New subgrid-scale models for large-eddy simulation of Rayleigh-Bénard convection

F. Dabbagh[1] F. X. Trias[1] A. Gorobets[1,2] and A. Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia, ETSEIAT, Terrassa, Spain
[2]Keldysh Institute of Applied Mathematics, 4A, Miusskaya Sq., Moscow 125047, Russia

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Centre Tecnològic de Transferència de Calor
Laboratori de Termotècnia i Energètica
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Outline

Rayleigh-Bénard convection (RBC)

- Definition: a convective cell heated from below and cooled from above.
- Flow dynamics are characterized by:

Rayleigh number:

$$Ra = \frac{g\alpha H^3 \Delta T}{\nu\kappa}$$

Prandtl number:

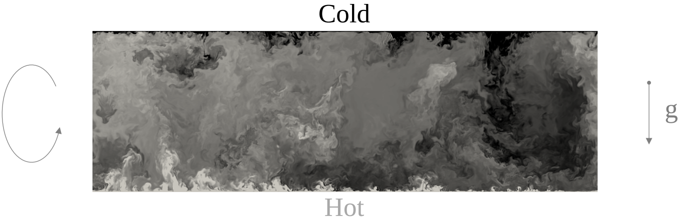
$$Pr = \frac{\nu}{\kappa}$$

cell aspect ratio Γ.

- system respond

$$Nu = \sqrt{RaPr}\langle wT\rangle - \frac{\partial\langle T\rangle}{\partial z}$$

- Clashes of Nu scaling in the ultimate regime $Ra > 10^{14}$.



Details of direct numerical simulation

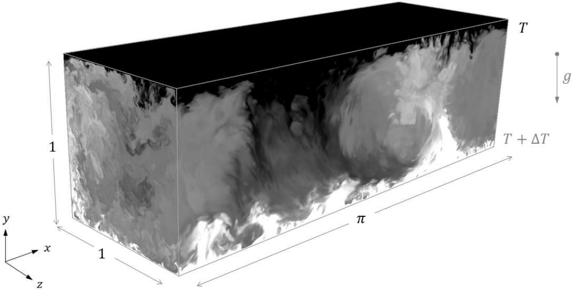
Dimensionless governing equations

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + (Pr/Ra)^{1/2}\nabla^2 \boldsymbol{u} + \boldsymbol{f}$$
$$\partial_t T + (\boldsymbol{u} \cdot \nabla) T = (RaPr)^{-1/2}\nabla^2 T$$

Parameters

$Pr = 0.7$ (air flow), adiabatic sidewalls in z direction and periodic boundaries in x-direction.

- Refinements on Grötzbach criterion [Grötzbach, 1983] and coarsing approach in x-dir.
- 4^{th}-order symmetry-preserving spatial discretizations [Verstappen & Veldman, J. Comp. Phys. 2003].



Case	Ra	$N_x \times N_y \times N_z$	N_{BL}	$t_{avg}[TU]$	Nu
DNS1	10^8	$400 \times 208 \times 208$	9	500	30.9
DNS2	10^{10}	$1024 \times 768 \times 768$	12	200	128.1

LES

- DNS is not feasible at very high *Ra*.

LES filtered equations

$$\partial_t \overline{\boldsymbol{u}} + (\overline{\boldsymbol{u}} \cdot \nabla)\overline{\boldsymbol{u}} = -\nabla \overline{p} + (Pr/Ra)^{1/2} \nabla^2 \overline{\boldsymbol{u}} + \overline{\boldsymbol{f}} - \nabla \cdot \tau(\overline{\boldsymbol{u}})$$
$$\partial_t \overline{T} + (\overline{\boldsymbol{u}} \cdot \nabla)\overline{T} = (RaPr)^{-1/2} \nabla^2 \overline{T} - \nabla \cdot q(\overline{\boldsymbol{u}}, \overline{T})$$

Where $\overline{(\cdot)}$ are the filtered resolved fields at filter length Δ.
$\tau(\overline{\boldsymbol{u}})$ is the SGS stress tensor and $q(\overline{\boldsymbol{u}}, \overline{T})$ the SGS heat flux vector.

Following the eddy-viscosity models and the gradient-diffusion hypothesis

- $\tau(\overline{\boldsymbol{u}}) \approx -2\nu_t \mathsf{S}(\overline{\boldsymbol{u}})$
- $q(\overline{\boldsymbol{u}}, \overline{T}) \approx -\kappa_t \nabla \overline{T}$

- $\mathsf{S}(\overline{\boldsymbol{u}}) = 1/2(\nabla \overline{\boldsymbol{u}} + \nabla \overline{\boldsymbol{u}}^t)$ is the rate-of-strain tensor.
- ν_t is the eddy viscosity.
- κ_t is the turbulent diffusivity.

