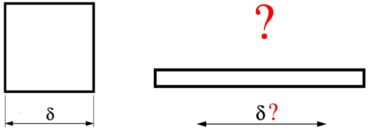

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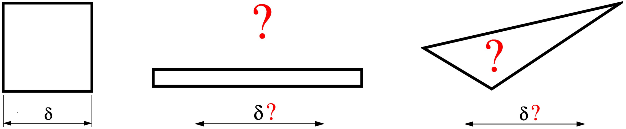

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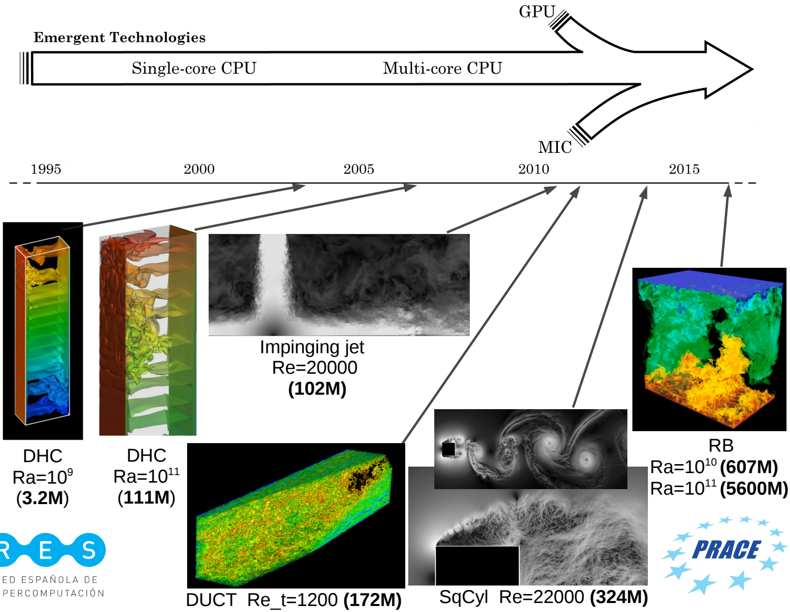
GPU
Emergent Technologies
Single-core CPU
Multi-core CPU
MIC
1995
2000
2005
2010
2015
Impinging jet
Re=20000
(102M)
DHC
Ra=10^9
(3.2M)
DHC
Ra=10^11
(111M)
RB
Ra=10^10 (607M)
Ra=10^11 (5600M)
DUCT Re_t=1200 (172M)
SqCyl Re=22000 (324M)
RES
RED ESPAÑOLA DE SUPERCOMPUTACIÓN
PRACE

Subgrid characteristic length for LES: state of the art

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nabla^2 \overline{u} - \nabla \overline{p} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_e S(\overline{u})$$

[1] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. **Physics of Fluids**, 27: 065103, 2015.

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$$\delta ?$$

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Subgrid characteristic length for LES: state of the art

- In the context of **LES**, most popular (by far) is:

$$\boxed{\delta_{\mathrm{vol}} = (\Delta x \Delta y \Delta z)^{1/3}} \Longleftarrow \textbf{Deardorff (1970)}$$

$$\delta_{\mathrm{Sco}} = f(a_1, a_2)\delta_{\mathrm{vol}}, \qquad \delta_{L^2} = \sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)/3}$$

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- In the context of **DES**:

$$\boxed{\delta_{\max} = \max(\Delta x, \Delta y, \Delta z)} \Longleftarrow \textbf{Sparlart et al. (1997)}$$

Recent flow-dependant definitions

$$\delta_{\omega} = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\omega|^2} \Longleftarrow \textbf{Chauvet et al. (2007)}$$

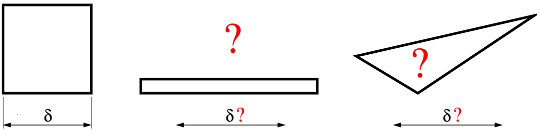
$$\tilde{\delta}_{\omega} = \frac{1}{\sqrt{3}} \max_{n,m=1,\dots,8} |l_n - l_m| \Longleftarrow \textbf{Mockett et al. (2015)}$$

$$\delta_{\mathrm{SLA}} = \tilde{\delta}_{\omega} F_{\mathrm{KH}}(VTM) \Longleftarrow \textbf{Shur et al. (2015)}$$

Building a new subgrid characteristic length for LES

Research question:

- Can we find a **simple and robust** definition of δ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?



