On a proper definition of the subgrid characteristic length for LES

F. X. Trias[a], A. Gorobets[a,b], A. Oliva[a]

[a]*Heat and Mass Transfer Technological Center, Technical University of Catalonia C/Colom 11, 08222 Terrassa, Spain*
[b]*Keldysh Institute of Applied Mathematics RAS, 4A, Miusskaya Sq., Moscow, 125047, Russia*

Abstract

Direct simulations of the incompressible Navier-Stokes equations are limited to relatively low-Reynolds numbers. Hence, dynamically less complex mathematical formulations are necessary for coarse-grain simulations. In this regard, eddy-viscosity models for Large-Eddy Simulation (LES) are probably the most popular example thereof. The present work focuses on the calculation of the subgrid characteristic length, a key element for any eddy-viscosity model. Namely, a new approach based on the Taylor-series expansion of the subgrid stress tensor in the computational space is proposed. Its simplicity and mathematical properties suggest that it can be a robust definition of the subgrid characteristic length that minimizes the effect of mesh anisotropies on the performance of LES models. The performance of the proposed models is successfully tested for decaying isotropic turbulence and a turbulent channel flow.

Keywords: subgrid characteristic length, eddy-viscosity, LES, turbulence

In this work, a novel definition of the subgrid characteristic length, δ, is proposed with the aim to answer the following research question: *can we find a simple and robust definition of δ that minimizes the effect of mesh anisotropies on the performance of subgrid scale (SGS) models?* In this regard, we consider the definition given by

$$\delta_{\text{lsq}} = \sqrt{\frac{\mathsf{G}_\delta \mathsf{G}_\delta^T : \mathsf{G}\mathsf{G}^T}{\mathsf{G}\mathsf{G}^T : \mathsf{G}\mathsf{G}^T}}, \tag{1}$$

as a very good candidate. Unlike the most common definitions in the context of LES that solely depend on geometrical properties of the mesh, *i.e.* $\Delta \equiv \text{diag}(\Delta x, \Delta y, \Delta z)$, it is also dependent on the local flow topology, $\mathsf{G} \equiv \nabla \overline{\boldsymbol{u}}$. The second-order tensor $\mathsf{G}_\delta \equiv \mathsf{G}\Delta$ can be viewed as a gradient in the so-called computational space. Actually, the definition of δ_{lsq} is obtained by minimizing (in a least-squares sense) the difference between these two tensors

$$\tau(\overline{\boldsymbol{u}}) = \frac{\delta^2}{12}\mathsf{G}\mathsf{G}^T + \mathcal{O}(\delta^4) \qquad \tau(\overline{\boldsymbol{u}}) = \frac{1}{12}\mathsf{G}_\delta \mathsf{G}_\delta^T + \mathcal{O}(\delta^4). \tag{2}$$

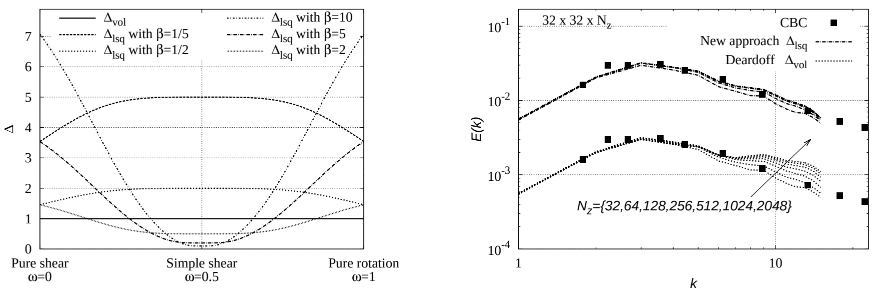

Figure 1: Comparison between the new definition δ_{lsq} proposed in Eq.(1) and the classical definition proposed by Deardorff [2], *i.e.* $\delta_{vol} \equiv (\Delta x \Delta y \Delta z)^{1/3}$. Left: 2D simple flow given by in Eq.(3). Right: energy spectra for decaying isotropic turbulence corresponding to the experiment of Comte-Bellot and Corrsin [3]. For clarity, latter results are shifted one decade.

They are the leading terms of the Taylor series expansion of the SGS tensor, $\tau(\overline{\boldsymbol{u}})$, for an isotropic and an anisotropic filter length, respectively. It fulfills a set of properties: locality, boundedness, low cost and sensitivity to flow orientation. To get a better understanding of δ_{lsq}, results for the following 2D case

$$\mathsf{\Delta} = \begin{pmatrix} \beta & 0 \\ 0 & \beta^{-1} \end{pmatrix} \qquad \mathsf{G} = \begin{pmatrix} 0 & 1 \\ 1-2\omega & 0 \end{pmatrix}, \tag{3}$$

are displayed in Figure 1 (left). Interestingly, for $\omega = 1/2$ (simple shear flow) $\delta_{lsq} = \beta^{-1}$. This situations mimics the typical quasi-2D grid-aligned flow in the initial region of a shear layer. As it could be expected, δ_{lsq} is equal to the grid size in the direction orthogonal to the shear layer. Finally, to show the adequacy of δ_{lsq} for highly anisotropic grids, LES have been computed using the Smagorinsky model for a set of (artificially) stretched meshes (see Figure 1, right). Notice that for increasing values of N_z, results with $\delta_{vol} = (\Delta x \Delta y \Delta z)^{1/3}$ tend to diverge whereas results with δ_{lsq} rapidly converge. Testing the performance of δ_{lsq} for wall-bounded flows using advanced LES [1] and DES models, also for unstructured meshes, is part of our research plans.

Acknowledgments: This work has been financially supported by the *Ministerio de Economía y Competitividad*, Spain (ENE2014-60577-R) and a *Ramón y Cajal* postdoctoral contract (RYC-2012-11996).

References

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