Eddy-viscosity models and turbulent Prandtl number

ν_t is calculated following S3PQR models recently proposed by [Trias, PoF, 2015]:

- $\nu_t^{S3PQ} = (C_{s3pq}\Delta)^2 P_{\mathsf{GG}^t}^{-5/2} Q_{\mathsf{GG}^t}^{3/2}$
- $\nu_t^{S3PR} = (C_{s3pr}\Delta)^2 P_{\mathsf{GG}^t}^{-1} R_{\mathsf{GG}^t}^{1/2}$
- $\nu_t^{S3QR} = (C_{s3qr}\Delta)^2 Q_{\mathsf{GG}^t}^{-1} R_{\mathsf{GG}^t}^{5/6}$

- P_{GG^t}, Q_{GG^t} and R_{GG^t} are the first, second and third invariants of GG^t, where $\mathsf{G} \equiv \nabla \overline{\boldsymbol{u}}$.
- C is the model constant.

κ_t is derived from ν_t by a constant turbulent Prandtl number $Pr_t = \nu_t / \kappa_t$

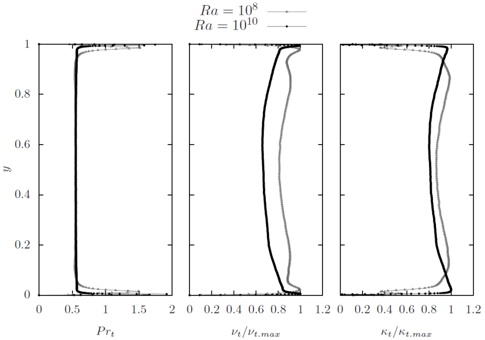
a priori Pr_t investigation

Using Taylor series expansion and the least square minimization technique:

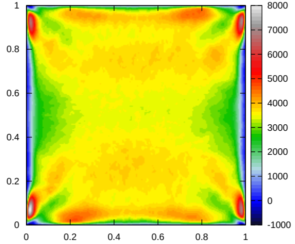
$$\nu_t \approx -\left\langle \frac{\mathsf{AA}^t : \mathsf{S}(\boldsymbol{u})}{2\mathsf{S}(\boldsymbol{u}) : \mathsf{S}(\boldsymbol{u})} \right\rangle \quad , \quad \mathsf{A} \equiv \nabla \boldsymbol{u}$$

$$\kappa_t \approx -\left\langle \frac{\mathsf{A}\nabla T \cdot \nabla T}{\nabla T \cdot \nabla T} \right\rangle$$

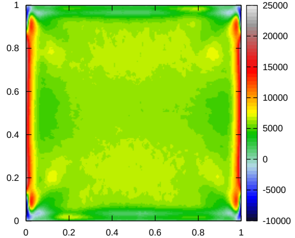
Turbulent Prandtl number



- Constant value of $Pr_t = 0.55$ in the bulk, independently of Ra.
- The turbulent wind is driven by the mean buoyant forces and not by the Reynolds stresses and ν_t is positive.



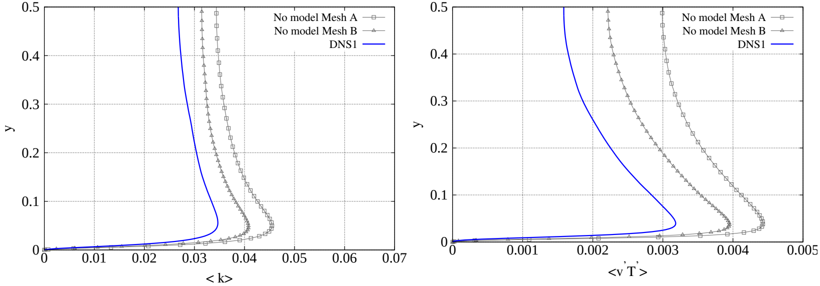
(a) Averaged t and x-dir ν_t



(b) Averaged t and x-dir κ_t

LES results at $Ra = 10^8$

	DNS1	Mesh A	Mesh B
Mesh	$400 \times 208 \times 208$	$120 \times 80 \times 80$	$168 \times 110 \times 110$



- Turbulent kinetic statistics are good predicted in near-walls and bulk regions. However, an overestimation of heat flux within the boundary layer.
- One improvement option is the dynamical calculation of Pr_t.