Building a new subgrid characteristic length for LES

Research question:

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Starting point:

$$\underbrace{G \equiv \nabla \overline{u}}_{\text{physical space}} \qquad \underbrace{G_\delta \equiv G\Delta}_{\text{computational space}}$$

$$\text{where for a Cartesian grid } \Delta \equiv \begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}$$

Building a new subgrid characteristic length for LES

Idea: δ, appears in a natural way when we consider the leading term of the Taylor series expansion of the subgrid stress tensor,

$$\underbrace{\tau(\overline{u}) = \frac{\delta^2}{12} GG^T + \mathcal{O}(\delta^4)}_{\text{physical space}} \qquad \underbrace{\tau(\overline{u}) = \frac{1}{12} G_\delta G_\delta^T + \mathcal{O}(\delta^4)}_{\text{computational space}}$$

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A **least-square minimization** leads to[3]

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}}$$

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Properties of new definition

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- Well-bounded: $\Delta x \leqslant \delta_{\mathrm{lsq}} \leqslant \Delta z$ (assuming $\Delta x \leqslant \Delta y \leqslant \Delta z$)

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- Sensitive to flow orientation, *e.g.* for rotating flows ($G = \Omega$)

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{\omega_x^2(\Delta y^2 + \Delta z^2) + \omega_y^2(\Delta x^2 + \Delta z^2) + \omega_z^2(\Delta x^2 + \Delta y^2)}{2|\omega|^2}}$$

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- Applicable to unstructured grid

Idea: $\dfrac{\partial \phi}{\partial x} \approx \dfrac{\phi_{i+1} - \phi_i}{\Delta x} \Longrightarrow$ if you can compute G; then, you can compute δ_{lsq}!

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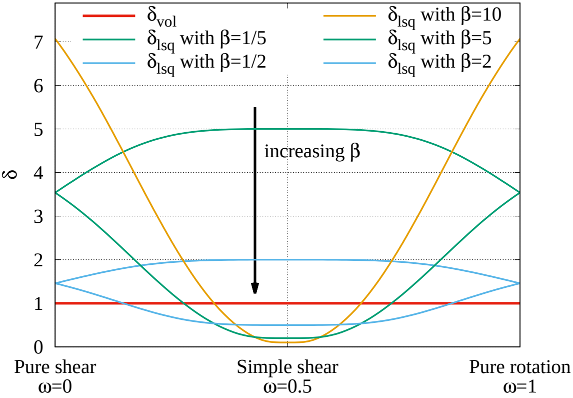
  Idea: $\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x} \implies$ if you can compute G; then, you can compute $\delta_{\rm lsq}$!

- Easy and cheap

Building a new subgrid characteristic length for LES

Properties of new definition

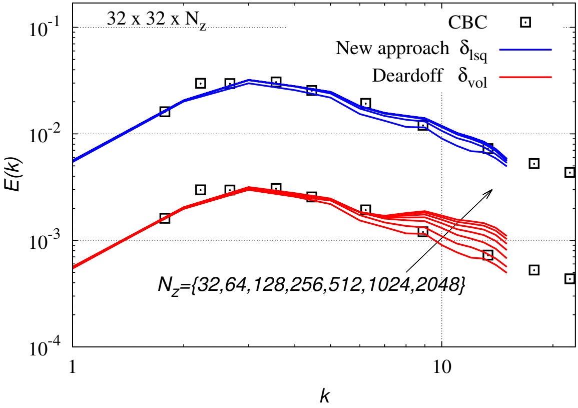
$$\Delta = \begin{pmatrix} \beta & 0 \\ 0 & \beta^{-1} \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 1 \\ 1-2\omega & 0 \end{pmatrix}$$