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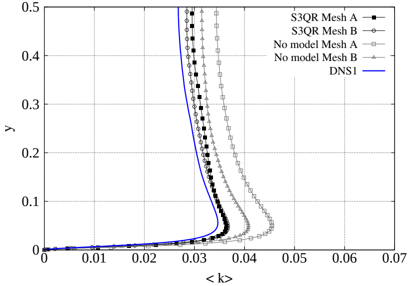

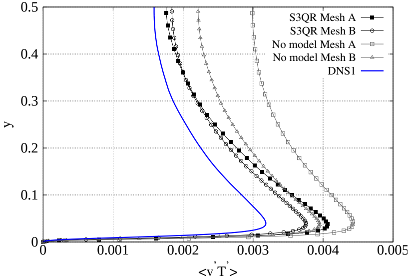

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Use of velocity-times-temperature gradient tensor

Direct following to the evolution of thermal plumes and buoyant production using invariants of tensor $\mathsf{A}_\theta = \nabla(\boldsymbol{u}T)$†:

- The square magnitude $|\mathsf{A}_\theta|^2 = \mathsf{A}_\theta : \mathsf{A}_\theta = tr(\mathsf{A}_\theta {\mathsf{A}_\theta}^t)$

Hence

$$\mathsf{G}_\theta \mathsf{G}_\theta^t = \overline{T}^2 \mathsf{G}\mathsf{G}^t + 2\overline{T}\overline{\boldsymbol{u}} \otimes \mathsf{G}\nabla\overline{T} + |\overline{\boldsymbol{u}}|^2 \nabla\overline{T} \otimes \nabla\overline{T}^t$$

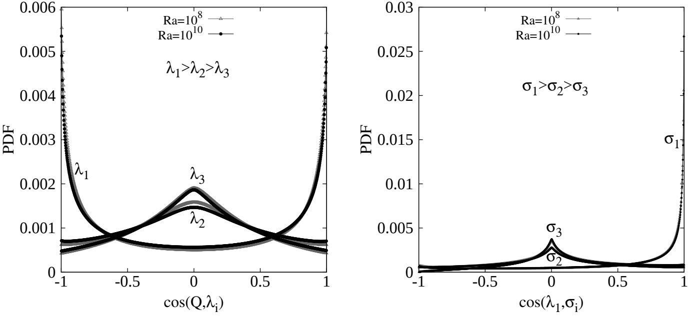
with $\mathsf{G}_\theta = \nabla(\overline{\boldsymbol{u}}\overline{T})$, can be used in modeling the SGS heat flux.

- In the gradient model framework, it includes terms of Reynolds stress $\mathsf{G}\mathsf{G}^t$, turbulent heat flux $\mathsf{G}\nabla\overline{T}$ and temperature variance $\nabla\overline{T} \otimes \nabla\overline{T}^t$.

† Dabbagh F., Trias F.X., Gorobets A. and Oliva A. *"On the evolution of flow topology in turbulent Rayleigh-Bénard convection"*. Physics of Fluids (submitted)

Use of velocity-times-temperature gradient tensor

- Symmetric second-order tensor with positive eigenvalues and invariants.
- Has interactions of the small and large scales effects (Buoyant and kinetic productions interchange)



Invariants of $\mathsf{G}_\theta \mathsf{G}_\theta^t$ from instantaneous DNS fields at $Ra = 10^8$

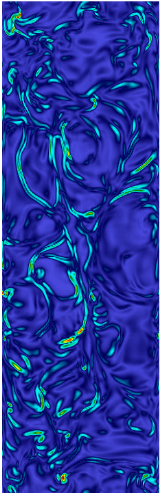


Figure: $\mathsf{G}\nabla T$

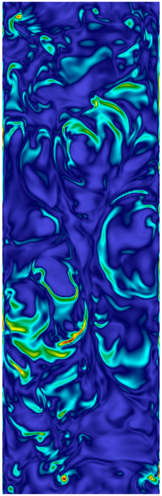


Figure: $P_{\mathsf{G}_\theta \mathsf{G}_\theta^t}$

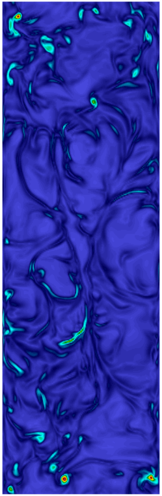


Figure: $Q_{\mathsf{G}_\theta \mathsf{G}_\theta^t}$

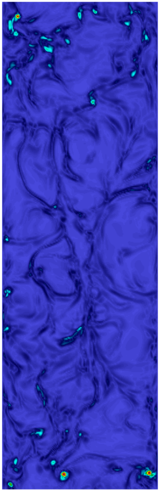


Figure: $R_{\mathsf{G}_\theta \mathsf{G}_\theta^t}$

conclusions

- DNS have been performed for turbulent RBC at $Ra = 10^8$ and 10^{10}.
- Turbulent Prandtl number has shown a constant bulk-dominated value $Pr_t = 0.55$ and the turbulent wind is driven by the mean buoyant forces at the sidewalls.
- The S3QR model proposed by [Trias, PoF, 2015], in LES frame modeling, has been applied in RBC.
- Good prediction of turbulent kinetics but overestimation of turbulent heat flux is overestimated near the wall.
- Encouraging use of $\mathsf{G}_\theta \mathsf{G}_\theta^t$ tensor in modeling the SGS heat flux.

Thanks for your attention!