Results for LES

Decaying isotropic turbulence

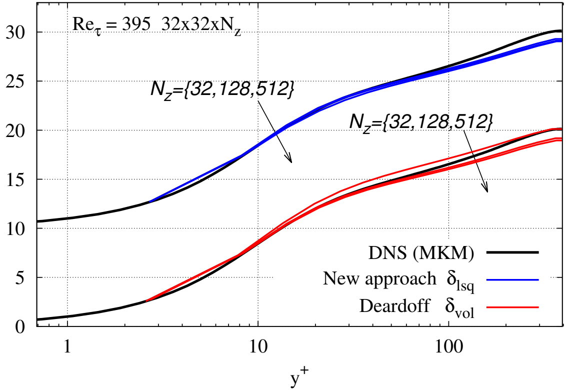
Comparison with classical Comte-Bellot & Corrsin (CBC) experiment



Results for LES

Turbulent channel flow

$Re_\tau = 395$ DNS Moser et al. LES $32 \times 32 \times N_z$



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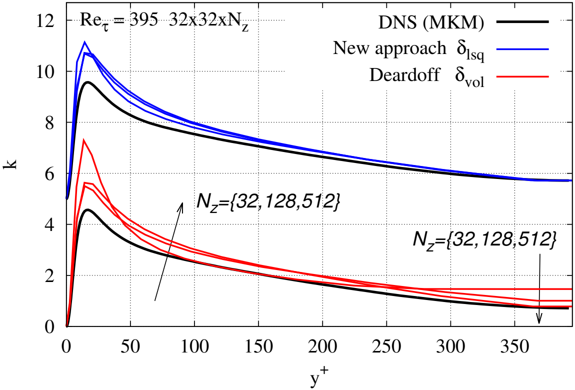
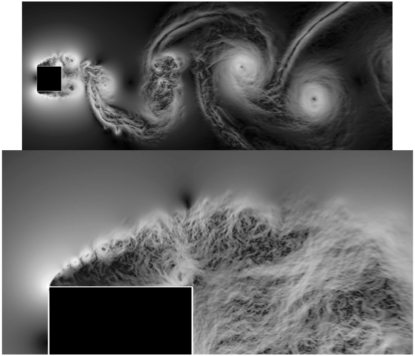


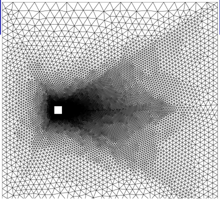
Results for LES

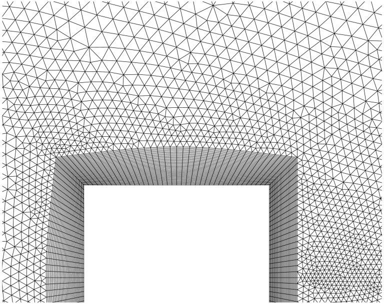
Turbulent flow around square cylinder at $Re = 22000$

DNS[4] 324M grid points

LES $19524 \times N_z$







[4]F.X.Trias, A.Gorobets, A.Oliva. *Turbulent flow around a square cylinder at Reynolds number 22000: a DNS study*, **Computers&Fluids**, 123:87-98, 2015.

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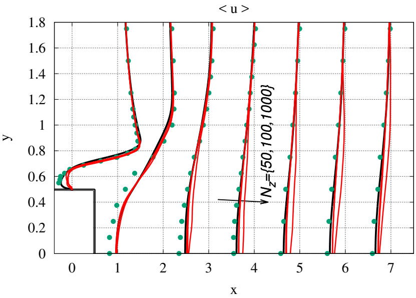
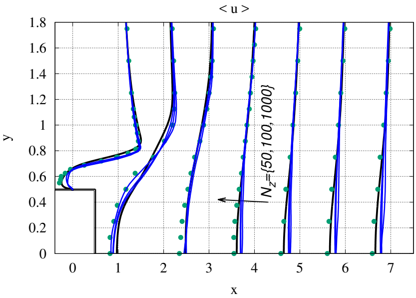
LES[5] $19524 \times N_z$



Deardorff δ_{vol}



New approach δ_{lsq}

[5] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. **Physics of Fluids**, 27: 065103, 2015.

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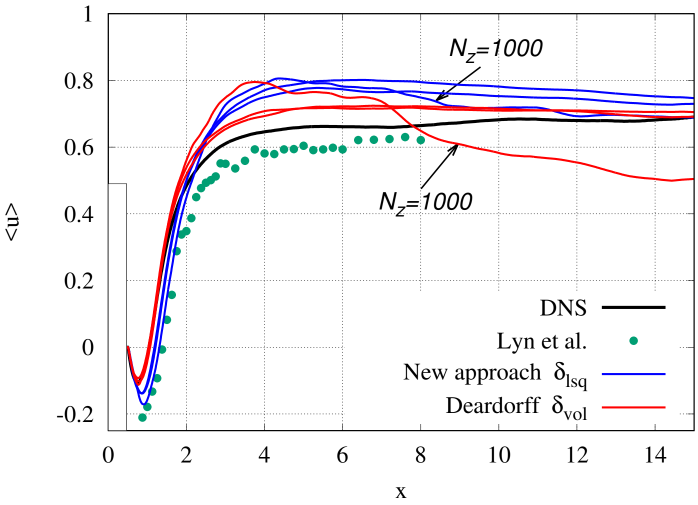

Concluding remarks

- A new definition for δ has been proposed

$$\delta_{\text{lsq}} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}}$$

- It is locally defined, well-bounded, cheap and easy to implement
- LES tests: HIT, turbulent channel flow ✓
- LES on unstructured grids: turbulent flow around square cylinder ✓

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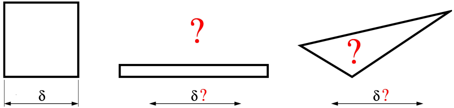

Takeaway message:

- Definition of δ can have a big effect on simulation results

Further reading...

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(Received 12 May 2017; accepted 6 November 2017; published online 29 November 2017)

Direct numerical simulations of the incompressible Navier-Stokes equations are not feasible yet for most practical turbulent flows. Therefore, dynamically less complex mathematical formulations are necessary for coarse-grained simulations. In this regard, eddy-viscosity models for Large-Eddy Simulation (LES) are probably the most popular example thereof. This type of models requires the calculation of a subgrid characteristic length which is usually associated with the local grid size. For isotropic grids, this is equal to the mesh step. However, for anisotropic or unstructured grids, such as the pancake-like meshes that are often used to resolve near-wall turbulence or shear layers, a consensus on defining the subgrid characteristic length has not been reached yet despite the fact that it can strongly affect the performance of LES models. In this context, a new definition of the subgrid characteristic length is presented in this work. This flow-dependent length scale is based on the turbulent, or subgrid stress, tensor and its representations on different grids. The simplicity and mathematical properties suggest that it can be a robust definition that minimizes the effects of mesh anisotropies on simulation results. The performance of the proposed subgrid characteristic length is successfully tested for decaying isotropic turbulence and a turbulent channel flow using artificially refined grids. Finally, a simple extension of the method for unstructured meshes is proposed and tested for a turbulent flow around a square cylinder. Comparisons with existing subgrid characteristic length scales show that the proposed definition is much more robust with respect to mesh anisotropies and has a great potential to be used in complex geometries where highly skewed (unstructured) meshes are present. *Published by AIP Publishing.* https://doi.org/10.1063/1.5012546

… and near future reading (hopefully)

New strategies for mitigating the Grey Area in DDES models

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This work presents a new approach for mitigating the unphysical delay in the RANS to LES transition, named *Grey Area*, which is a classical issue for hybrid RANS-LES turbulence models such as Delayed-Detached Eddy Simulation (DDES). An existing methodology designed for improving the LES performance in complex flow areas has been adapted and tested. In particular, LES and DDES suffer from an excessive diffusion in critical areas where the flow does not strictly behave in a fully turbulent manner, such as free shear layers. In these situations, dissipation needs to be reduced in order to enable more physically accurate development of turbulence, and thus, the overall meanflow field. In this context, the following paper assesses a recent 2D sensitive turbulent model and a new subgrid length scales, initially developed for LES applications, as a new solution for mitigating the *Grey Area*. The new approach has been assessed for two classic cases and compared with standard methodologies in both incompressible and compressible flow. Moreover, two different codes have been used, *OpenFOAM* **and** *NOISEtte*, **for cross-validation purposes. Encouraging results have been obtained with the new approach, supporting its suitability as a good candidate for addressing the *Grey Area* numerical issue.**

Thank you for your attention